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Power Plastics: A Hybrid Lagrangian/Eulerian Solver for Mesoscale Inelastic Flows

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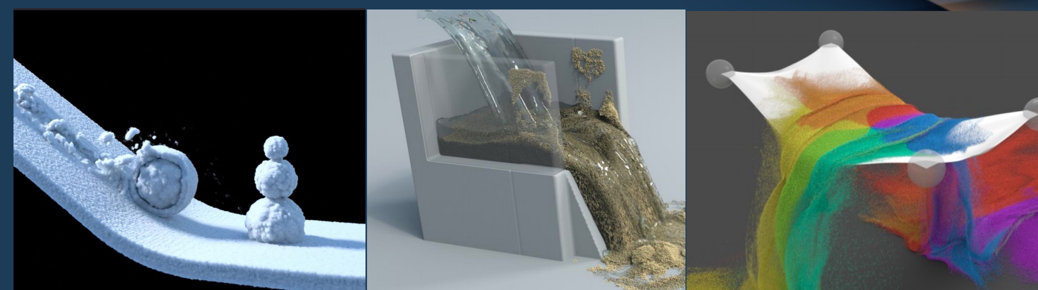
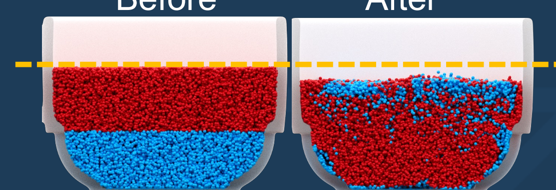
MOTIVATION

- MPM (Material Point Method) has been very popular over the past decade, it can simulate many different materials and coupling between them, e.g. snow, fluid, sand, cloth etc.



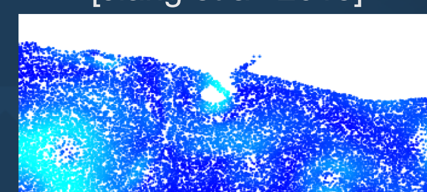
[Stomakhin et al. 2012] [Pradhana et al. 2017] [Jiang et al. 2017]

- Some limitations of MPM:
  - volume loss
  - particle clumping/voiding
  - minimum PPC requirement
  - particles are homogeneous



Volume loss

[Jiang et al. 2015]



Particles clumping/voiding

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MOTIVATION

What if we represent each particle as a bubble with different volume in MPM?

- ❑ Incorrect dynamics
- ❑ Particles are leaking through

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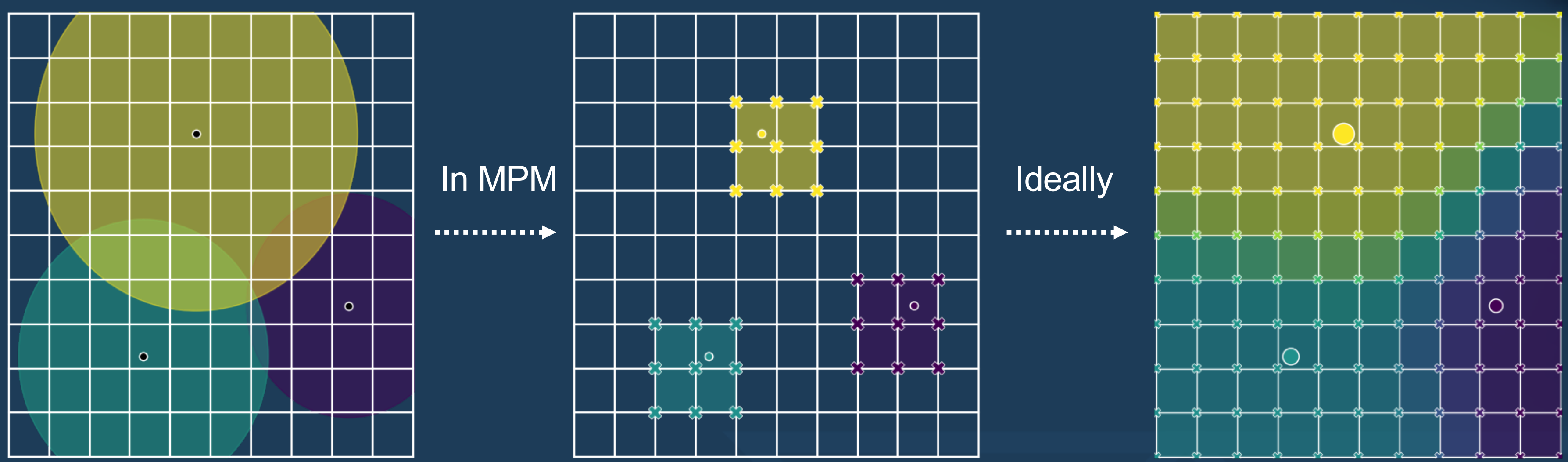
MOTIVATION



