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Subspace-Preconditioned GPU Projective Dynamics with Contact for Cloth Simulation

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Introduction

Motivation

A fine and high-resolution discretization is often needed for rich and vivid effects like detailed wrinkles, folds, and creases.

Low-res Simulation

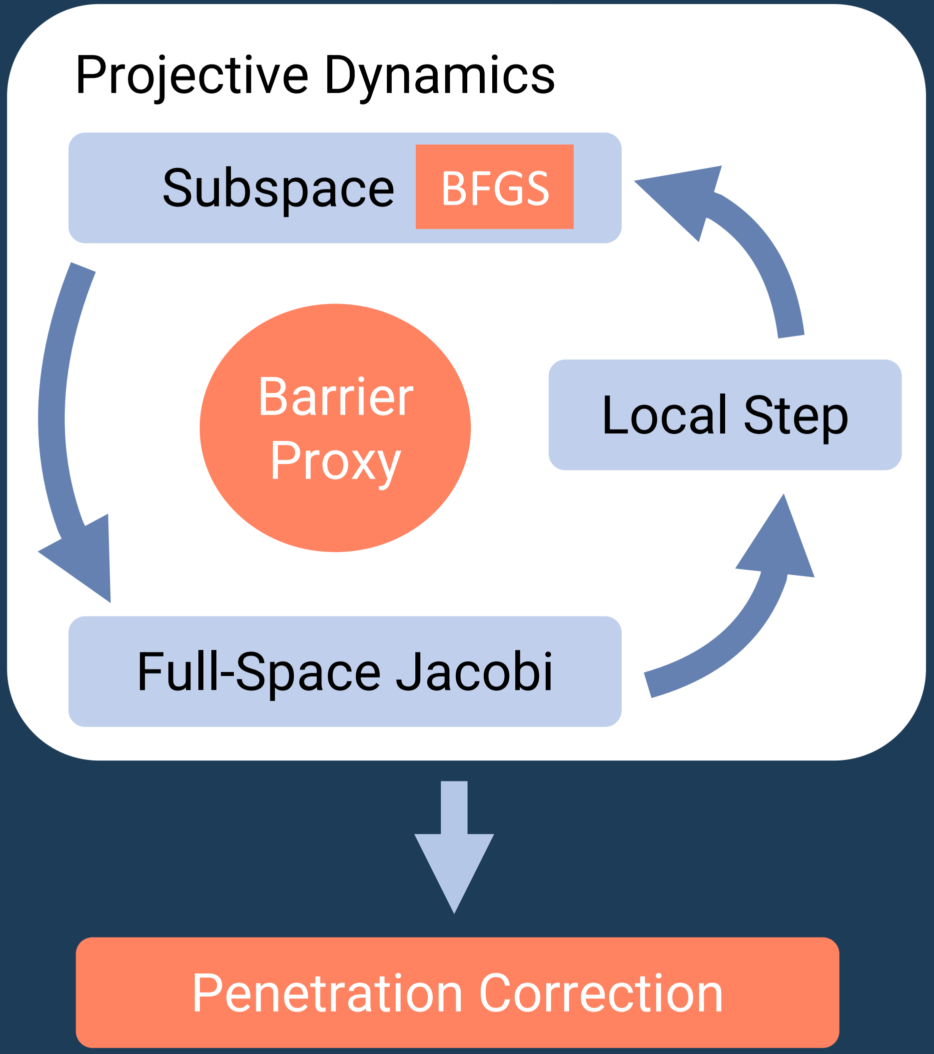
High-res Simulation

Challenges

- **Nonlinear cloth dynamics** require significant computation costs for high-resolution simulation.

- **Collisions and self-collisions** are ubiquitous in cloth simulation. The state-of-the-art contact modeling, Incremental Potential Contact (**IPC**) significantly increase the stiffness of the nonlinear cloth dynamics.

Overview



A novel subspace-preconditioned projective dynamics (PD) framework to accelerate cloth nonlinear dynamics.

- Low-frequency motion modes are captured within a designed subspace.
- High-frequency details are resolved by parallel Jacobi relaxation.

Adopt a time-splitting scheme [Xie et al. 2023] to accelerate contact handling.

- Subspace BFGS progressively integrates the quadratic contact proxy into the subspace matrix.
- Penetration issues are resolved through a penetration correction step.

