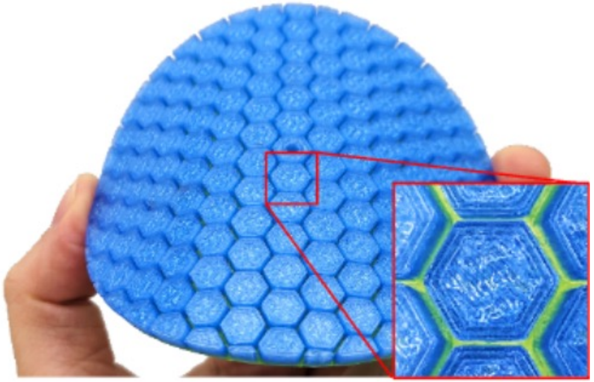
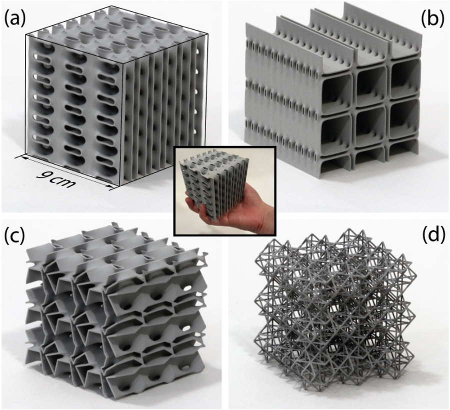
Computational Modeling and Design of Nonlinear Materials and Structures

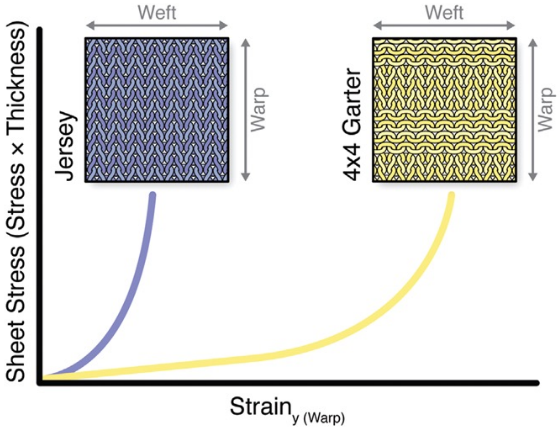
Pengbin Tang

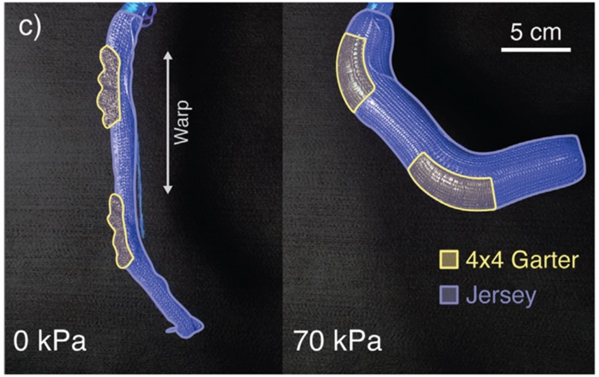
ETH zürich

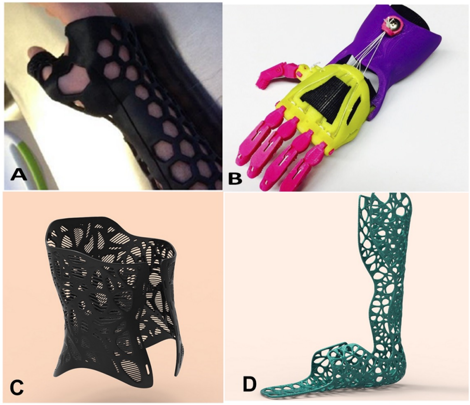
Design of Materials



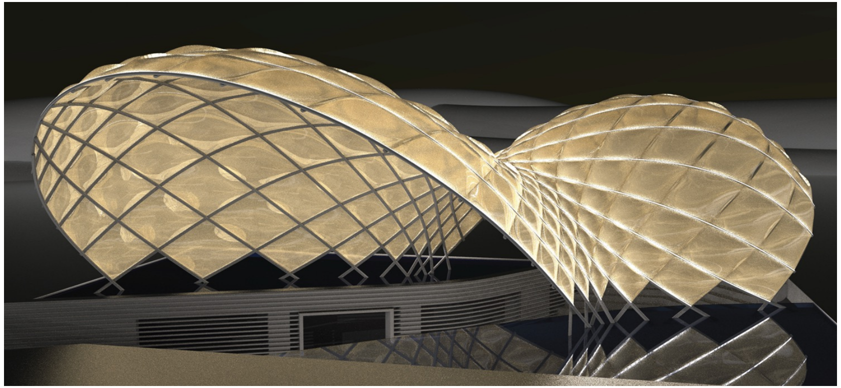


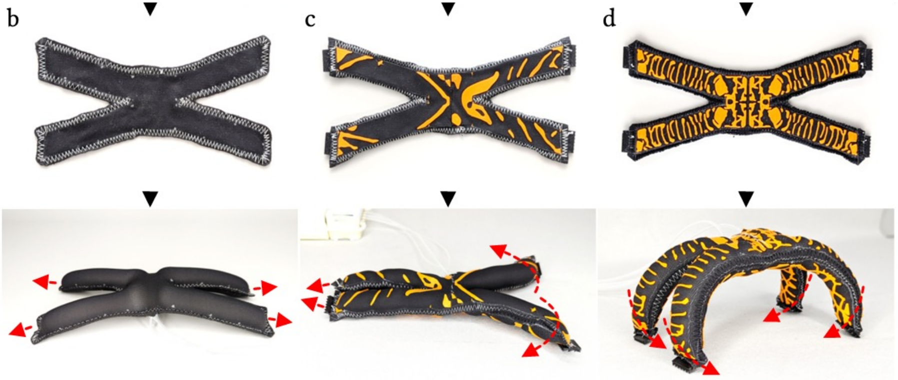






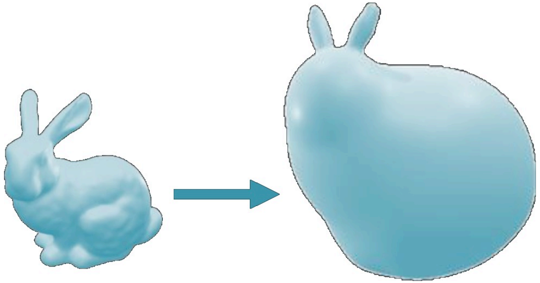
Design of Structures



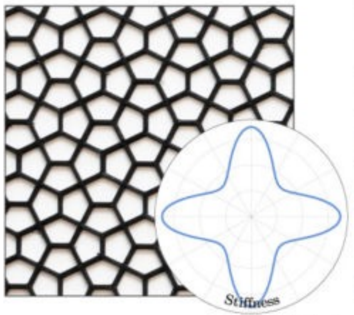


Challenges

- Hard to predict and characterize.



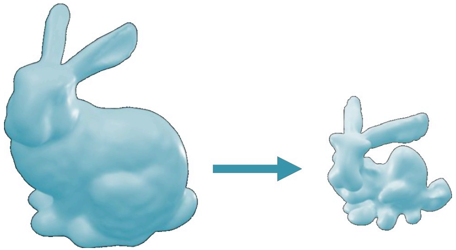
Large deformations



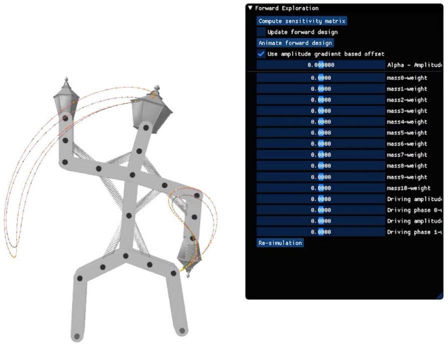
Strong anisotropy

Contacts

- Inverse design



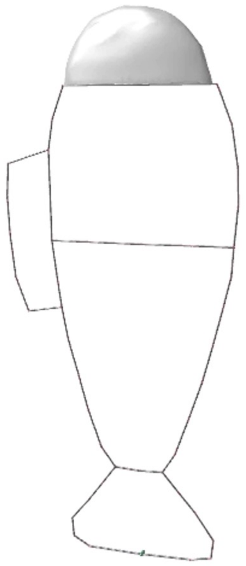
Nonlinear effect



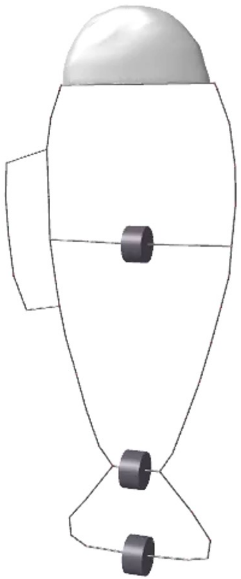
high-dimensional design space

Designing for Large-Amplitude Oscillations

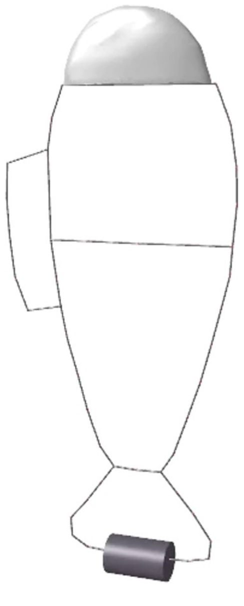
Design 1
(0g, 0g, 0g)



Design 2
(40g, 40g, 40g)

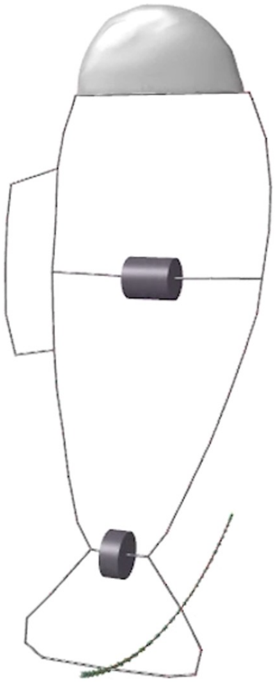


Design3
(0g, 0g, 120g)

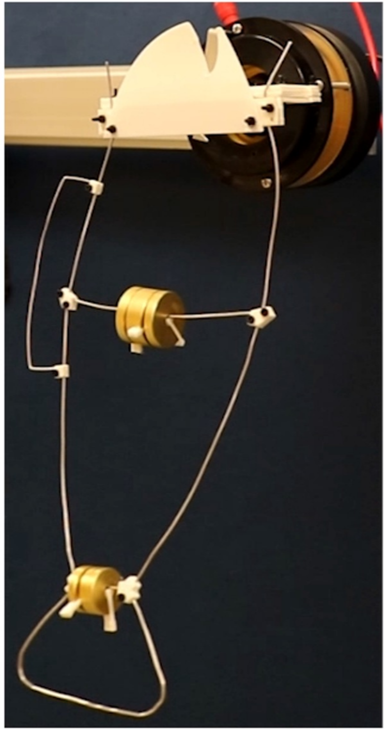


Designing for Large-Amplitude Oscillations

Optimized Design
(80g, 31.4g, 0g)

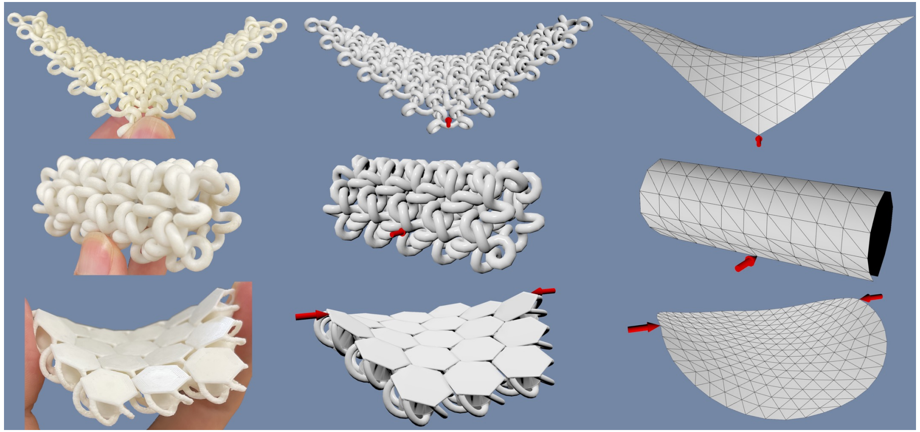


Physical Prototype

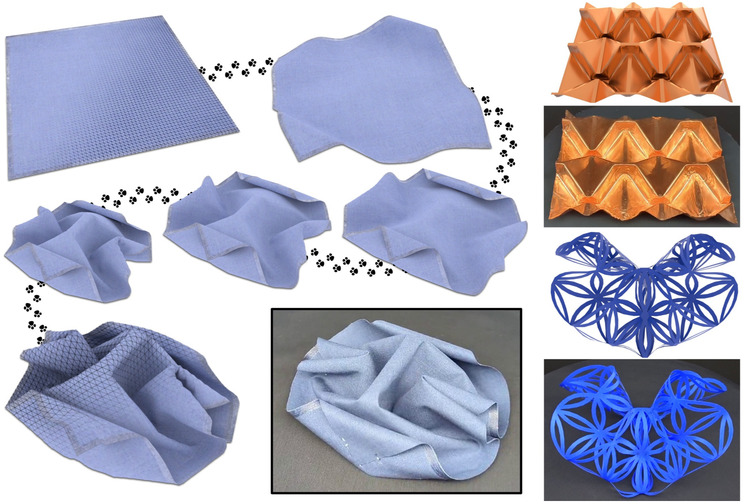


Overview

Beyond Chainmail: Computational Modeling of Discrete Interlocking Materials

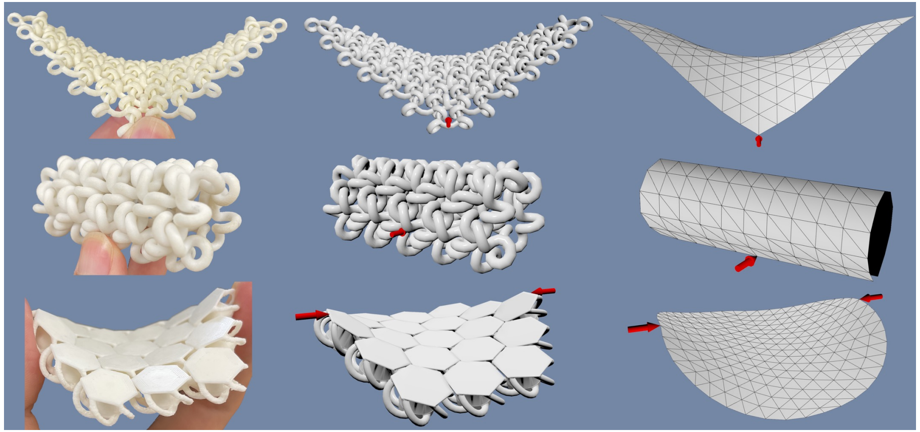


Modal Folding: Discovering Smooth Folding Patterns for Sheet Materials using Strain-Space Modes



Overview

Beyond Chainmail: Computational Modeling of Discrete Interlocking Materials



Modal Folding: Discovering Smooth Folding Patterns for Sheet Materials using Strain-Space Modes

SIGGRAPH 2023
LOS ANGELES+ 6-10 AUG

Beyond Chainmail: Computational Modeling of Discrete Interlocking Materials

Pengbin Tang[1,2], Stelian Coros[2], Bernhard Thomaszewski[2]

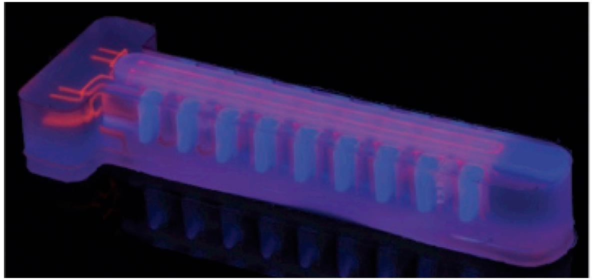
[1] Université de Montréal

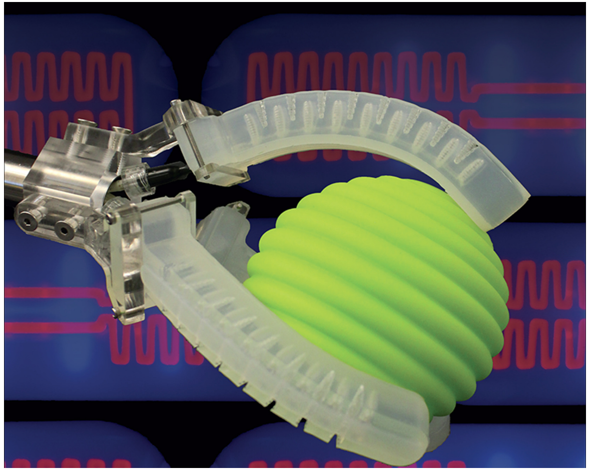
[2] ETH zürich

Application of Materials – Soft Robotics

[Pascali et al. 2022]

[Jeong et al. 2018]

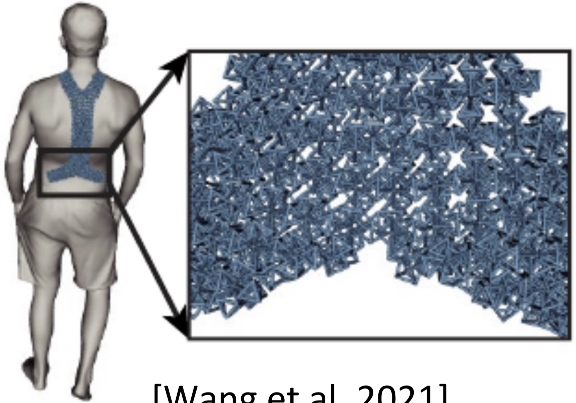




[Truby et al. 2018]