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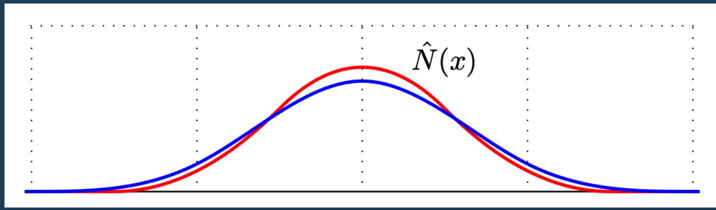
BACKGROUND: MLS-MPM [Hu et al. 2018]

Summary: MLS-MPM discretizes governing equations with *MLS shape function*

MLS-MPM approximates the continuous equation with $\mathbf{v}(\mathbf{x}) = \sum_i \Phi^i(\mathbf{x})\mathbf{v}_i,$

where MLS shape function $\Phi_i^n(\mathbf{x}) = w_{pi}^n \left(\mathbf{x} - \mathbf{x}_p^n\right)^T M^{-1} \left(\mathbf{x}_i - \mathbf{x}_p^n\right),$

If we chose weighting function w_{pi}^n to be B-spline, MLS-MPM is equivalent to APIC/PolyPIC



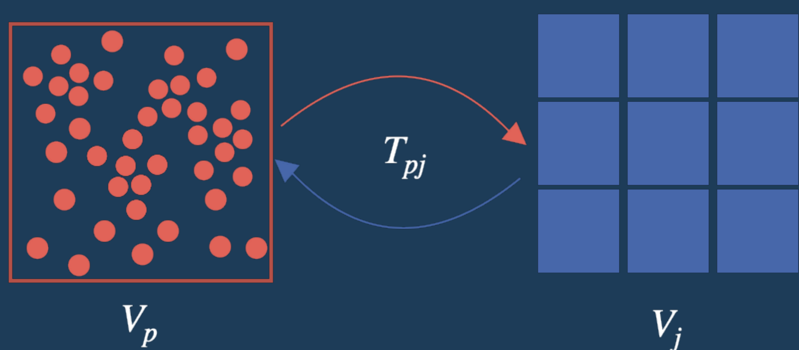
Cubic(blue) and quadratic(red) B-spline, [Jiang et al. 2016]

B-spline properties:

- Compact/fixed support
- Partition of unity
- Ease of computation

BACKGROUND: Power PIC [Qu et al. 2022]

Summary: reformulate particle-grid transfer as a *regularized optimal transport problem*



Positive, partition of unity

Volume aware

Discretized regularized power diagram

Only tested in fluid solver

Power Kernel $\chi_p^\epsilon(x_j) = T_{pj} / \sum_q T_{qj} = T_{pj} / V_j$

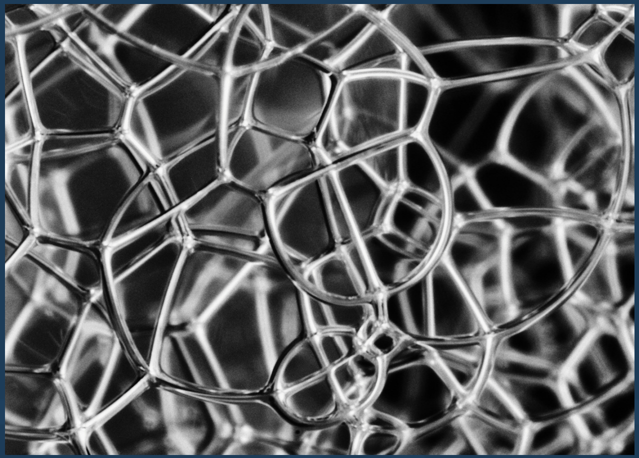
Can we bring power kernel to MPM?