Application of Materials – Wearables

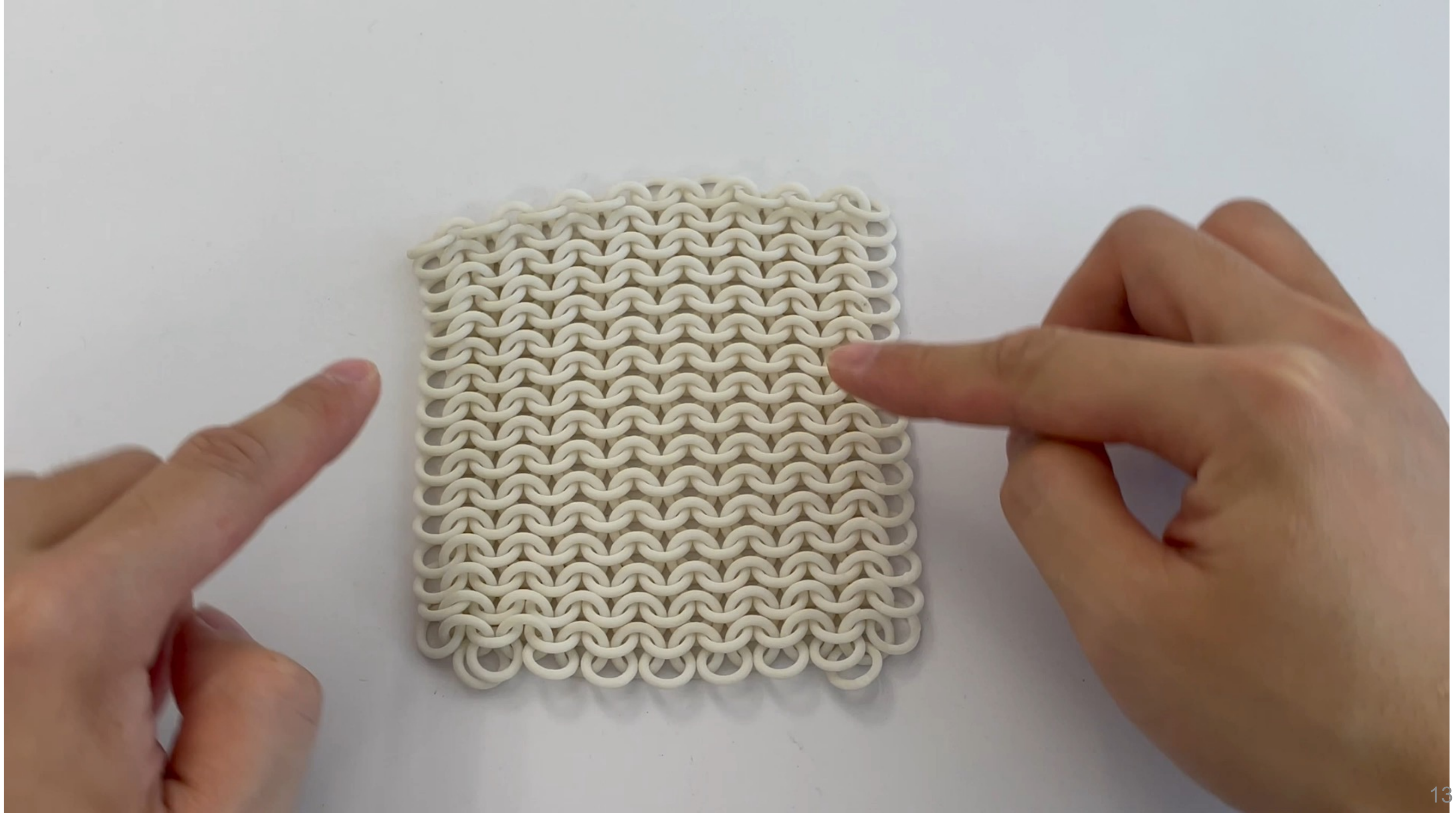
[Fitzpatrick et al. 2017]



[Wang et al. 2021]

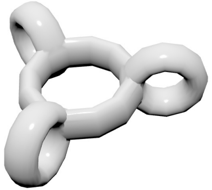
[Luo et al. 2022]

Discrete Interlocking Materials (DIM)

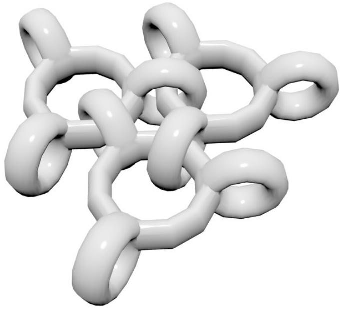


Discrete Interlocking Materials (DIM)

Element Shape + Connectivity

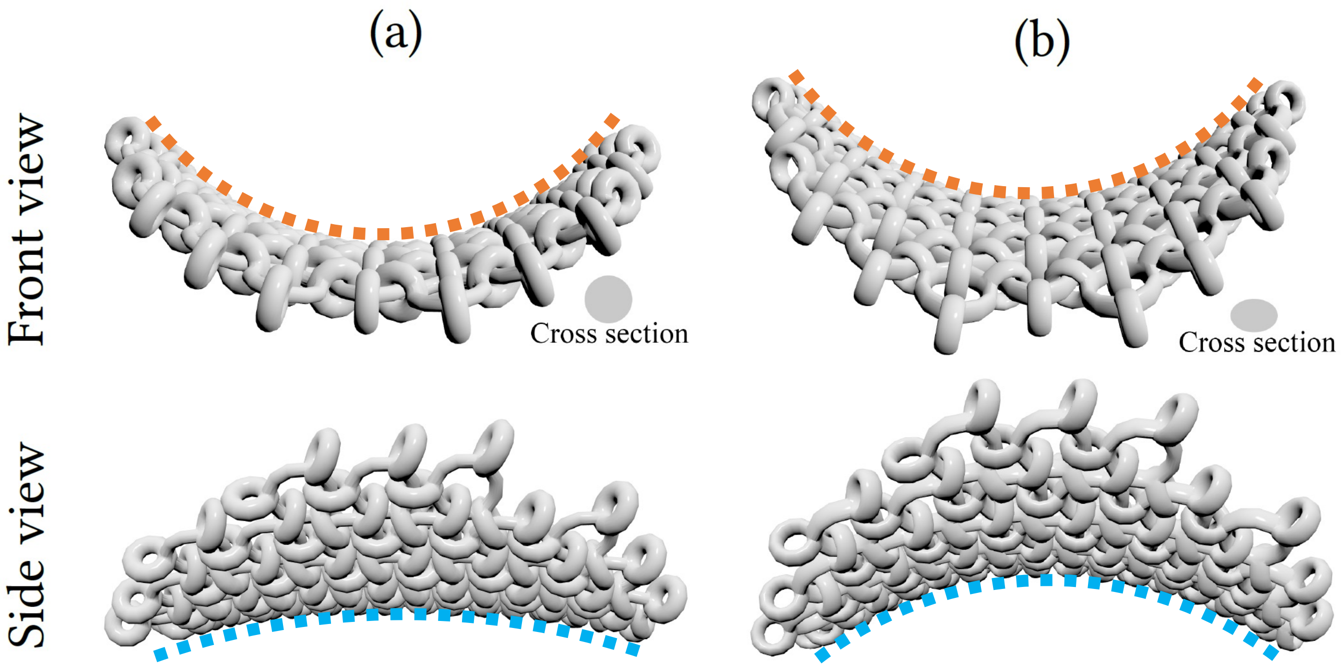


?



Macromechanical Properties

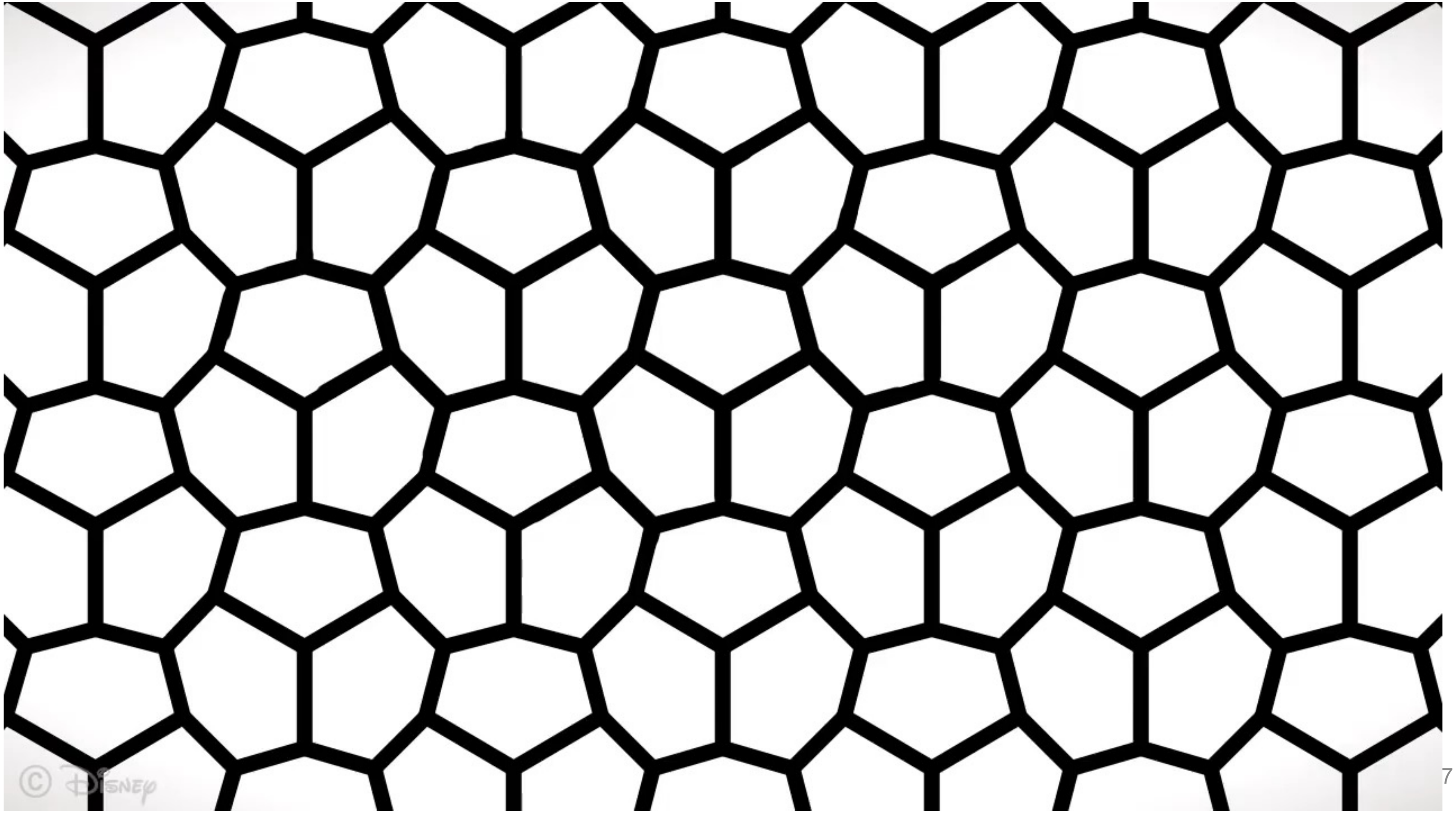
Discrete Interlocking Materials (DIM)



Goal

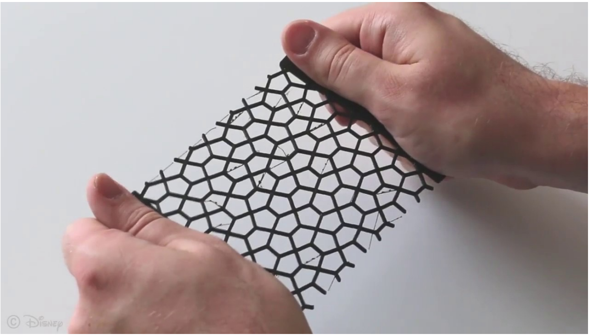
A new computational framework for modeling and characterizing DIM composed of quasi-rigid elements.

Homogenization

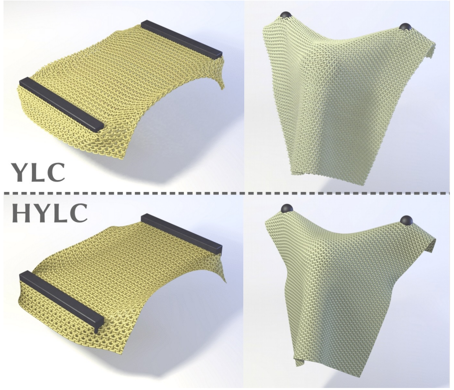


Homogenization

[Schumacher et al. 2015]

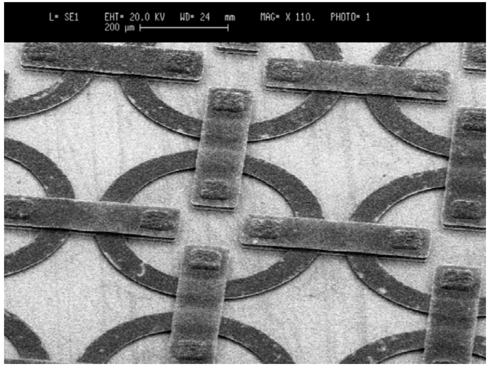


[Schumacher et al. 2018]

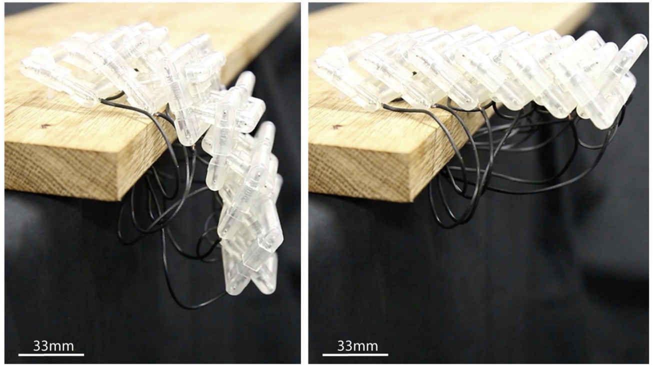


[Sperl et al. 2020]

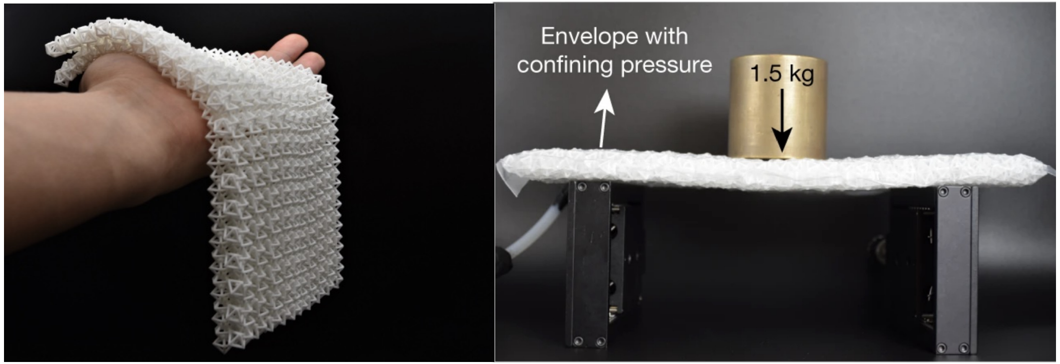
Interlocking Materials



[Engel and Liu 2007]

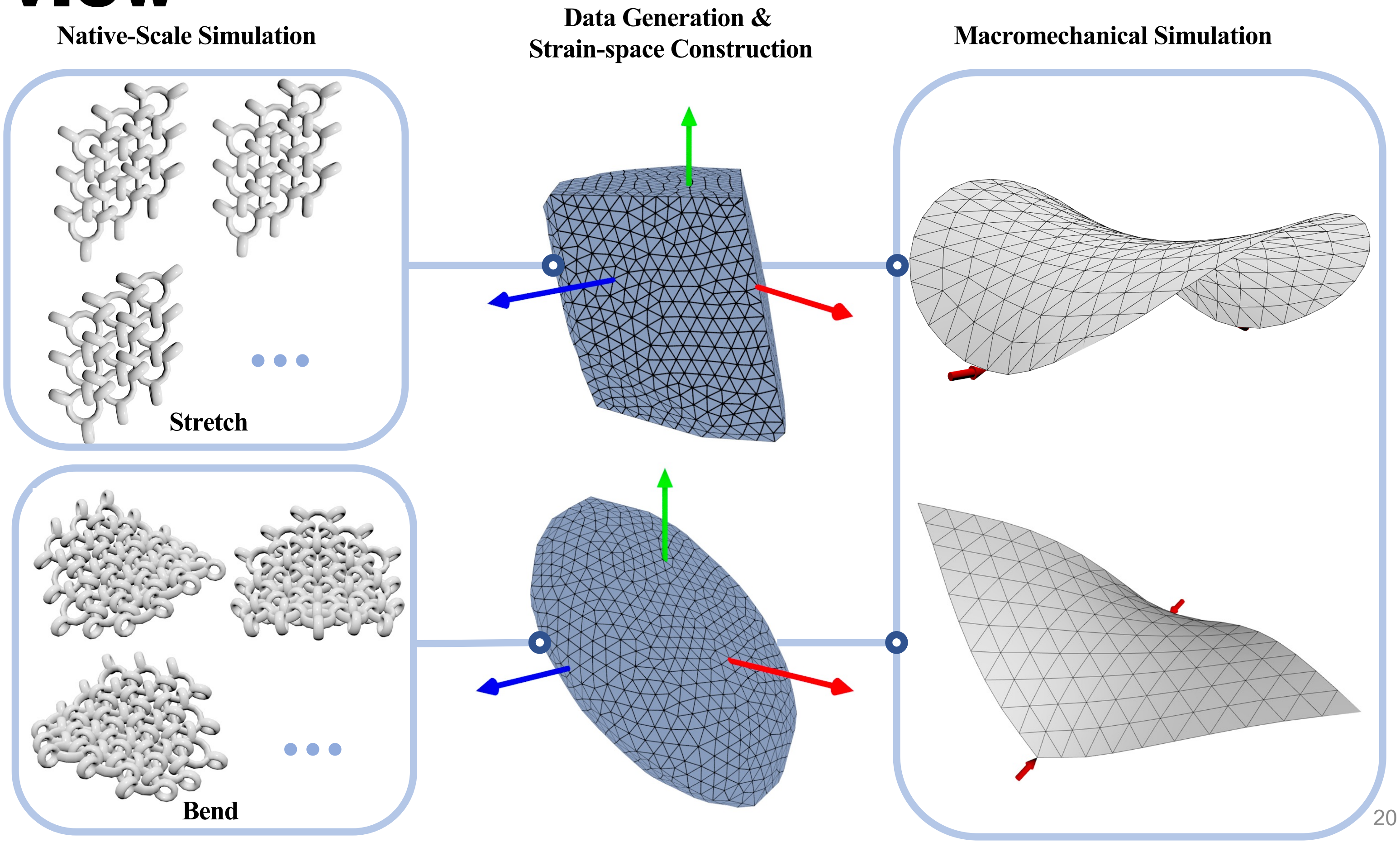


[Ransley et al. 2017]