BACKGROUND: Bubble/Foam simulation

Target: volume-varying mesoscale dry bubbles/foam reached Plateau's equilibrium

Plateau's Law

- Soap films are made of entire (unbroken) smooth surfaces
- The mean curvature of a portion of a soap film is everywhere constant on any point on the same piece of soap film.
- Soap films always meet in threes at angle of arccos(−1/2) = 120°
- These Plateau borders meet in fours at angle of arccos(−1/3) ≈ 109.47°

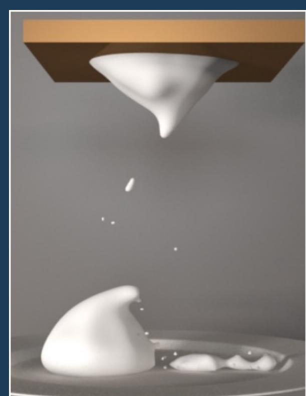


Bubbles in a foam of soap, wikipedia

BACKGROUND: Bubble/Foam simulation

Target: volume-varying mesoscale dry bubbles/foam reached Plateau's equilibrium

[Busaryev et al. 2012]



[Yue et al. 2015]

	Geometry	Versatility	Generalizability
[Busaryev et al]	👍👍👍	👍	👍
[Yue et al]	👍	👍👍	👍👍
Ours	👍👍👍	👍👍👍	👍👍

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CONSTITUTIVE MODEL

Assumption: our mesoscale bubbles can be still modeled as a continuum

Bubble/Foam: Herschel-Bulkley Model [Yue et al. 2015]

Fluid: compressible fluid

$$\psi(F) = -\frac{\lambda}{2}(J-1)^2,$$

Sand: St. Venant-Kirchhoff (StVK) [Klar et al. 2016]

$$\psi(F_E) = \mu \mathrm{tr}((\ln \Sigma)^2) + \frac{\lambda}{2}(\mathrm{tr}(\ln \Sigma))^2,$$

Bubble dynamics can be controlled with Herschel-Bulkley model

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METHODOLOGY: Discretization

Updated Lagrangian: deformation gradient update through

$$\mathbf{F}_p^{n+1} = \left(\mathbf{I} + \Delta t \frac{\partial \mathbf{v}^{n+1}}{\partial \mathbf{x}}(\mathbf{x}_p^n)\right)\mathbf{F}_p^n,$$

Similarly material velocity can be evaluated with MLS as

$$\mathbf{v}^{n+1}(\mathbf{x}) = \sum_i \Phi_i^n(\mathbf{x})\mathbf{v}_i^{n+1},$$

Where we use the MLS shape function

$$\Phi_i^n(\mathbf{x}) = w_{pi}^n \mathbf{P}^T(\mathbf{x} - \mathbf{c}_p^n)^T M^{-1}\mathbf{P}(\mathbf{x}_i - \mathbf{c}_p^n),$$

METHODOLOGY: Discretization

MLS shape function $\Phi_i^n(\mathbf{x}) = w_{pi}^n \mathbf{P}^T(\mathbf{x} - \mathbf{c}_p^n)^T M^{-1} \mathbf{P}(\mathbf{x}_i - \mathbf{c}_p^n),$

Differences compared with MLS-MPM

- MLS shape function is centered at *centroid* $\mathbf{c}_p = \frac{1}{\sqrt{V_p}} \int_\Omega \chi_p^\varepsilon(\mathbf{x}) d\mathbf{x} \approx \frac{1}{\sqrt{V_p}} \sum_j T_{pj} \mathbf{x}_j,$
- Weighting function use *Power Weights* $w_{pi} = \frac{1}{\sqrt{V_p}} \int_\Omega \chi_p^\varepsilon(\mathbf{x}) N_i(\mathbf{x}) d\mathbf{x} \approx \frac{1}{\sqrt{V_p}} \sum_j T_{pj} N_i(\mathbf{x}_j),$

METHODOLOGY: Discretization

Velocity gradient $\frac{\partial \mathbf{v}^{n+1}}{\partial \mathbf{x}}(\mathbf{x}_p^n) = \mathbf{A}_p^{n+1} = \mathbf{B}_p^{n+1}\left(\mathbf{D}_p^n\right)^{-1},$ Power APIC [Qu et al. 2022]

Deformation gradient update $\mathbf{F}_p^{n+1} = \left(\mathbf{I} + \Delta t \mathbf{A}_p^{n+1}\right)\mathbf{F}_p^n,$

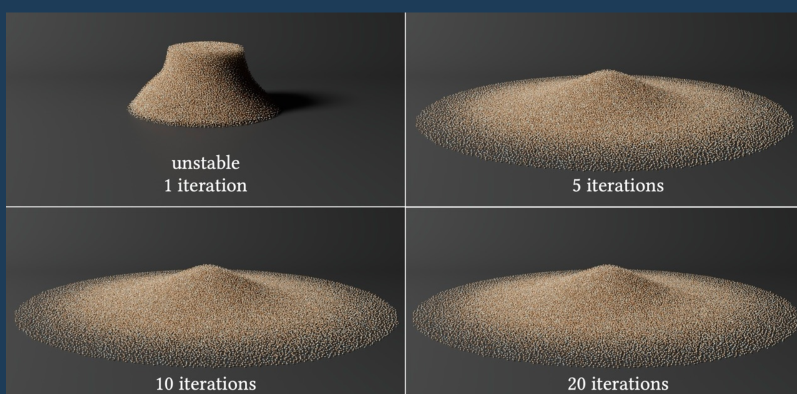
Volume update $V_p^{n+1} = \det(\mathbf{F}_p^{n+1}) V_p^0,$

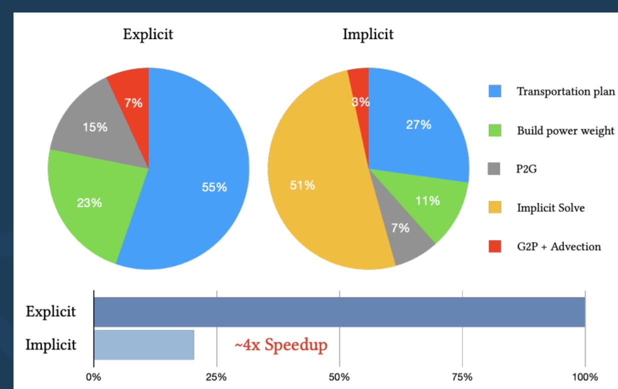
Force computation $\mathbf{f}_i = -\sum_p w_{pi} \mathbf{V}_0^p \mathbf{D}_p^{-1} \frac{\partial \Psi}{\partial \mathbf{F}_p}(\mathbf{F}_p^n)\mathbf{F}_p^{nT}(\mathbf{x}_i - \mathbf{c}_p^n).$

METHODOLOGY: Implicit Time Integrator

Gauss-Seidel Solver: like X-PBD [Macklin et al. 2016], we can define energy potential constraint per particle

$$V_0^p \Psi(\mathbf{F}_p^{n+1}) = \frac{1}{2\alpha}\left[C_p(\mathbf{F}_p^{n+1})\right]^2$$





METHODOLOGY: Algorithm

1. Compute transportation plan and power weights
2. Particle to grid transfer
3. Update grid momentum with implicit time integrator
4. Grid to particle transfer
5. Update deformation gradient, particle volume and plasticity projection
6. Particles advection