[Wang et al. 2021]

Overview



Native-Scale Simulation
Data Generation &
Strain-space Construction
Macromechanical Simulation
Stretch
Bend

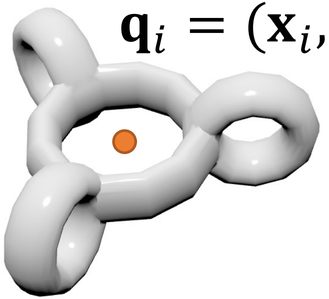
Theory

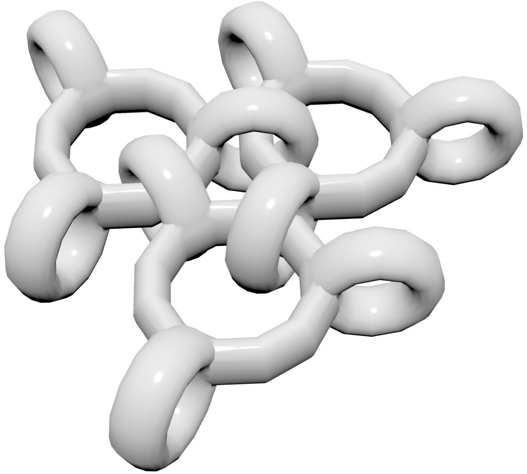
Native-Scale Model

- We simulate static equilibrium states of DIM as rigid bodies with contact by an unconstrained minimization problem [Ferguson et al. 2021] as

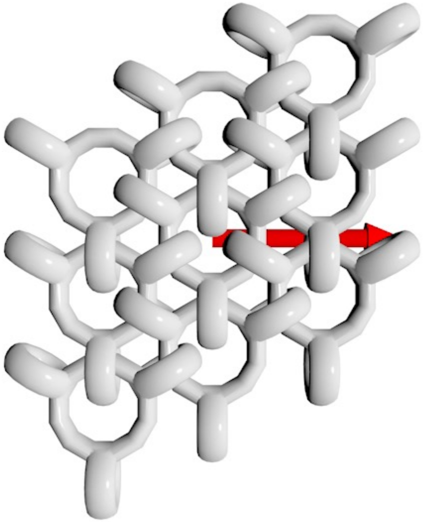
$$\min_{\mathbf{q}} E_{\mathrm{Ext}}(\mathbf{q}) + E_{\mathrm{Coll}}(\mathbf{q})$$