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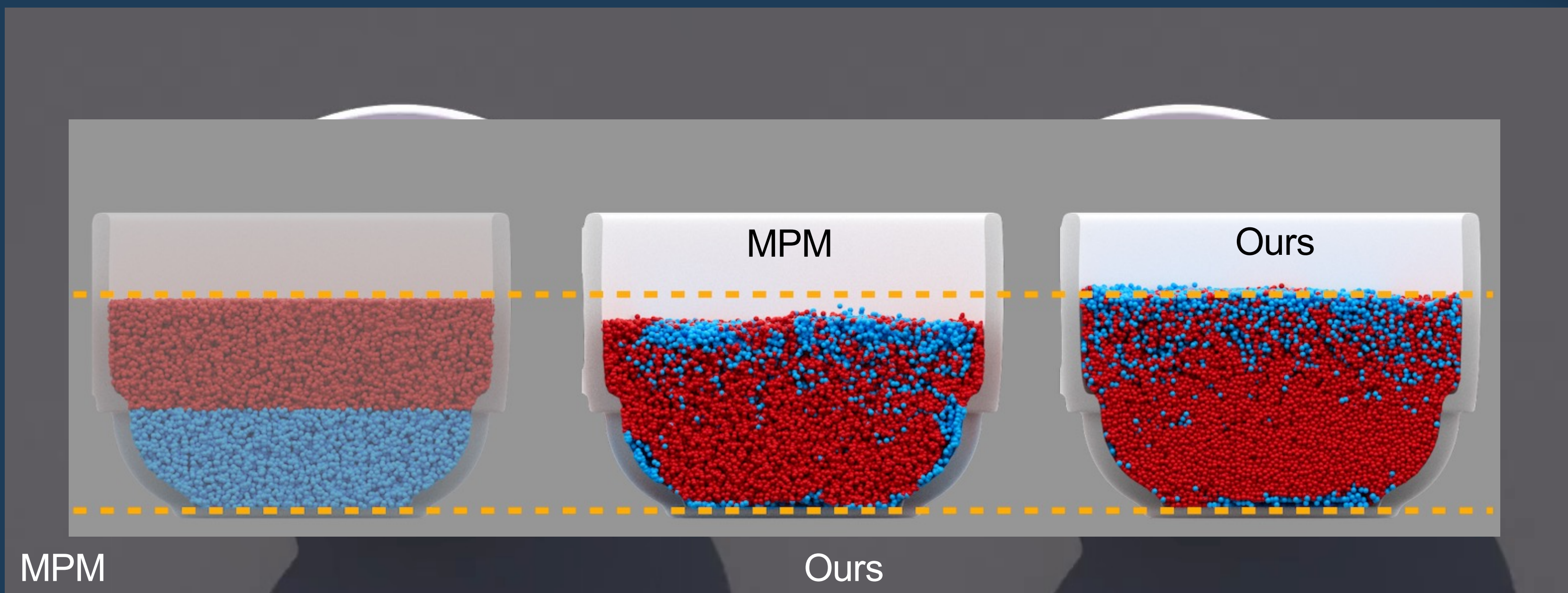
RESULTS:



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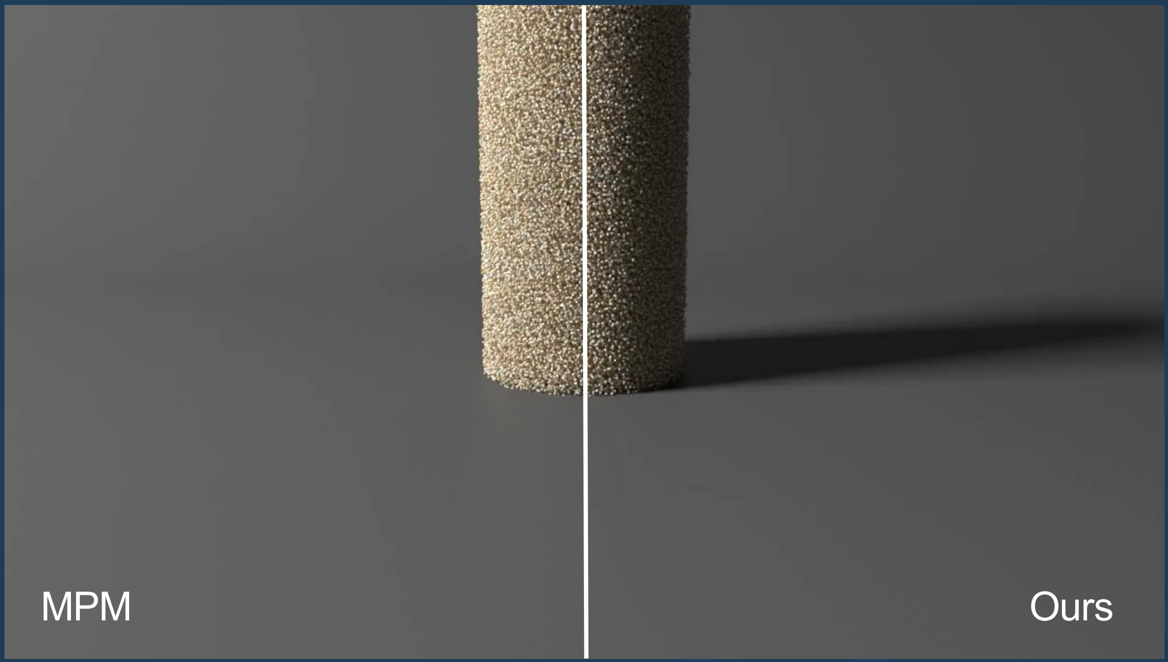

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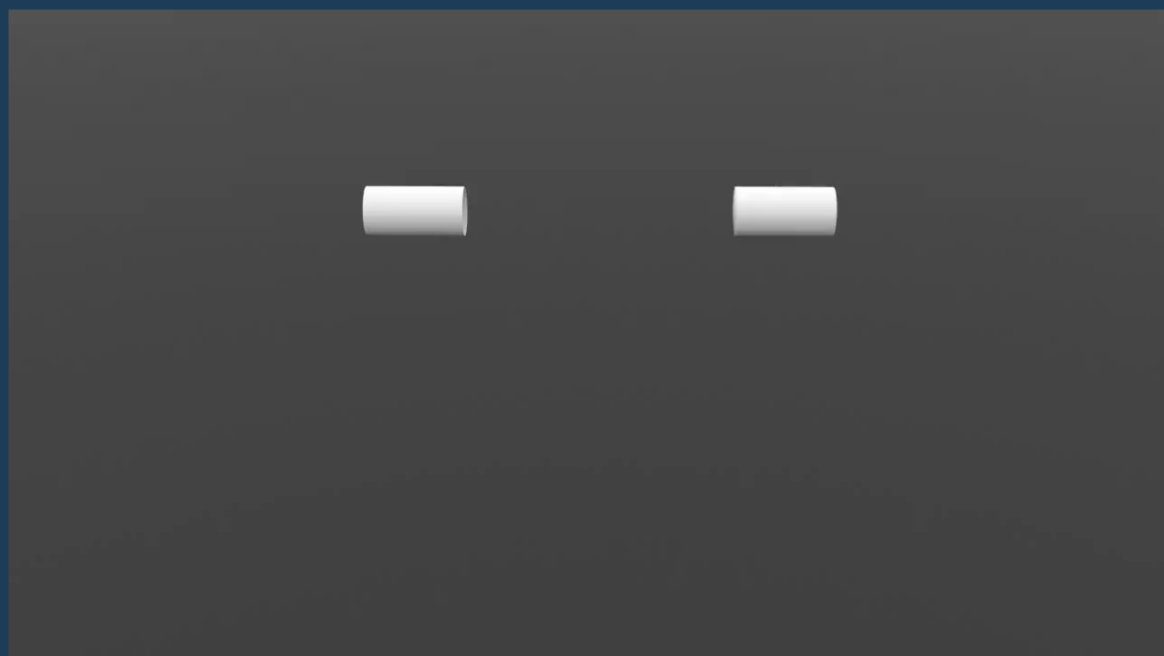
RESULTS:



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RESULTS:

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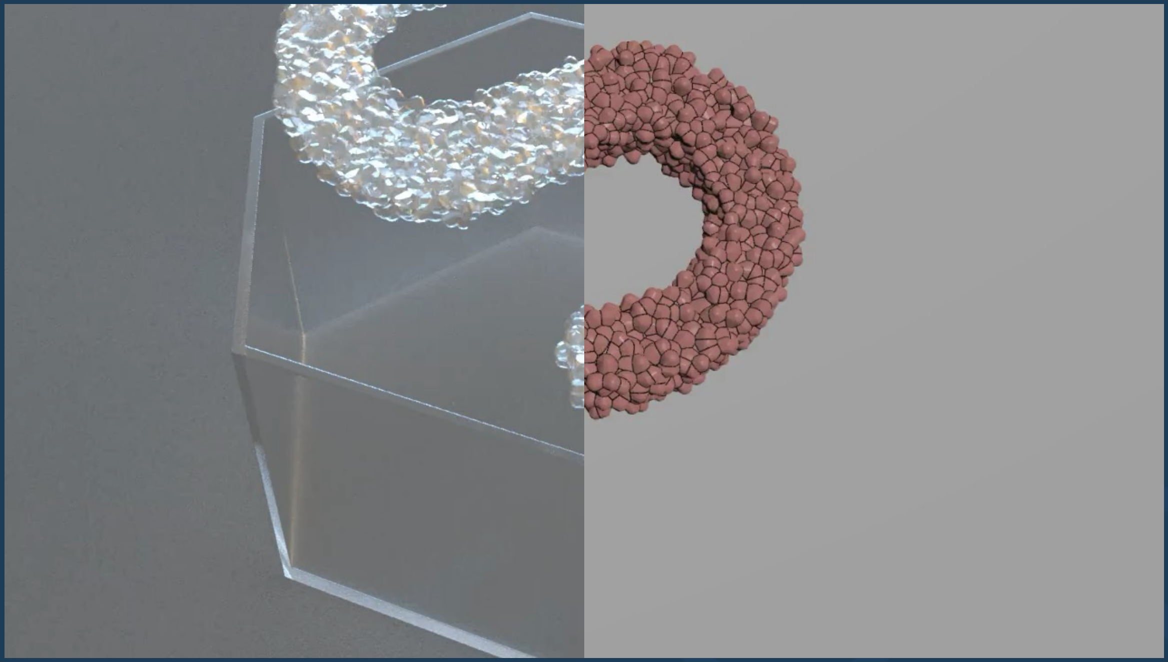

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RESULTS:

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SUMMARY:

- A hybrid Lagrangian/Eulerian solver capable of capturing the geometry of both macroscale and mesoscale materials
- We extend Power Particle-In-Cell Method with updated Lagrangian discretization of inelastic deformations
- We extend MLS-MPM incorporating power weights, removing any particle-per-cell restrictions
- An implicit solver like X-PBD for faster time integration of inelastic flows within a MPM simulation

LIMITATION:

- When there is a significant variation in the volume of particles, achieving effective load balancing can present difficulties
- Pure elastic material can show instability due to the centroid update
- Transportation plan and power weights computation is still bottleneck

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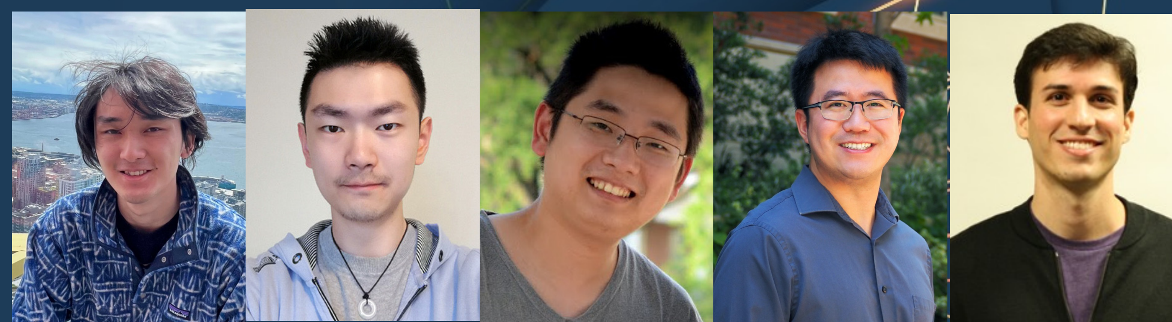
Thank you!



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