$$\mathbf{q}_i = (\mathbf{x}_i, \boldsymbol{\omega}_i)$$

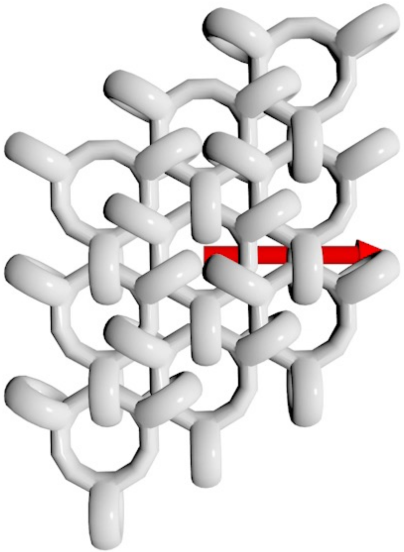




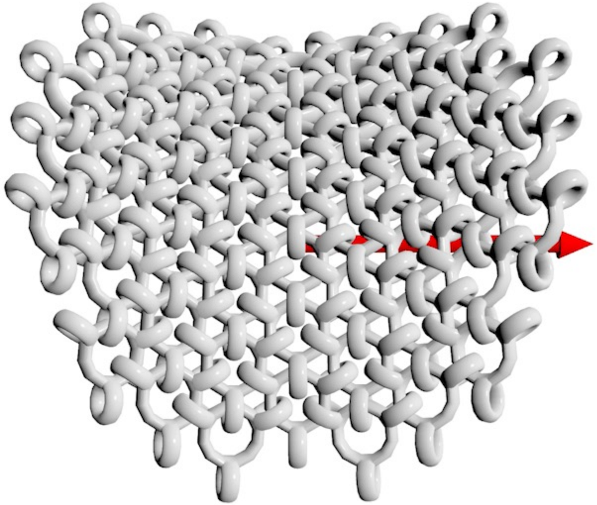
Native-Scale Simulations



Stretching

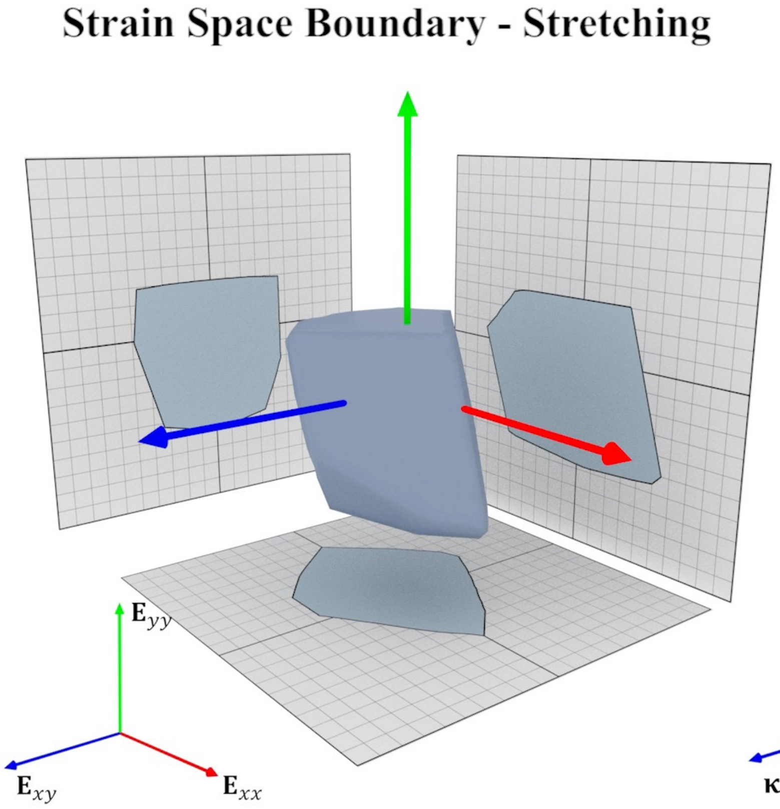


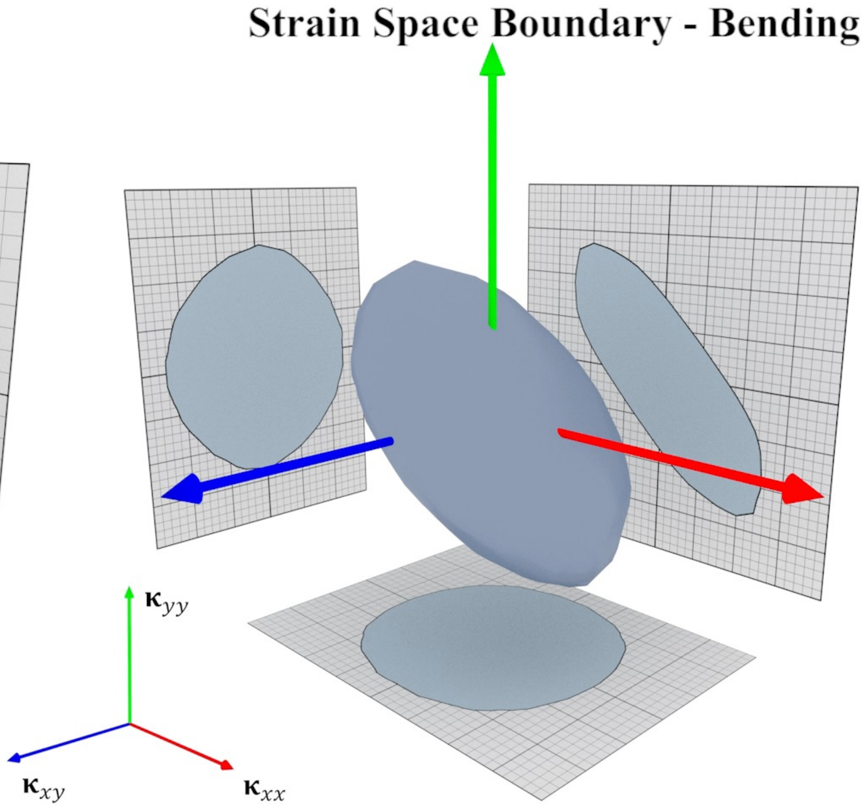
Compression



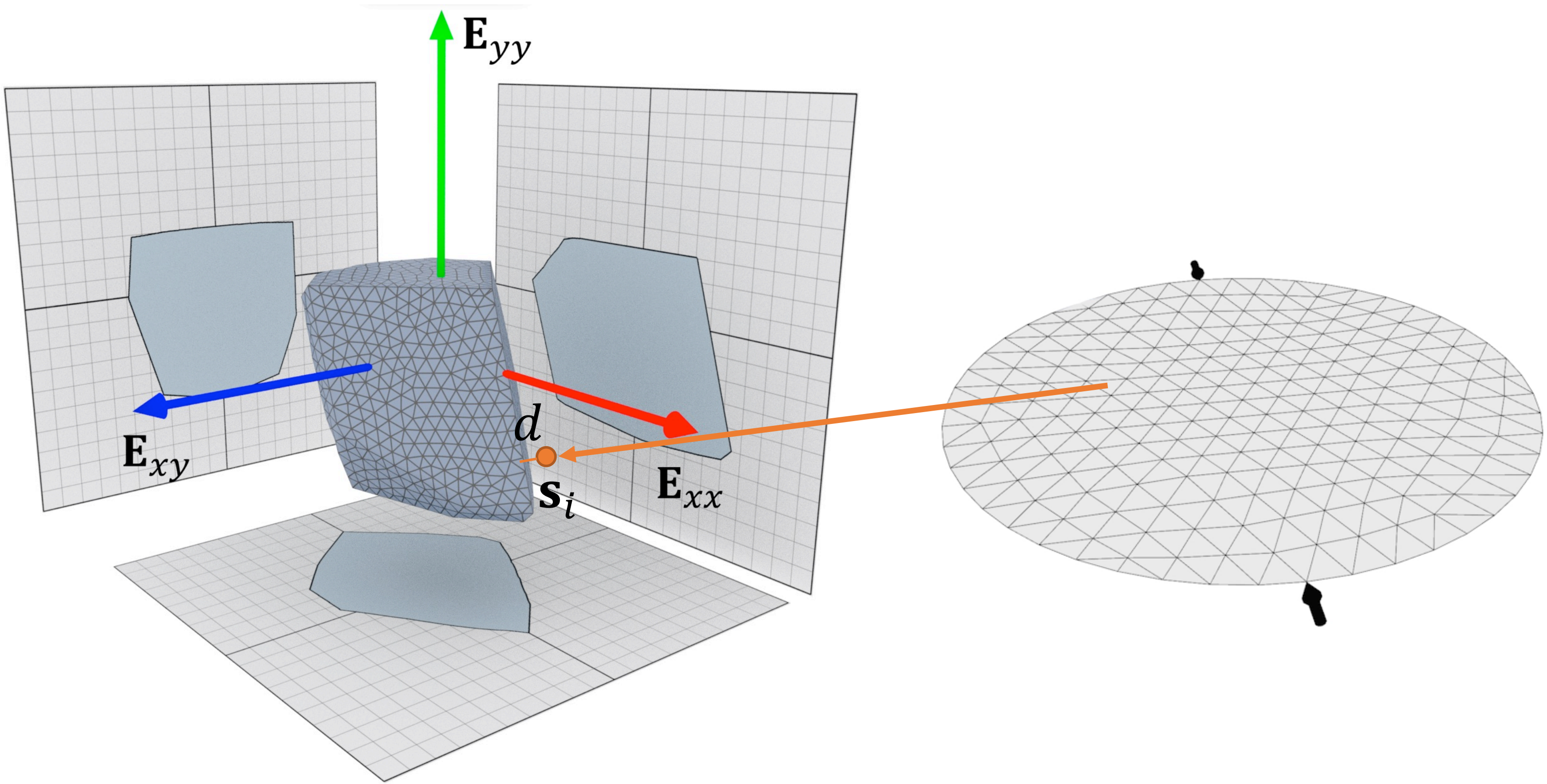
Bending

Strain-Space Representation

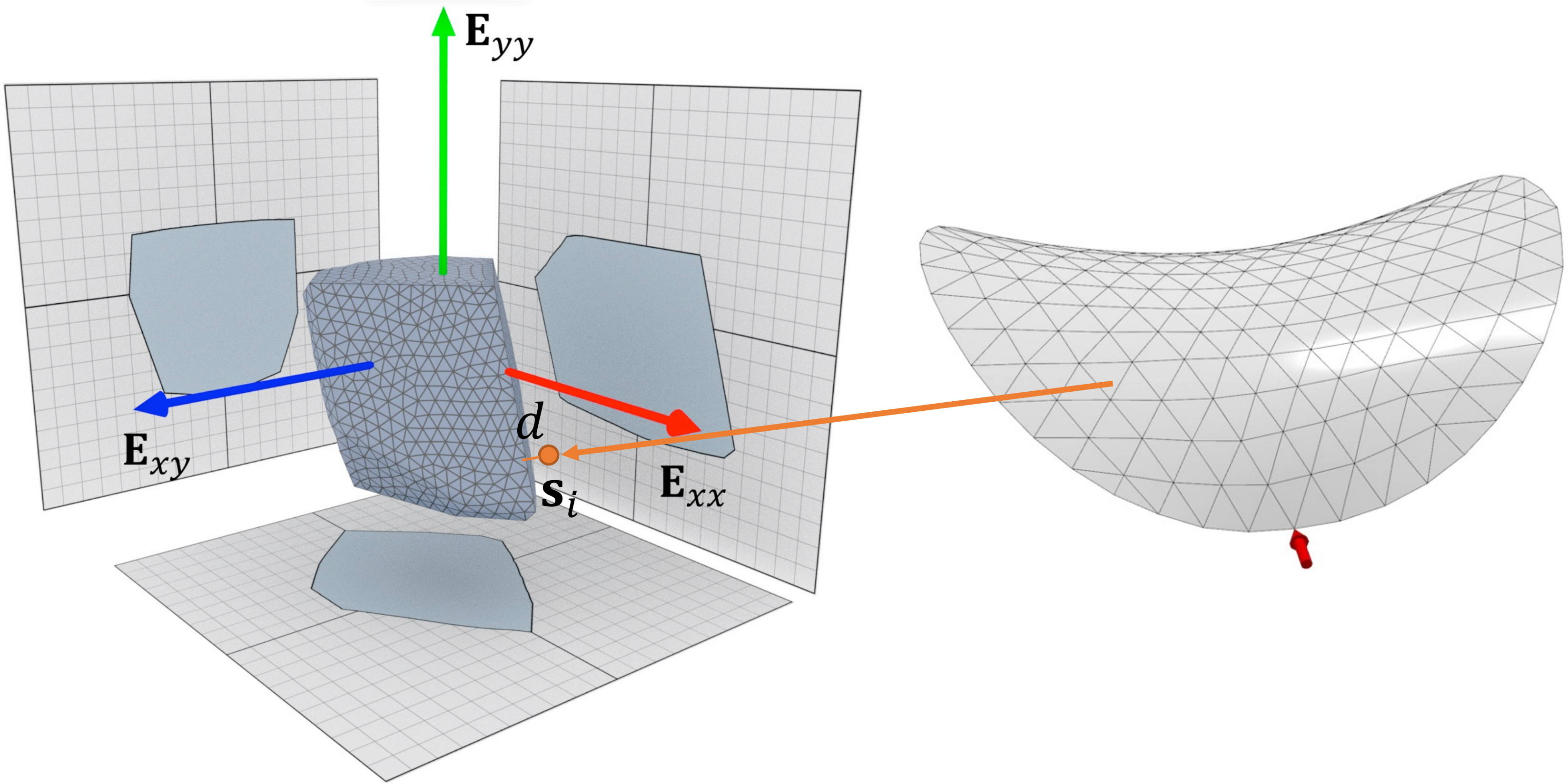




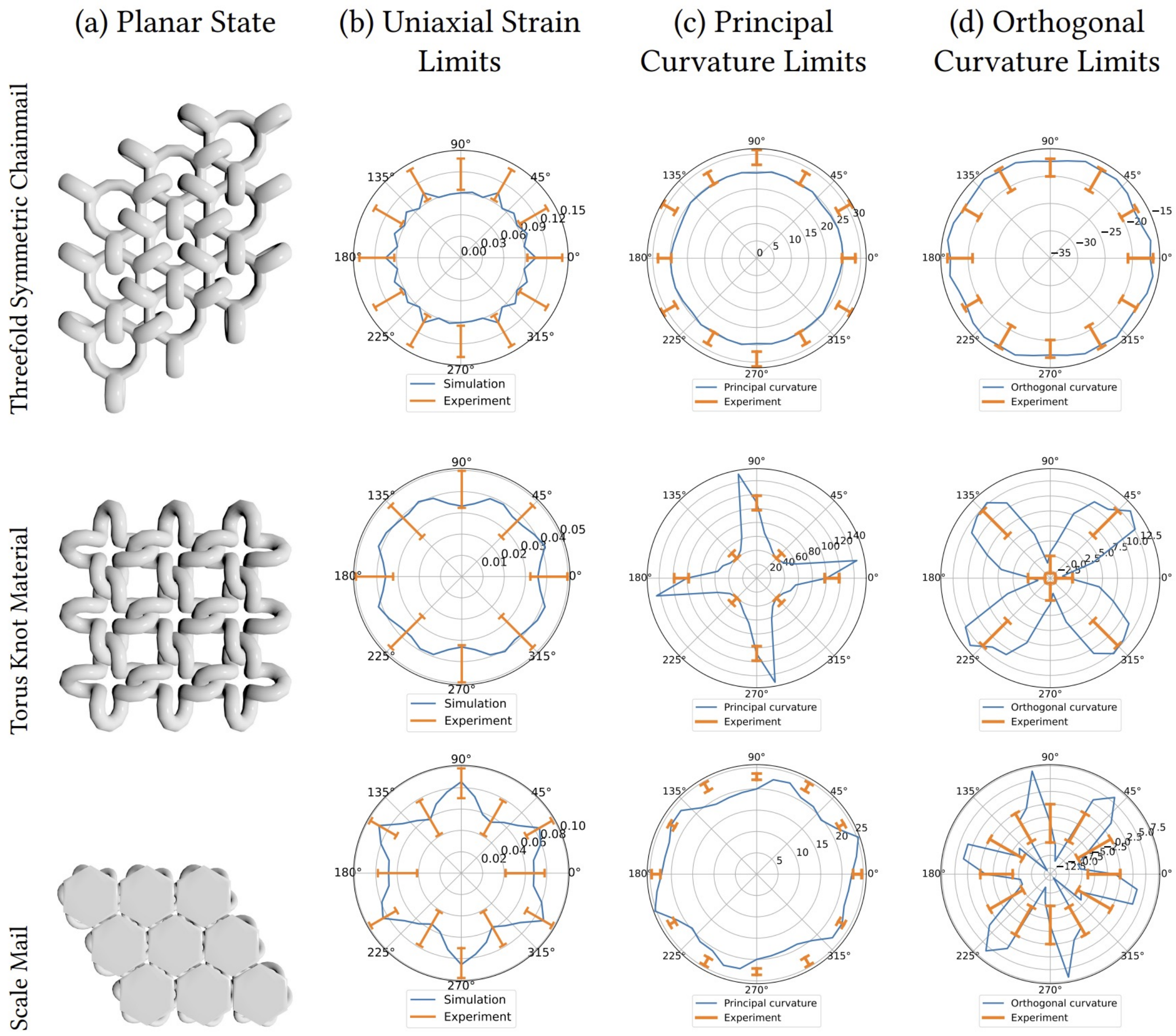
Macro-Scale Model



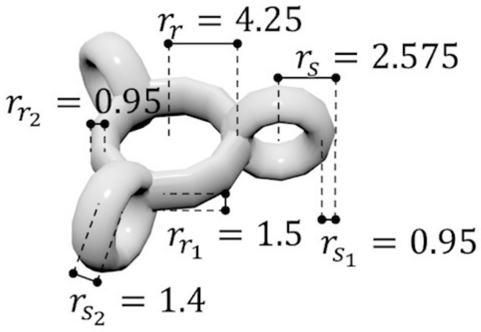
Macro-Scale Model

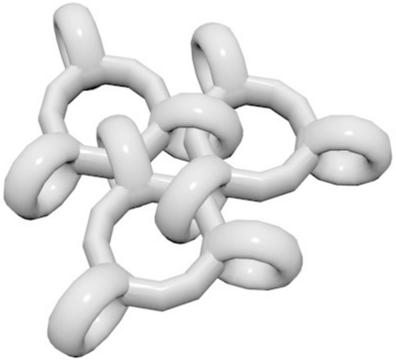


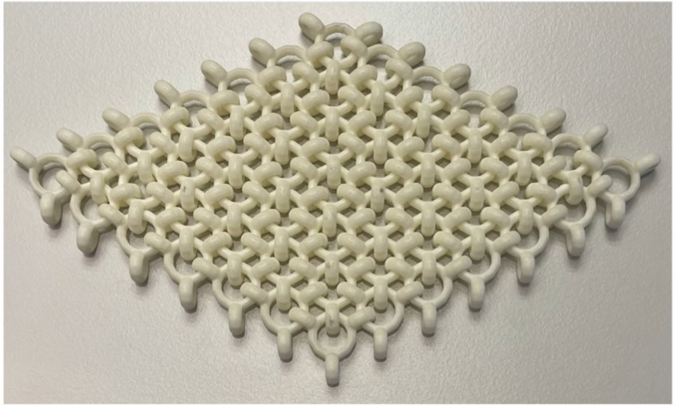
Results


(a) Planar State
(b) Uniaxial Strain Limits
(c) Principal Curvature Limits
(d) Orthogonal Curvature Limits
Threefold Symmetric Chainmail
Torus Knot Material
Scale Mail
Simulation
Experiment
Principal curvature
Experiment
Orthogonal curvature
Experiment

Threefold Symmetric Chainmail

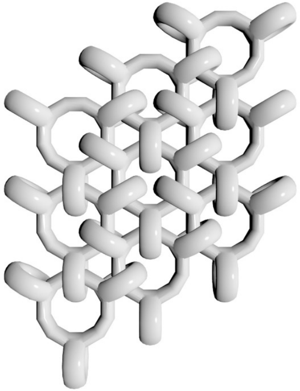




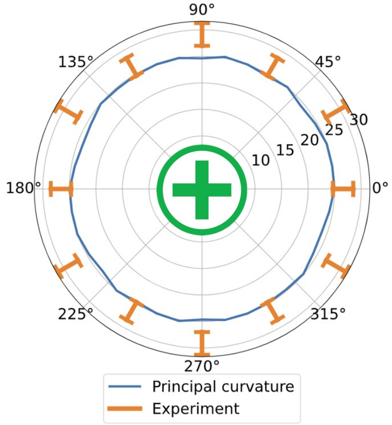


Threefold Symmetric Chainmail

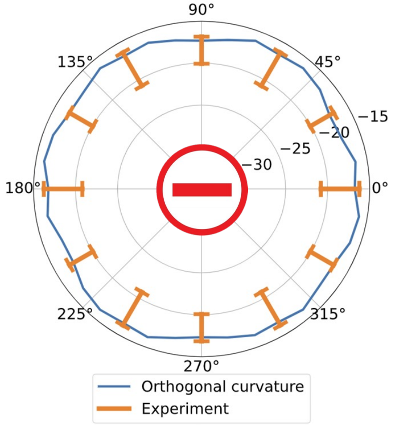
Planar State

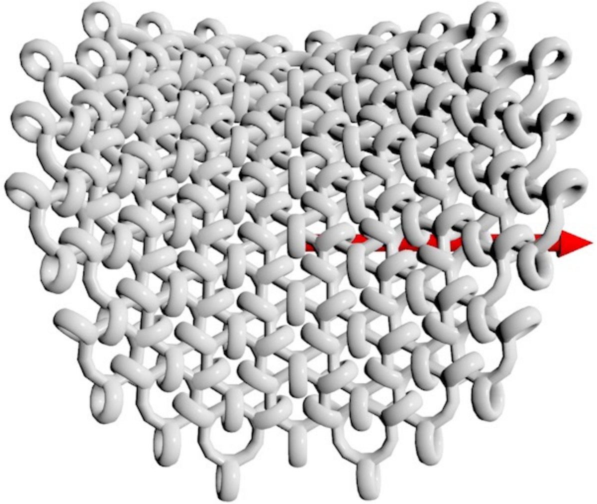


Principal Curvature Limits



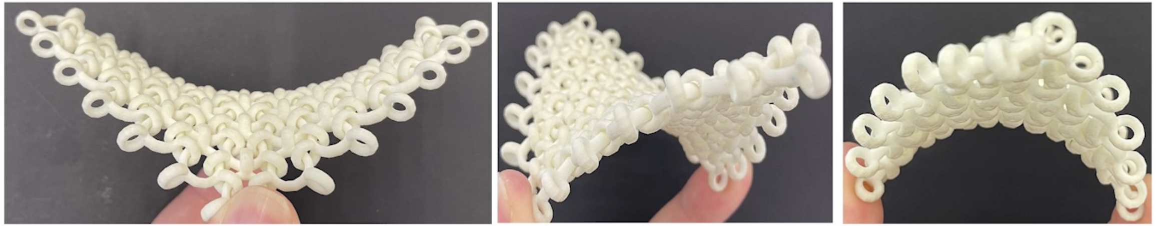
Orthogonal Curvature Limits



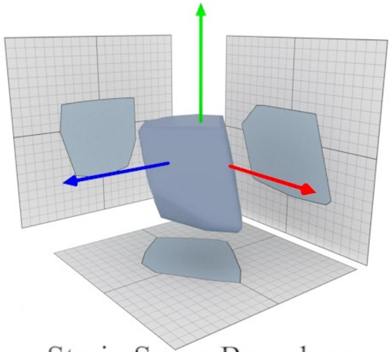


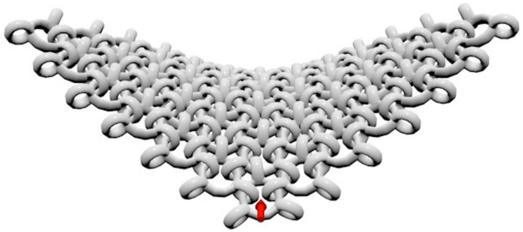
Bending

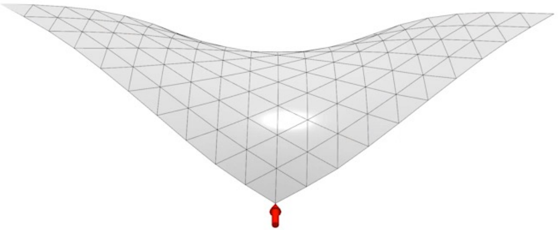
Threefold Symmetric Chainmail



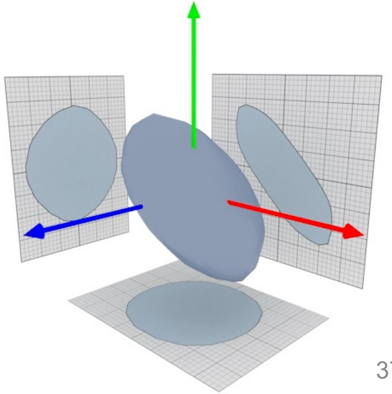
Strain Space Boundary
(Stretching)



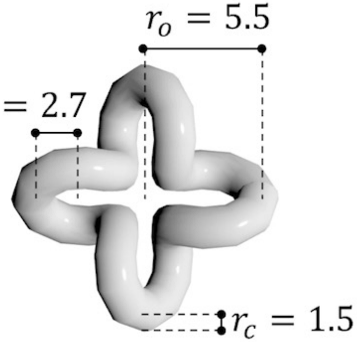


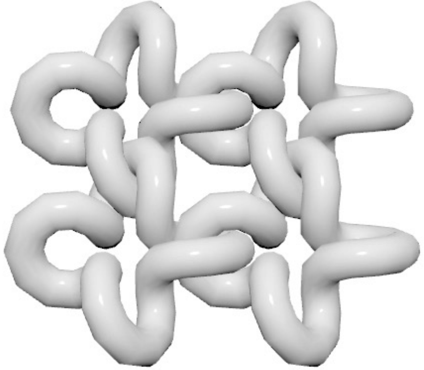


Strain Space Boundary
(Bending)



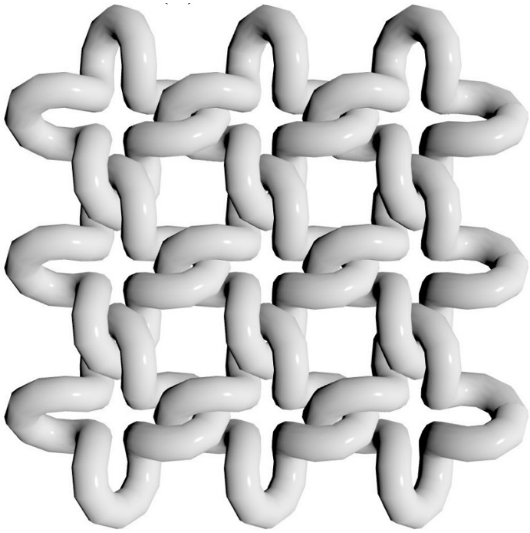
Torus Knot Material



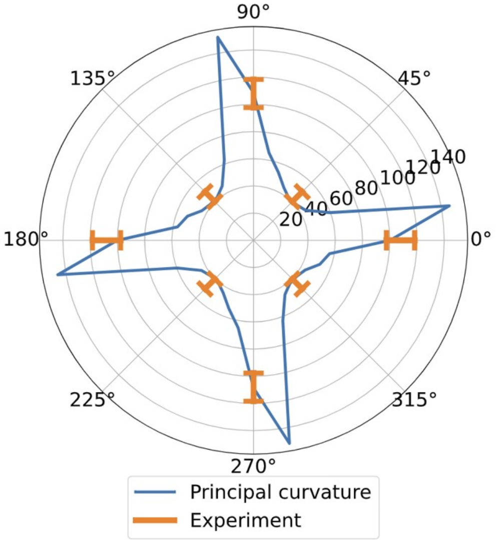


Torus Knot Material

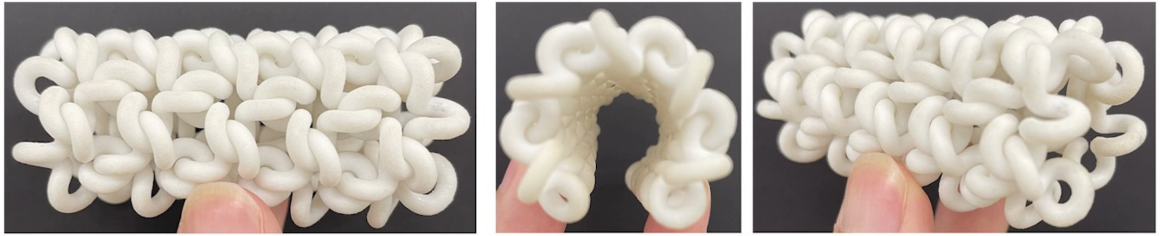
Planar State

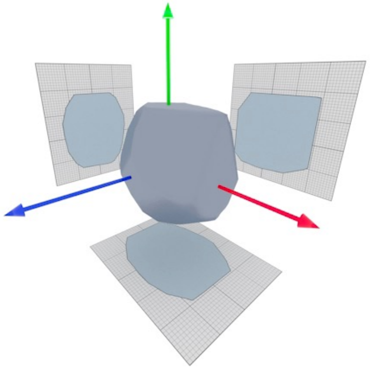


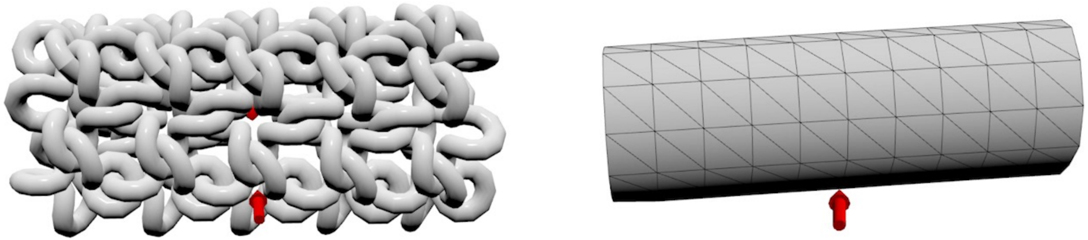
Principal Curvature Limits

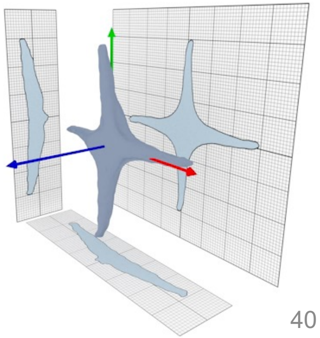


Torus Knot Material

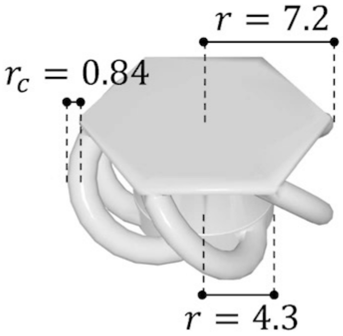


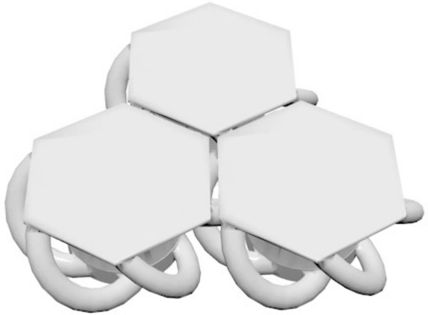


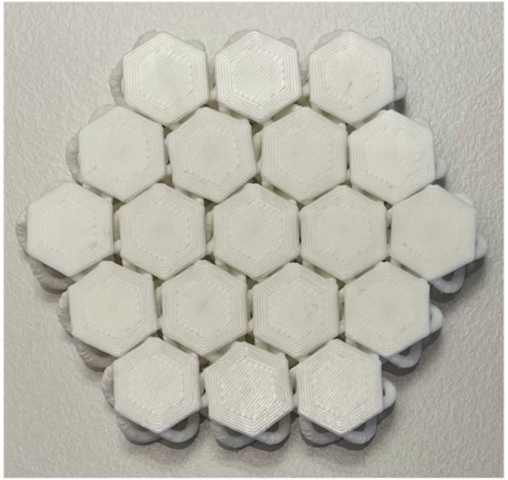




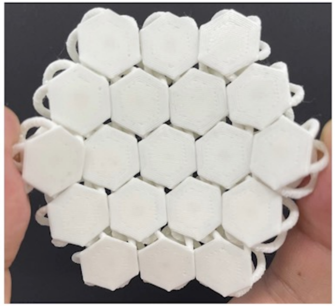
Scale Mail

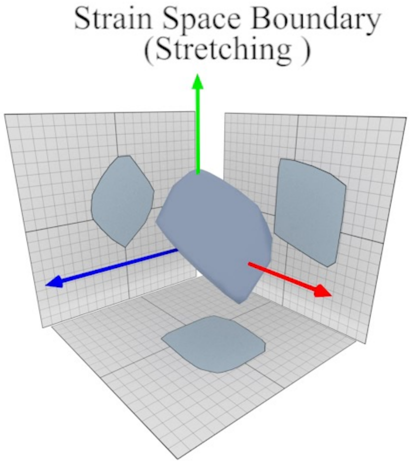


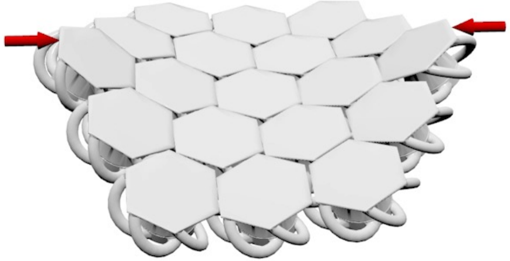


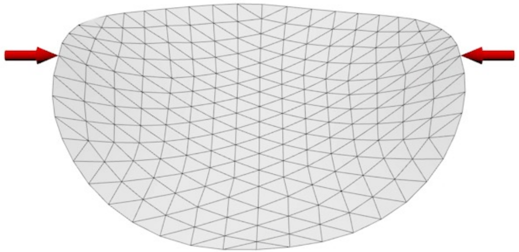


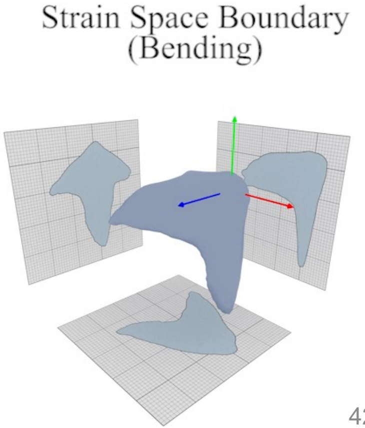
Scale Mail




Strain Space Boundary
(Stretching)

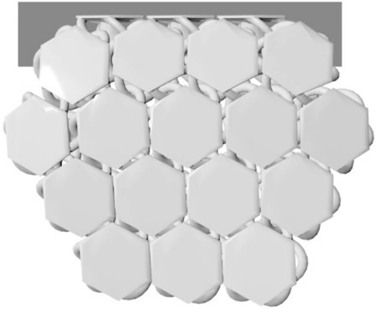


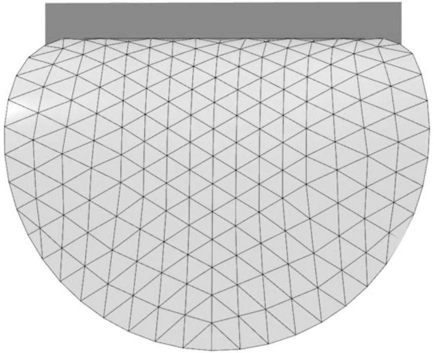



Strain Space Boundary
(Bending)

Scale Mail

Bending Under Gravity - Side 1

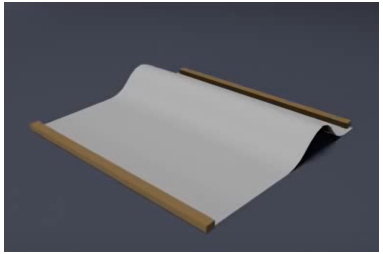




Limitations & Future Work

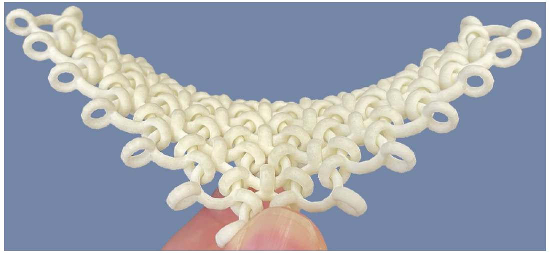
- Element shapes admitting twist/screw-like motions.

- No friction.



[Miguel et al. 2013]

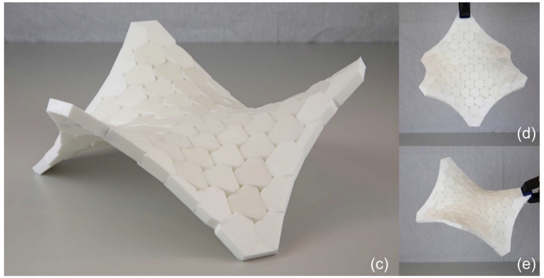
- no elastic deformation



- Adding geometric detail.

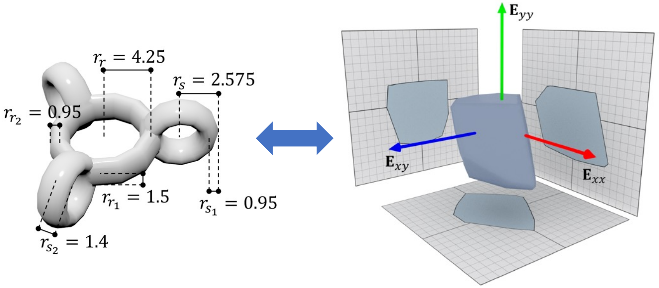
[Sperl et al. 2021]

- Heterogeneous materials with curved rest shapes.



[Wang et al. 2019]

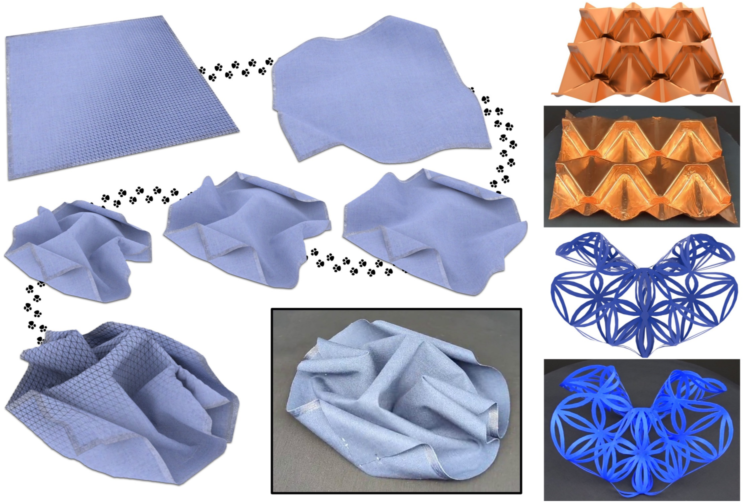
- Inverse design of DIM.



Overview

Beyond Chainmail: Computational Modeling of Discrete Interlocking Materials

Modal Folding: Discovering Smooth Folding Patterns for Sheet Materials using Strain-Space Modes



SIGGRAPH 2024
DENVER+ 28 JUL — 1 AUG

Modal Folding: Discovering Smooth Folding Patterns for Sheet Materials using Strain-Space Modes

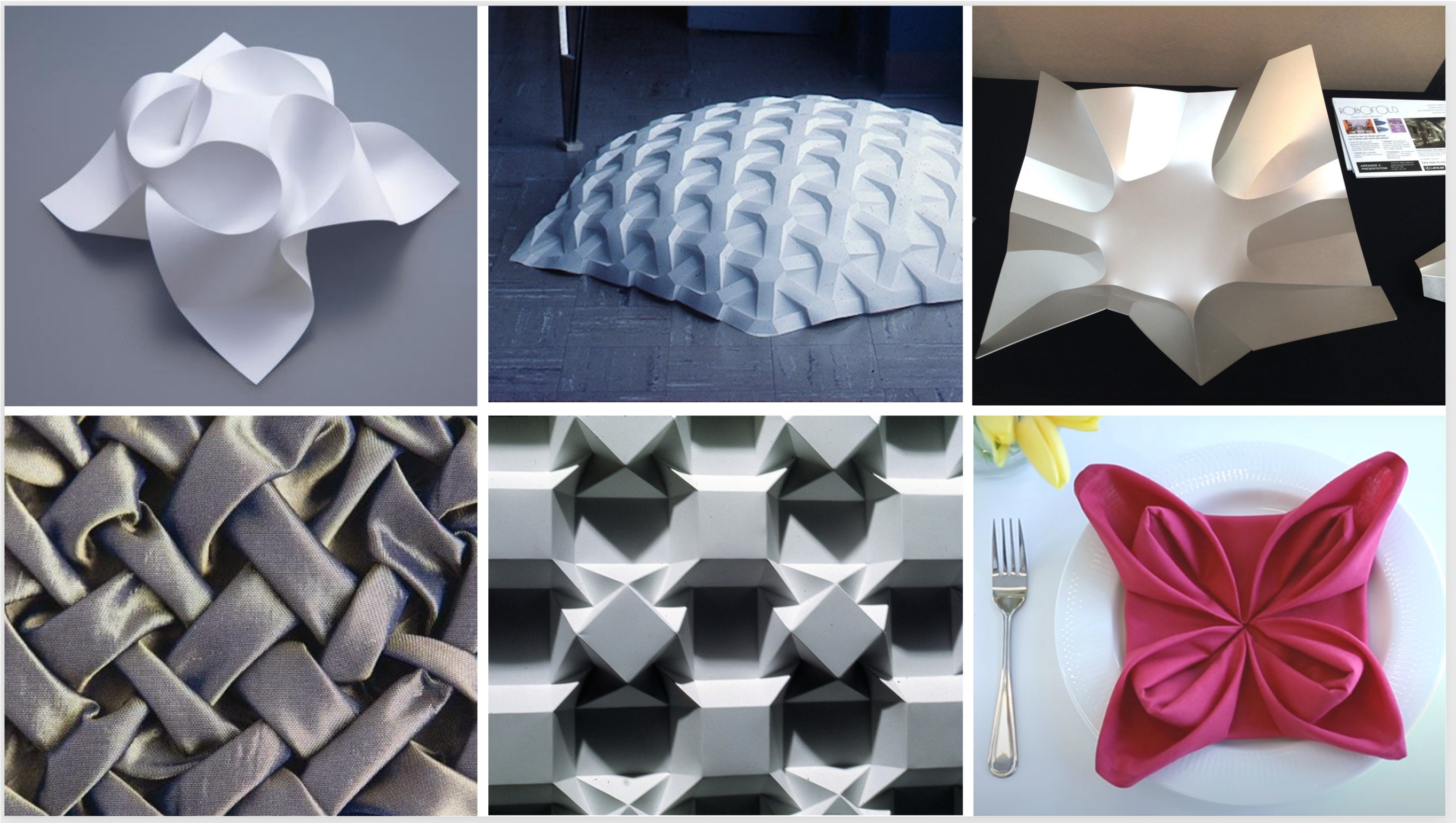
Pengbin Tang[1,2], Ronan Hinchet[1], Roi Poranne[1,3],
Bernhard Thomaszewski[1] , Stelian Coros[1]

1 ETH zürich

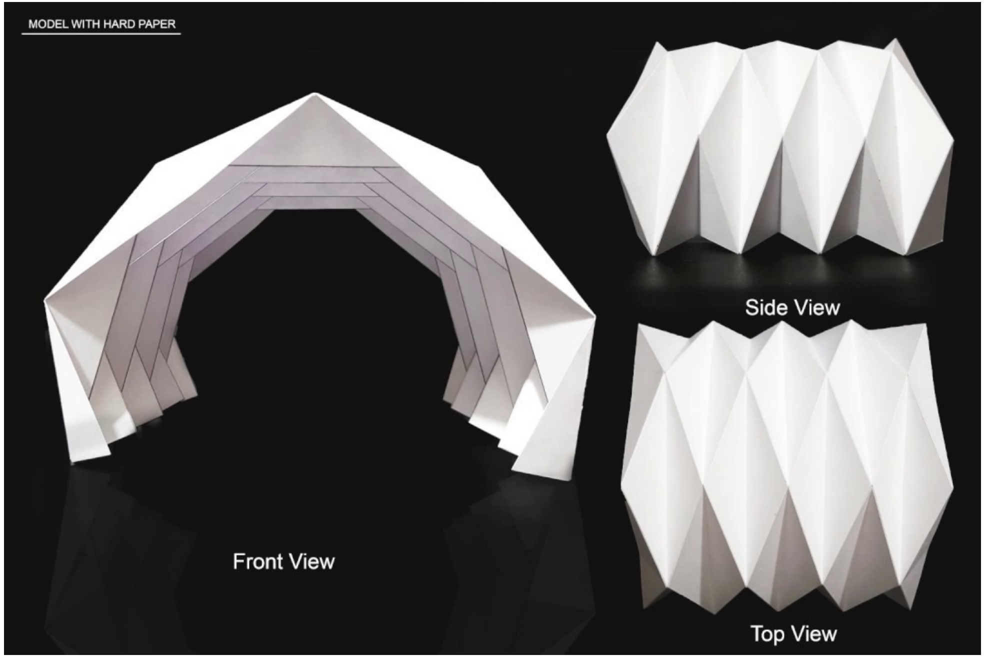
2 Université de Montréal

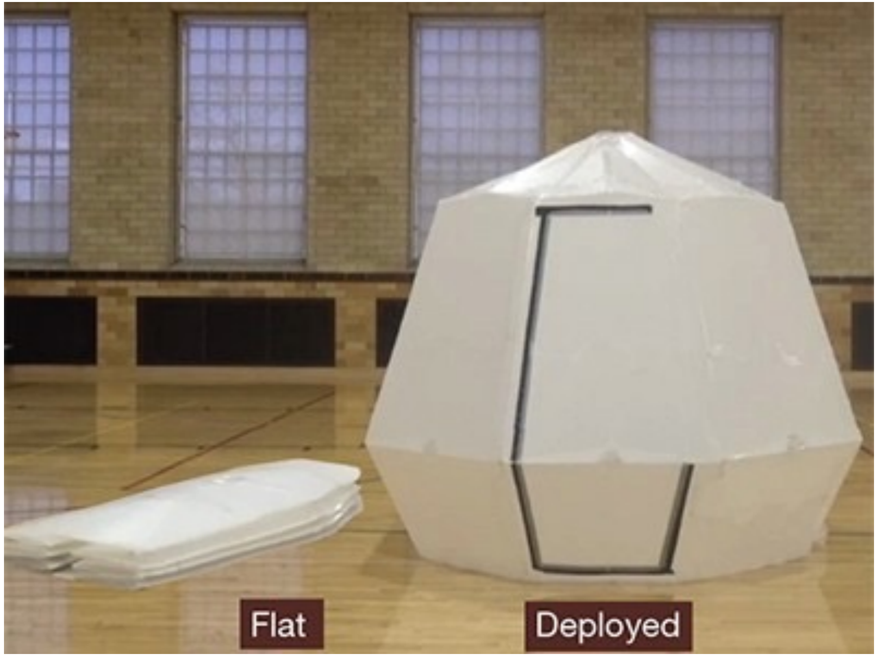
3 אוניברסיטת חיפה UNIVERSITY OF HAIFA جامعة حيفا

Folding patterns



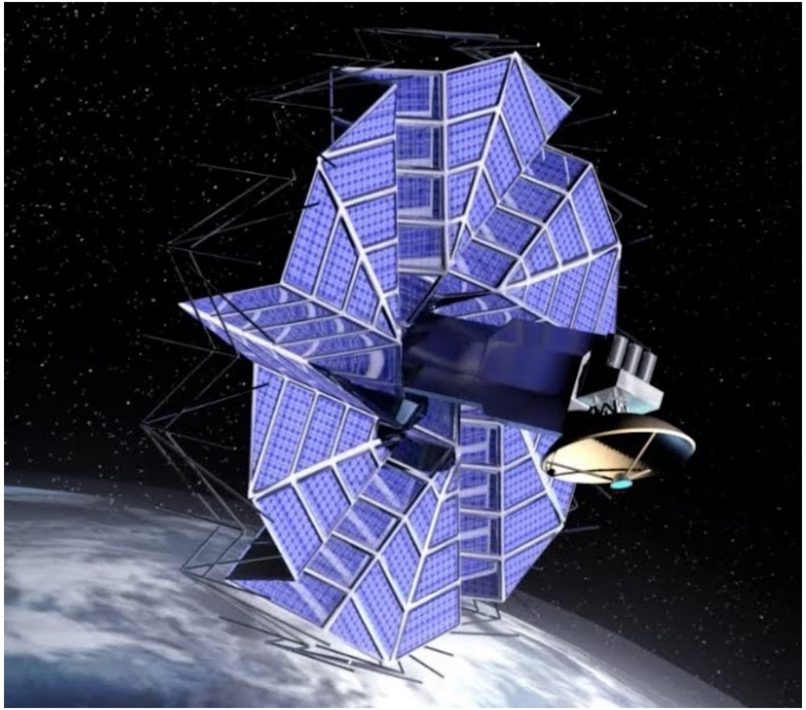
Applications of Folding Patterns





Deployable shading panels

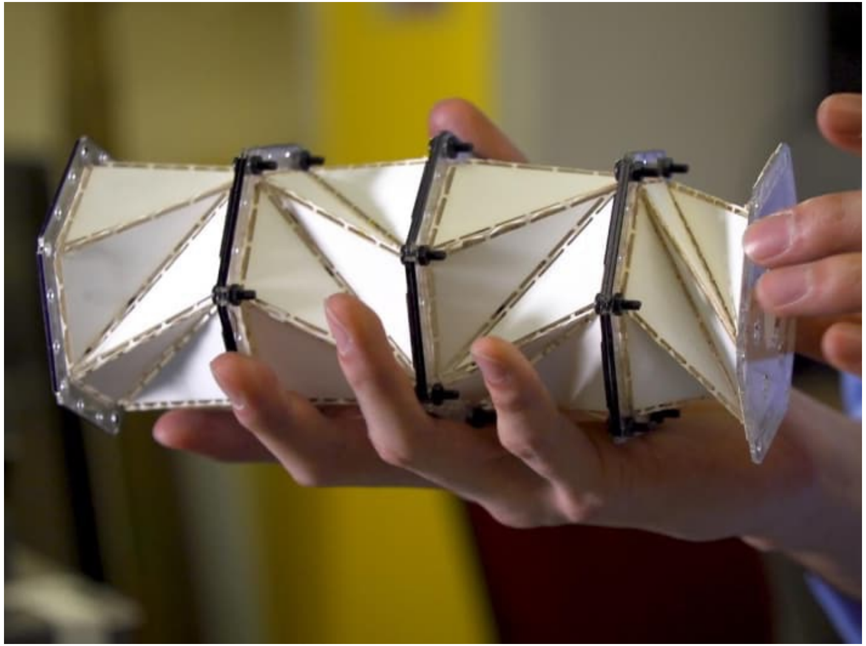
Applications of Folding Patterns



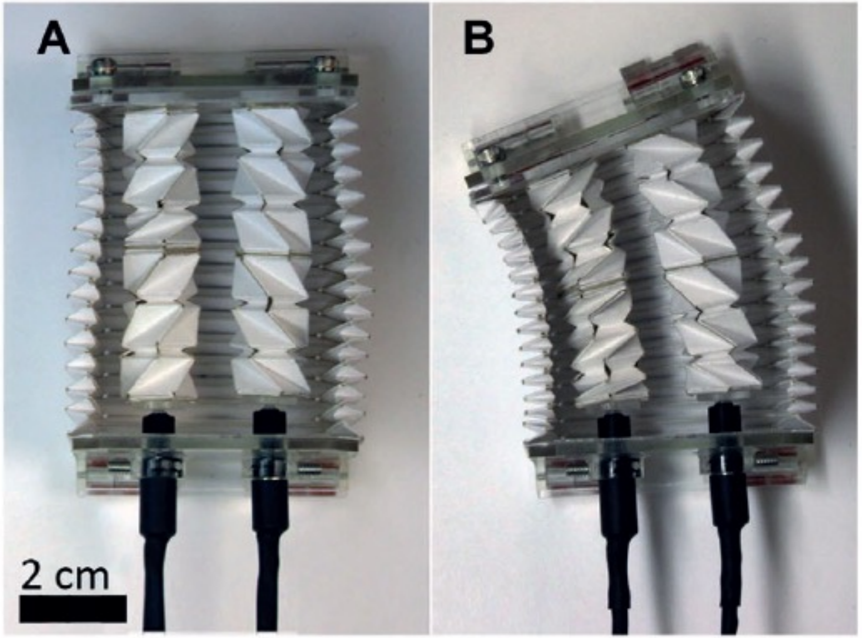
Origami solar panels

Carbon-fiber layups

Applications of Folding Patterns

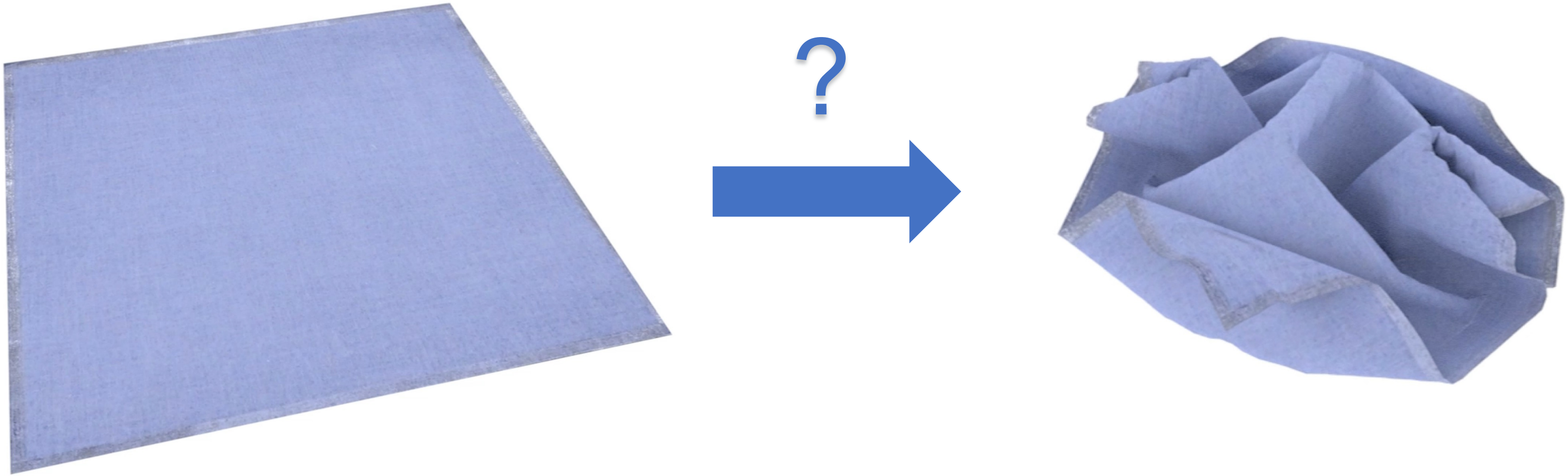


Origami metamaterials



Soft robotics

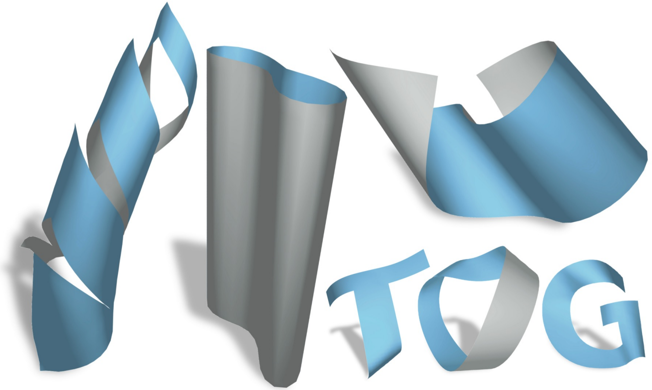
Folding Transformation



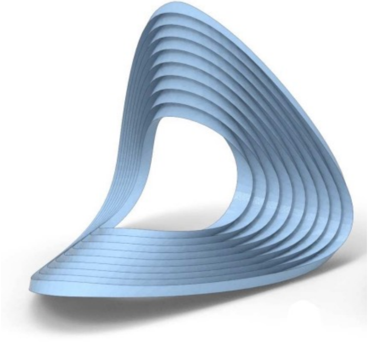
Goal

A new computational framework for automatically discovering folding patterns of sheet materials.

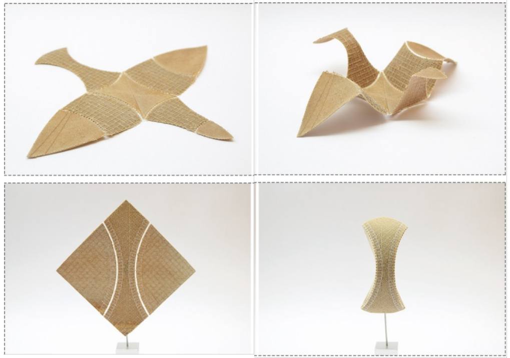
Folding Thin Sheets



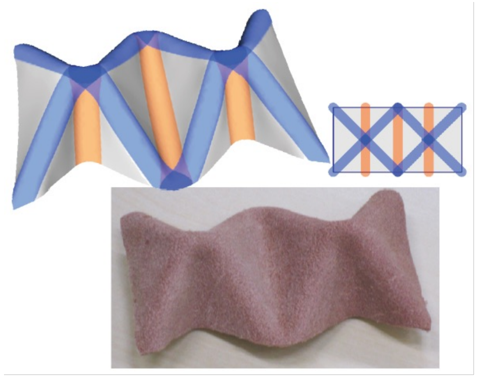
[Rabinovich et al. 2018]



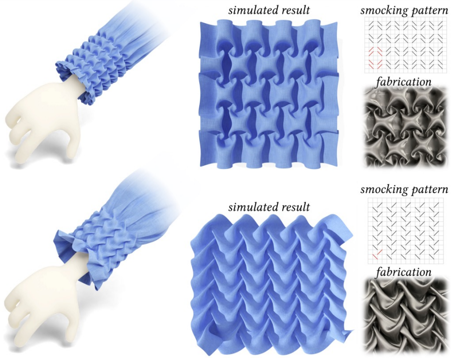
[Jiang et al. 2019]



[Tahouni et al. 2018]

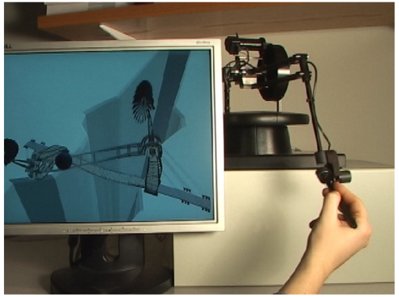


[Zhu et al. 2013]



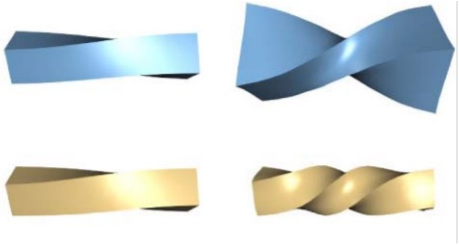
[Ren et al. 2024]

Modes in Animation and Design



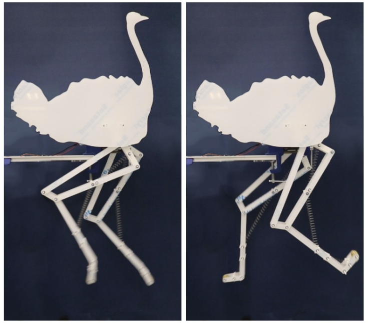
[Barbic and James 2005]

[Choi and Ko 2005]

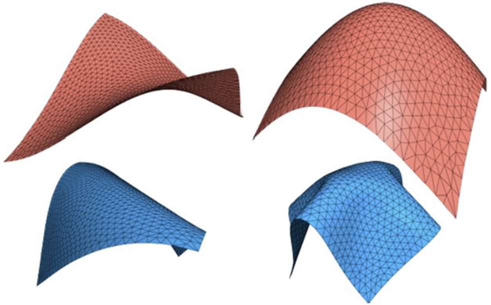


[Huang et al. 2011]

[Zehnder et al. 2016]



[Tang et al. 2016]



[Duenser et al. 2022]

Strain-Space Modes

Folding transformations

$\Downarrow$

Large deformations with little mechanical work

$\Downarrow$

Find configurations

$$\min_{\mathbf{x}} W(\mathbf{x}) \quad \text{s.t.} \quad C(\mathbf{x}) = 0$$

Linear Modes

- For deformations around the rest state, a second-order expansion of the elastic energy yields

$$W(\mathbf{X}+\mathbf{u}) = \underbrace{W(\mathbf{X})}_{\mathbf{0}} + \underbrace{\nabla_{\mathbf{x}} W(\mathbf{X},\mathbf{x})\mathbf{u}}_{\mathbf{0}} + \frac{\mathbf{1}}{\mathbf{2}}\mathbf{u}^{\mathrm{T}}\mathbf{H}\mathbf{u} + O\left(\mathbf{u}^{\mathbf{3}}\right)$$

$$\approx \frac{\mathbf{1}}{\mathbf{2}}\mathbf{u}^{\mathrm{T}}\mathbf{H}\mathbf{u}$$

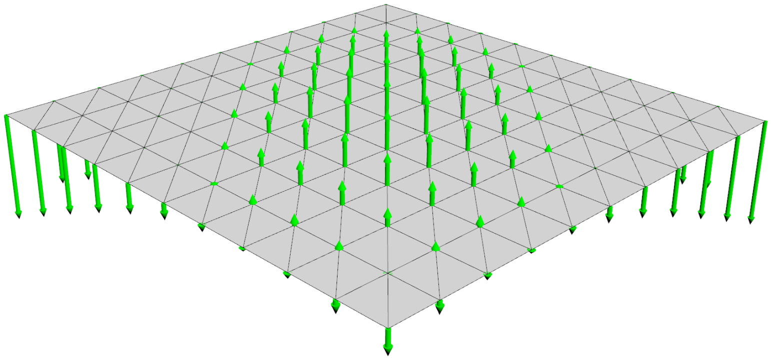
- Leading to a constrained optimization problem

$$\min \frac{\mathbf{1}}{\mathbf{2}}\mathbf{u}^{\mathrm{T}}\mathbf{H}\mathbf{u} \quad \text{s.t.} \quad \frac{\mathbf{1}}{\mathbf{2}}\mathbf{u}^{\mathrm{T}}\mathbf{M}\mathbf{u} = b^2$$

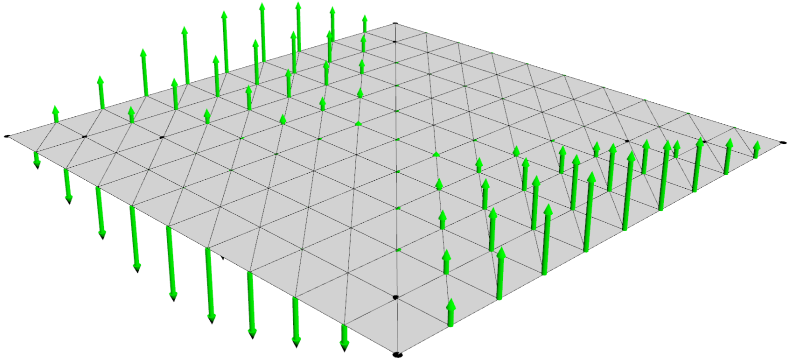
- The first-order optimality conditions of this problem require that

$$\mathbf{H}\mathbf{u} + \lambda\mathbf{M}\mathbf{u} = \mathbf{0}$$

Linear Modes



Mode 6

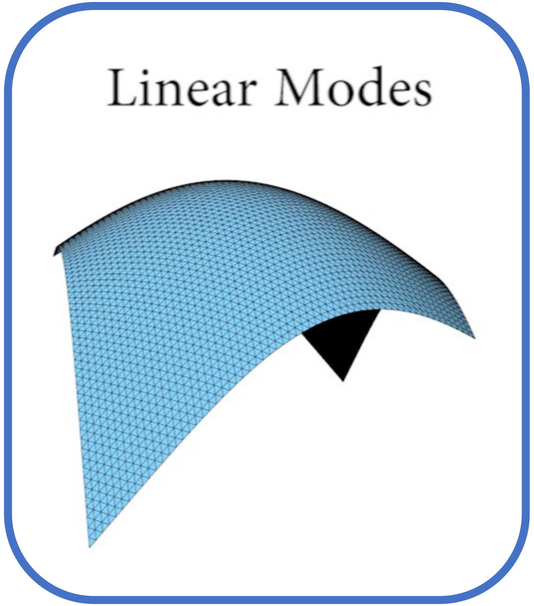


Mode 7

Linear Modes

$$\dot{\boldsymbol{\kappa}}_i = \frac{d\boldsymbol{\kappa}}{d\mathbf{x}}\mathbf{e}_i$$

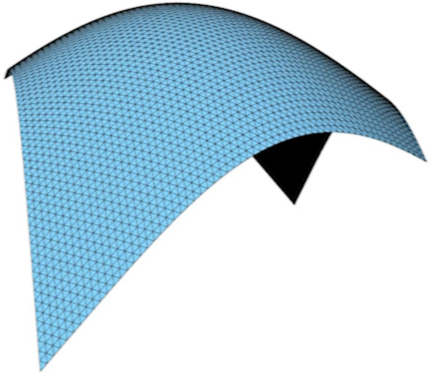
$$\boldsymbol{\Psi}_i^{\mathrm{LM}}(t) = \mathbf{X} + t\mathbf{e}_i$$

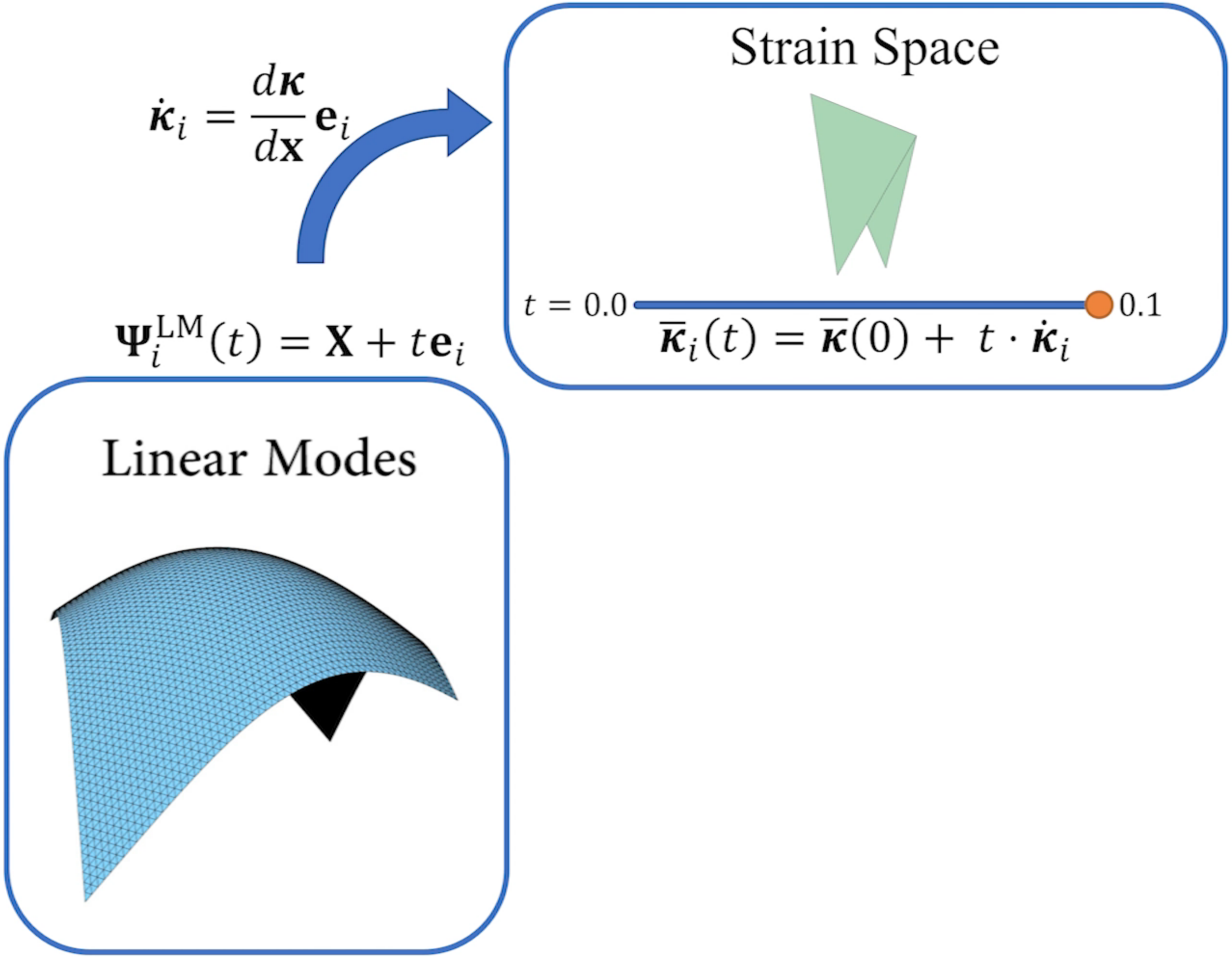


$$\dot{\boldsymbol{\kappa}}_i = \frac{d\boldsymbol{\kappa}}{d\mathbf{x}}\mathbf{e}_i$$

$$\boldsymbol{\Psi}_i^{\mathrm{LM}}(t) = \mathbf{X} + t\,\mathbf{e}_i$$

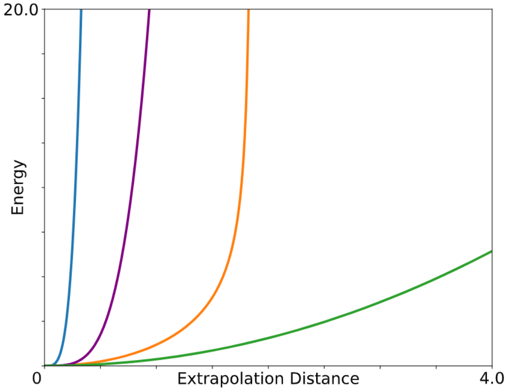
Linear Modes

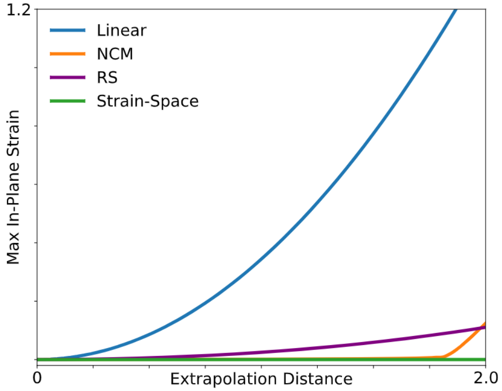


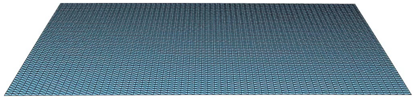


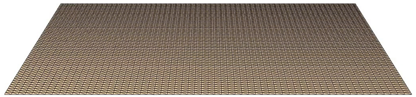
Strain Space
$\dot{\boldsymbol{\kappa}}_i = \frac{d\boldsymbol{\kappa}}{d\mathbf{x}}\mathbf{e}_i$
$t = 0.0$
0.1
$\overline{\boldsymbol{\kappa}}_i(t) = \overline{\boldsymbol{\kappa}}(0) + t \cdot \dot{\boldsymbol{\kappa}}_i$
$\boldsymbol{\Psi}_i^{\mathrm{LM}}(t) = \mathbf{X} + t\mathbf{e}_i$
Linear Modes

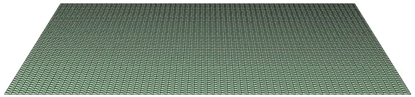
Comparing Modes











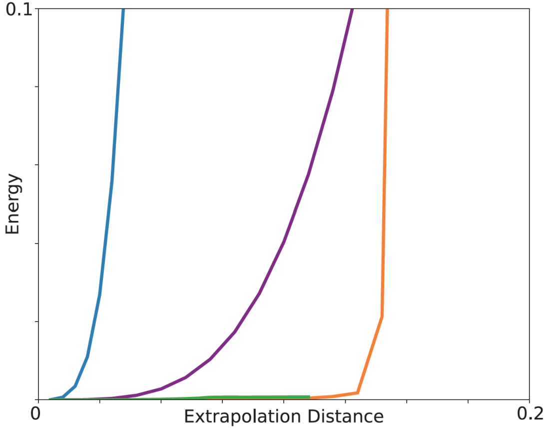
Linear Modes

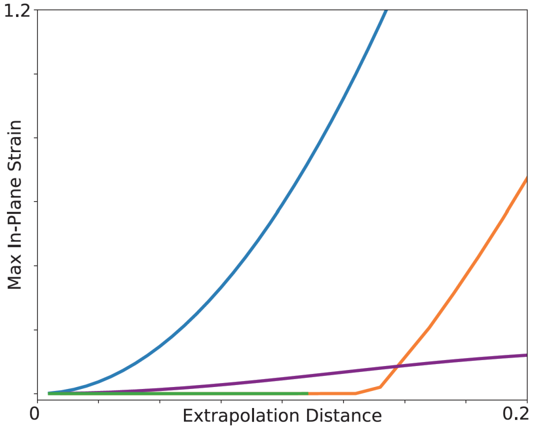
Nonlinear Compliant Modes
[Duenser et al. 2022]

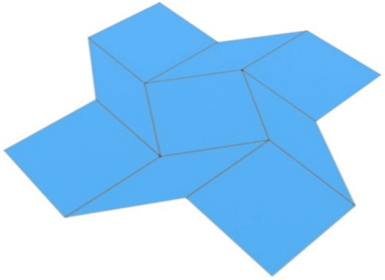
Rotation-Strain Space Modes
[Huang et al. 2011]

Ours

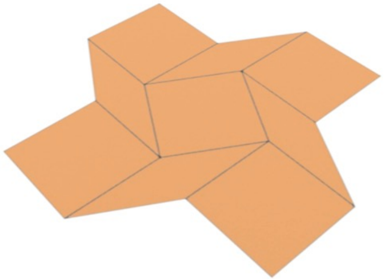
Square Twist Origami



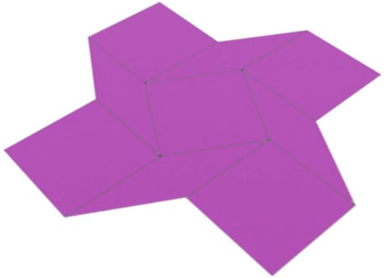




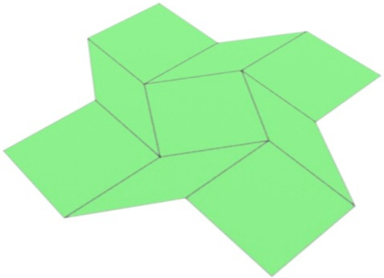
Linear Modes



Nonlinear Complaint Modes
[Duenser et al. 2022]



Rotation-Strain Space Modes
[Huang et al. 2011]

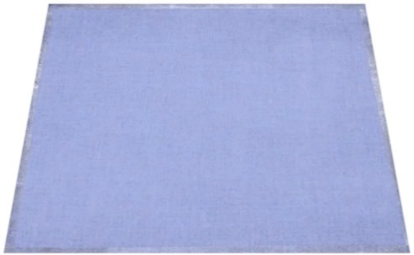


Ours

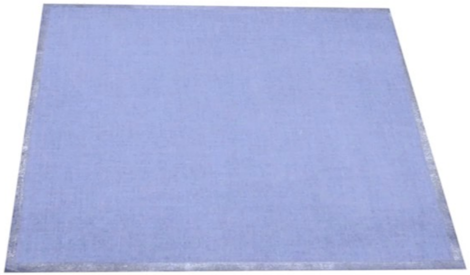
Exploring Strain-Space Modes

A Square Sheet

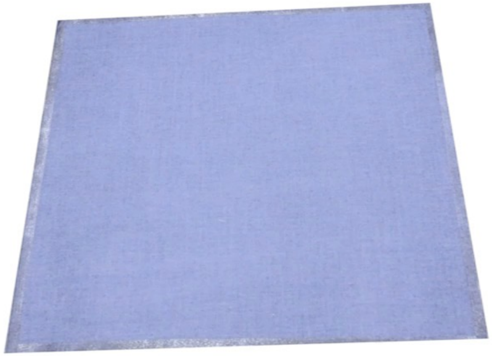
Mode 25



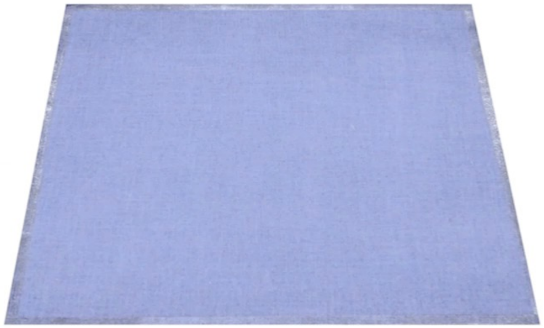
Mode 35



Mode 50

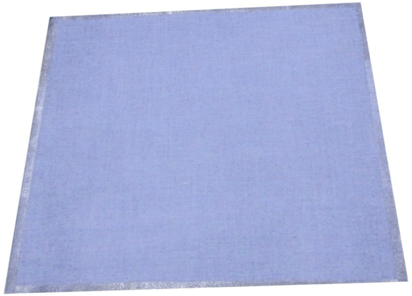


Mode 57



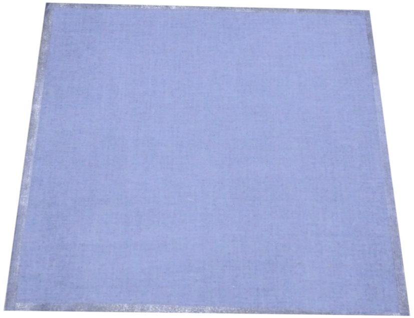
A Square Sheet

Mode 30



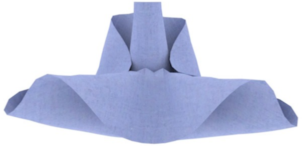
Mode 32

Mode 43



A Disc-shaped Sheet

Mode 16

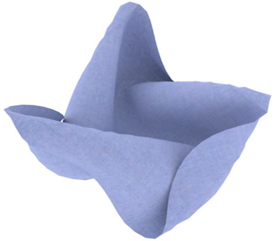


Mode 20

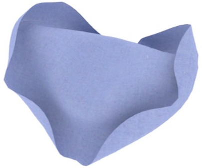
Mode 21

Mode 24

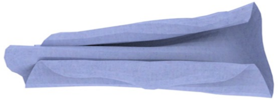
Mode 25



Mode 30

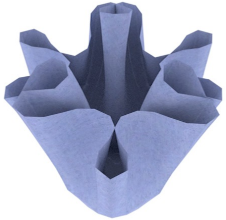


Mode 32



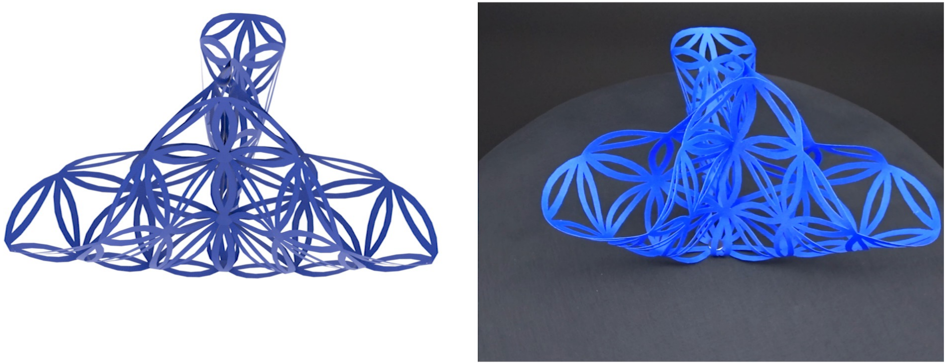
Mode 33

Mode 36

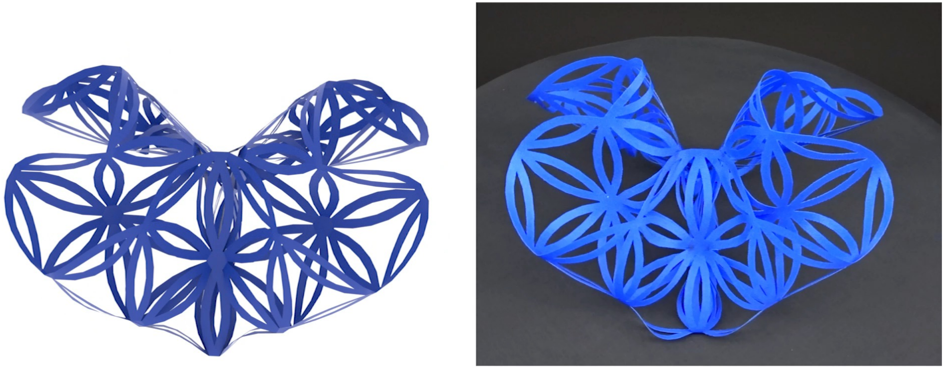


A Life-flower Pattern

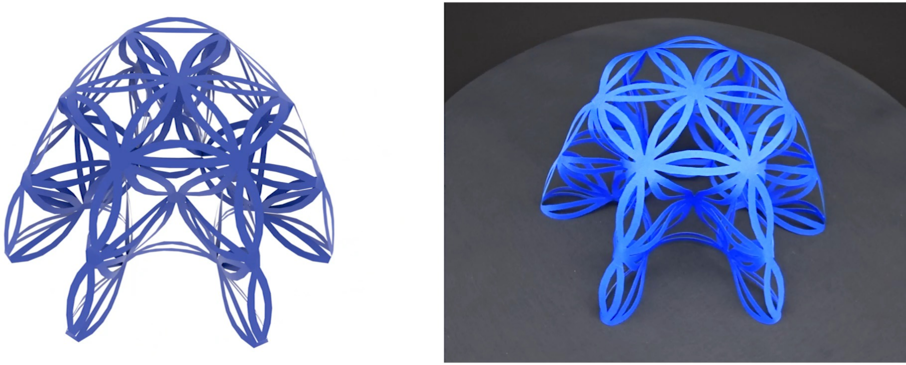
Mode 9



Mode 13

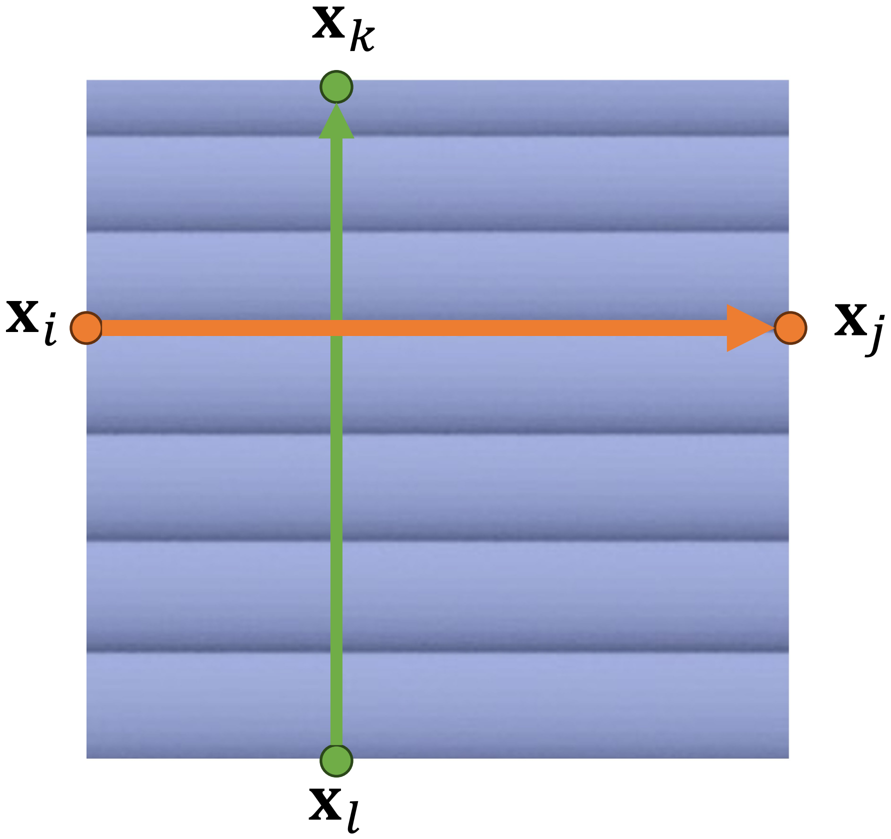


Mode 24



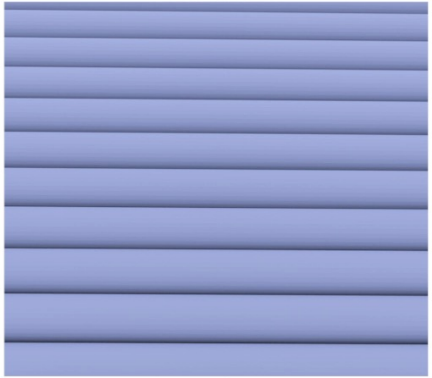
Periodic Folding Patterns

Translational Tiling

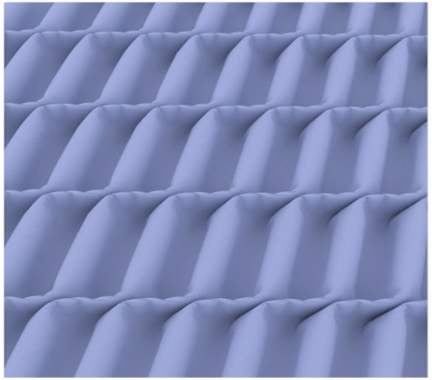


Translational Tiling

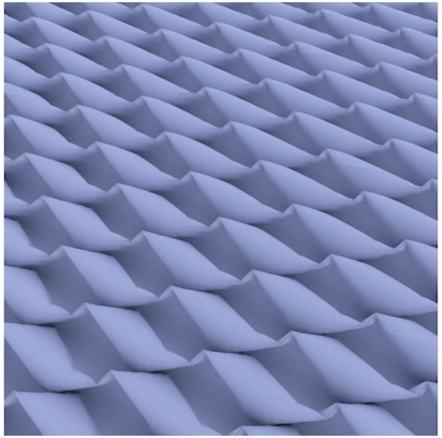
Mode 6



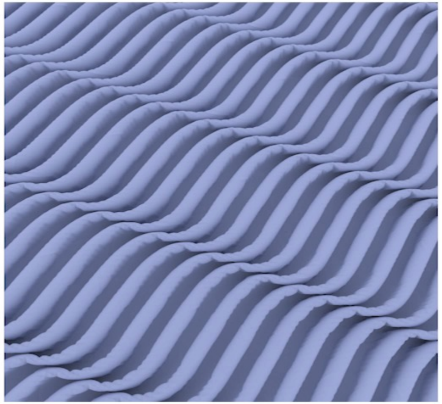
Mode 21



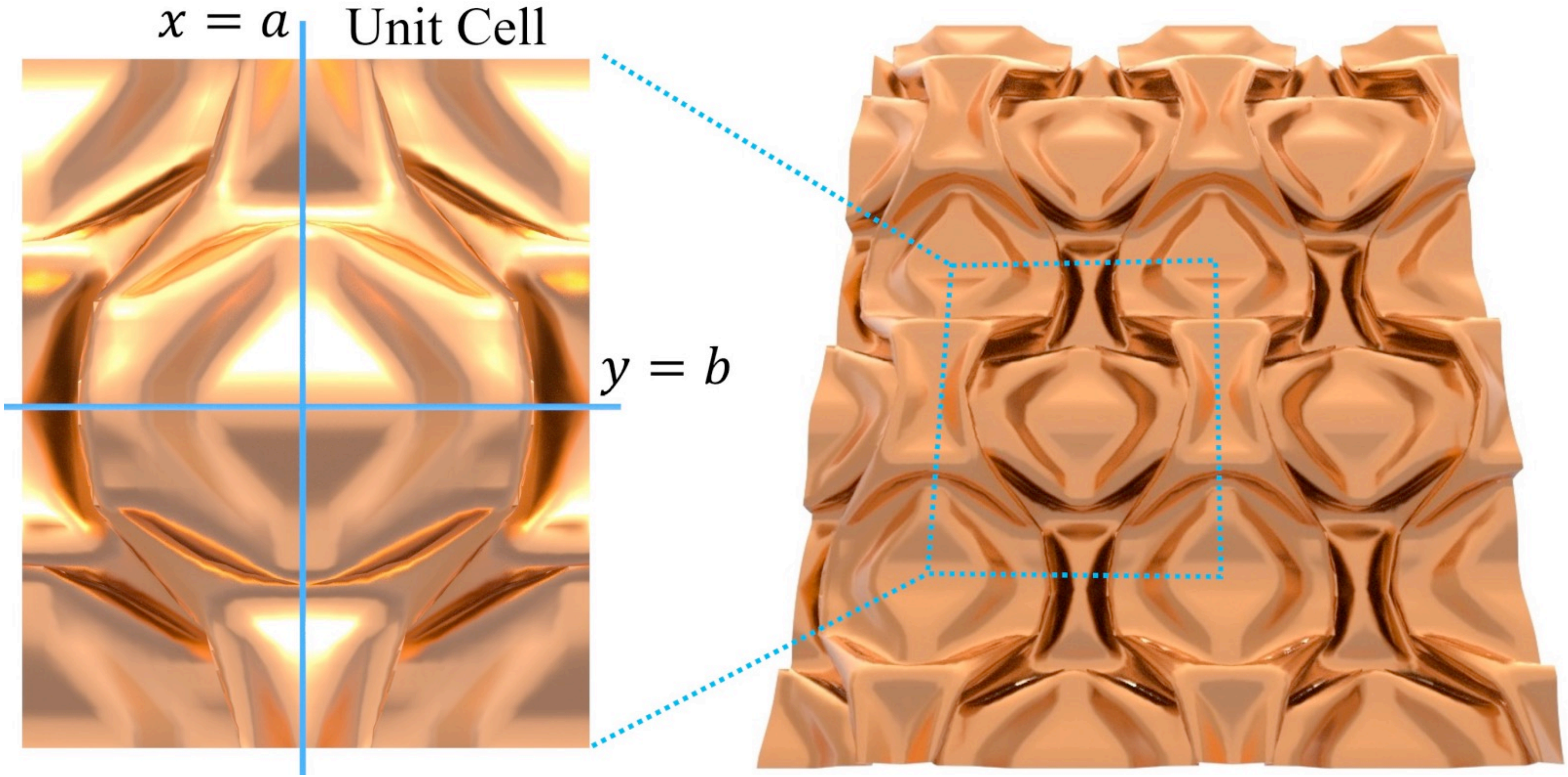
Mode 43



Mode 57



Reflection Tiling



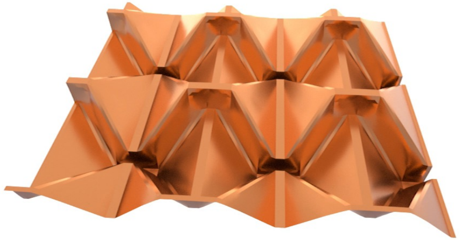
Reflection Tiling

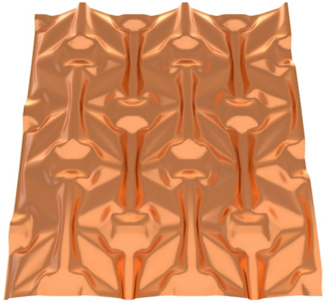
Reflection Tiling

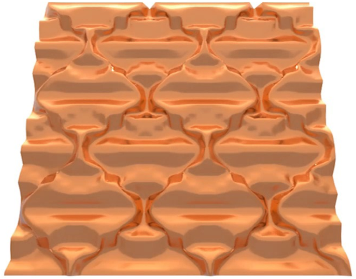
Mode 8

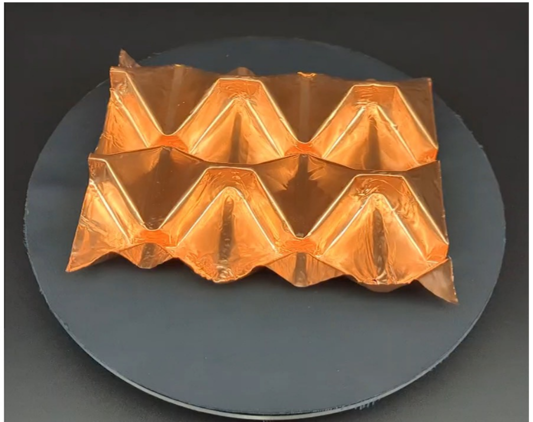
Mode 15

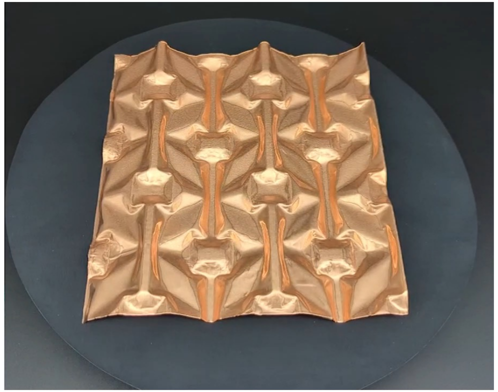
Mode 19

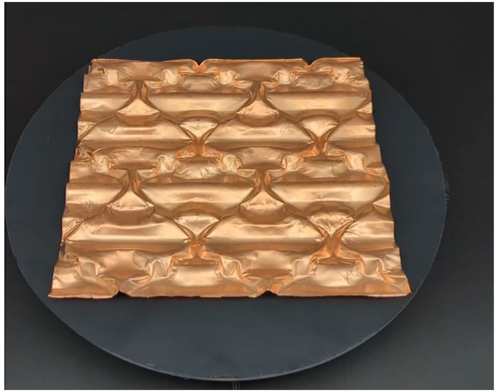












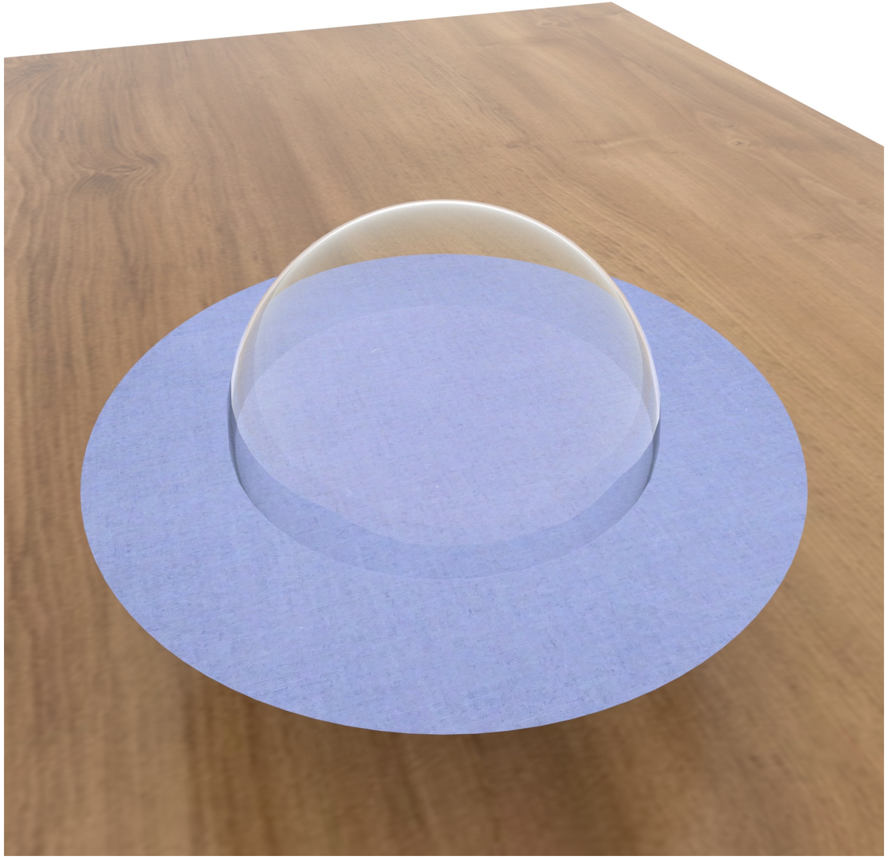
Inverse Design

- Multi-Dimensional Extension

$$\bar{\kappa} = \bar{\kappa}(0) + \sum_j c_j \mathbf{J}\mathbf{e}_j$$

- Inverse Design

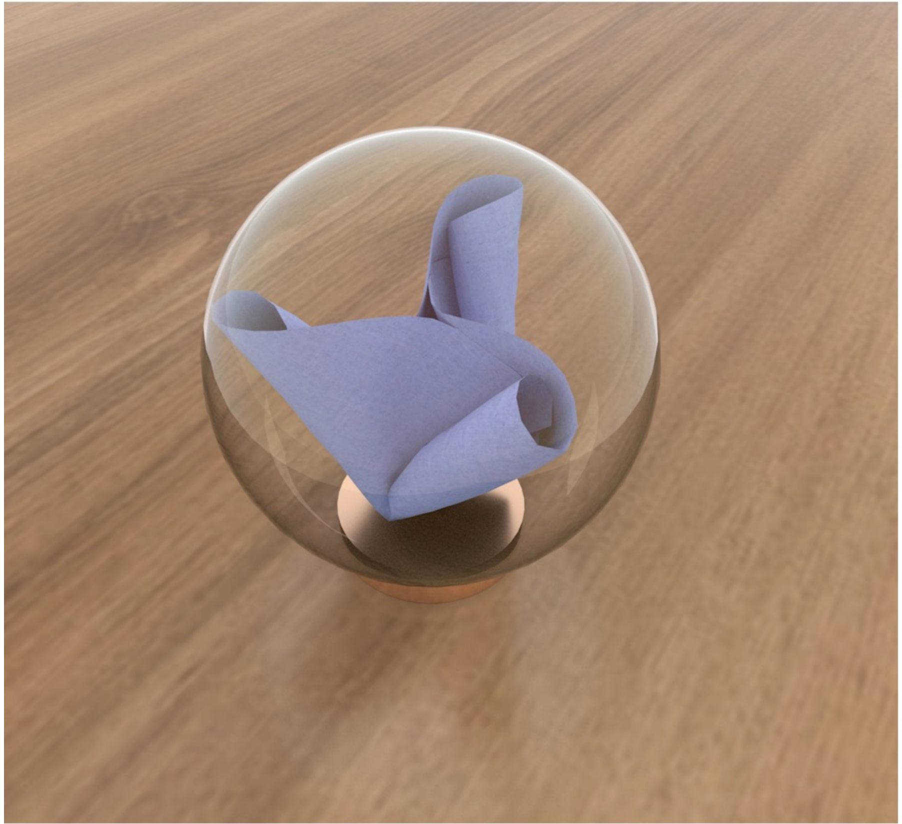
$$\min_{\mathbf{c}} T(\mathbf{c})$$
$$s.t. \quad \mathbf{f}(\mathbf{c}) = \mathbf{0}$$



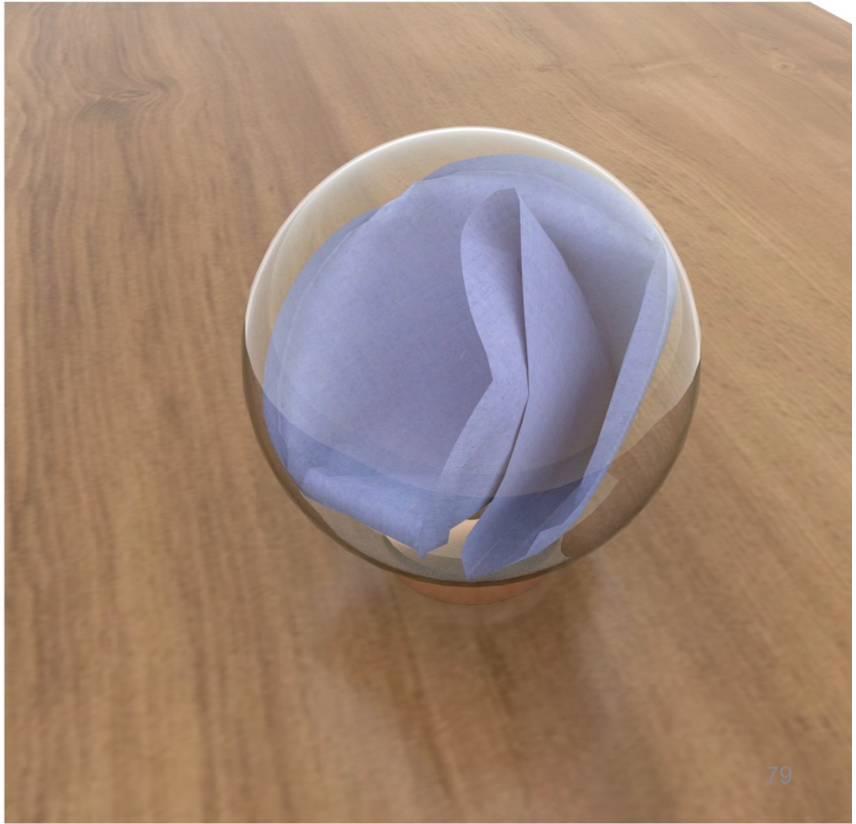
Fitting into a sphere

Inverse Design

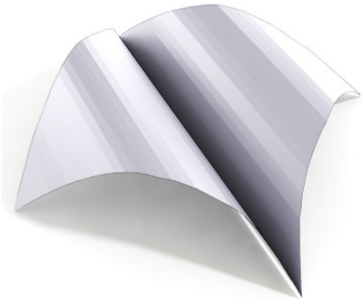
Strain-space modes



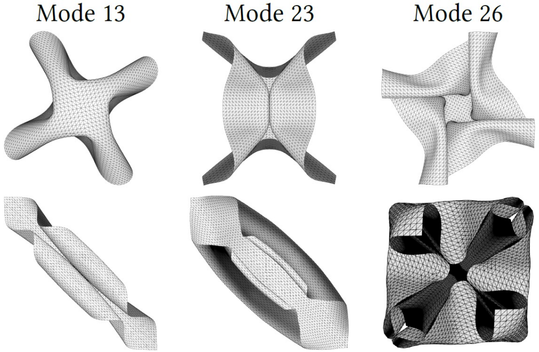
Contact-induced compaction



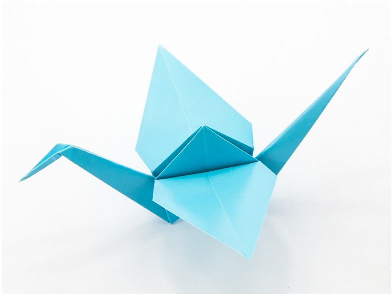
Limitations & Future Work



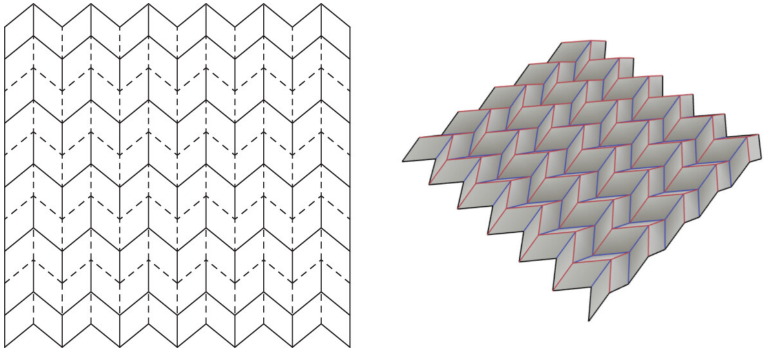
Not developable



Mesh Dependence



Modes Blending



Generating Sharp creases

Thank you!

Questions?