An efficient GPU implementation

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Technical Details

Time Integration

Backward Euler Time Integration

$$\boldsymbol{M}(\boldsymbol{v}^{n+1} - \boldsymbol{v}^{n}) = h(-\nabla\Psi_{\text{mem}}(\boldsymbol{x}^{n+1}) - \nabla\Psi_{\text{bend}}(\boldsymbol{x}^{n+1}) + \boldsymbol{M}\boldsymbol{g})$$

$$\boldsymbol{x}^{n+1} = \boldsymbol{x}^{n} + h\boldsymbol{v}^{n+1}$$

$\tilde{\boldsymbol{x}} = \boldsymbol{x}^* + h\boldsymbol{v}^* + h^2\boldsymbol{g}$ - Predicted position under inertia

$\boldsymbol{F}_t$ - In-plane deformation gradient

$\boldsymbol{R}(\boldsymbol{F})$ - Closest isometry

Optimization Time Integration

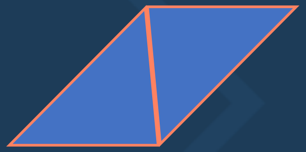
$$\underbrace{\min_{\boldsymbol{x}} \frac{1}{2h^2}\|\boldsymbol{x}-\tilde{\boldsymbol{x}}\|_M^2}_{\text{Inertia}} + \underbrace{\frac{E_{\text{mem}}}{2}\sum_t \|\boldsymbol{F}_t - \boldsymbol{R}(\boldsymbol{F}_t)\|^2}_{\text{ARAP Stretching}} + \underbrace{\frac{E_{\text{bend}}}{2}\sum_e \|\boldsymbol{x}\|_{Q_e}^2}_{\text{Quadratic Bending}}$$

Solved by Newton, which requires recomputing Hessian matrices

$$\boldsymbol{v}^{n+1} = \frac{1}{h}(\boldsymbol{x}^{n+1} - \boldsymbol{x}^{n})$$

Stretching stencil

Bending stencil



Projective Dynamics (PD)

Optimization Time Integration

$$\min_{x} \frac{1}{2h^2}\|\boldsymbol{x}-\tilde{\boldsymbol{x}}\|_M^2+\frac{E_{\text{mem}}}{2}\sum_t \|\boldsymbol{F}_t-\boldsymbol{R}(\boldsymbol{F}_t)\|^2+\frac{E_{\text{bend}}}{2}\sum_e \|\boldsymbol{x}\|_{Q_e}^2$$

Projective Dynamics [Bouaziz et al. 2014]

For k = 1, 2, 3, …

Constant

Global Step $$\min_{\boldsymbol{x}^k} \frac{1}{2h^2}\|\boldsymbol{x}^k-\tilde{\boldsymbol{x}}\|_M^2+\frac{E_{\text{mem}}}{2}\sum_t \|\boldsymbol{F}_t^k-\boldsymbol{R}_t^k\|^2+\frac{E_{\text{bend}}}{2}\sum_e \|\boldsymbol{x}^k\|_{Q_e}^2$$

Local Step $$\boldsymbol{R}_t^{k+1}=\boldsymbol{R}(\boldsymbol{F}_t^k)$$

Solving Full-Order PD

- The global step is a quadratic problem with a fixed system hessian

$$\min_{\boldsymbol{x}^k} \frac{1}{2h^2}\|\boldsymbol{x}^k - \tilde{\boldsymbol{x}}\|_M^2 + \frac{E_{\text{mem}}}{2}\sum_t \|\boldsymbol{F}_t^k - \boldsymbol{R}_t^k\|^2 + \frac{E_{\text{bend}}}{2}\sum_e \|\boldsymbol{x}^k\|_{Q_e}^2$$

Subject to solving a linear system:

$$\Rightarrow \boldsymbol{H}\boldsymbol{u}^k = \boldsymbol{b}^{k-1} \qquad \boldsymbol{u}^k = \boldsymbol{x}^k - \boldsymbol{x}^{k-1}$$

Current Residual New increment

System matrix can be prefactorized if it is small

Otherwise, a fixed number of Jacobi iterations are applied

- The local step can be executed on triangles in parallel
- The global-local alternation can be accelerated by Chebyshev acceleration [Wang 2015]

Subspace PD

Rationale

- Decrease #DOF for efficient prefactorization.
- Low-frequency modes dominate the overall motions.

Subspace Solve for Global Step

$\mathcal{P}$ - Basis matrix: each column is one basis for the displacement

Subspace solve: $$\min_{\boldsymbol{y}^k} \frac{1}{2h^2}\|\boldsymbol{x}^k - \tilde{\boldsymbol{x}}\|_M^2 + \frac{E_{\mathrm{mem}}}{2}\sum_t \|\boldsymbol{F}_t^k(\boldsymbol{x}^k) - \boldsymbol{R}_t^k\|^2 + \frac{E_{\mathrm{bend}}}{2}\sum_e \|\boldsymbol{x}^k\|_{Q_e}^2, \quad \text{s.t. } \boldsymbol{x}^k - \boldsymbol{x}^{k-1} = \mathcal{P}\boldsymbol{y}^k$$

Linear form: $$\underbrace{\mathcal{P}^T \boldsymbol{H}\mathcal{P}}_{\text{Can be prefactorized}}\boldsymbol{y}^k = \mathcal{P}^T \boldsymbol{b}^{k-1} \qquad \boldsymbol{u}^k = \mathcal{P}\boldsymbol{y}^k = \boldsymbol{x}^k - \boldsymbol{x}^{k-1}$$

Can be prefactorized

B-Spline Subspace for Cloth

Cloth has sewing patterns. Each patch has a UV-plate.

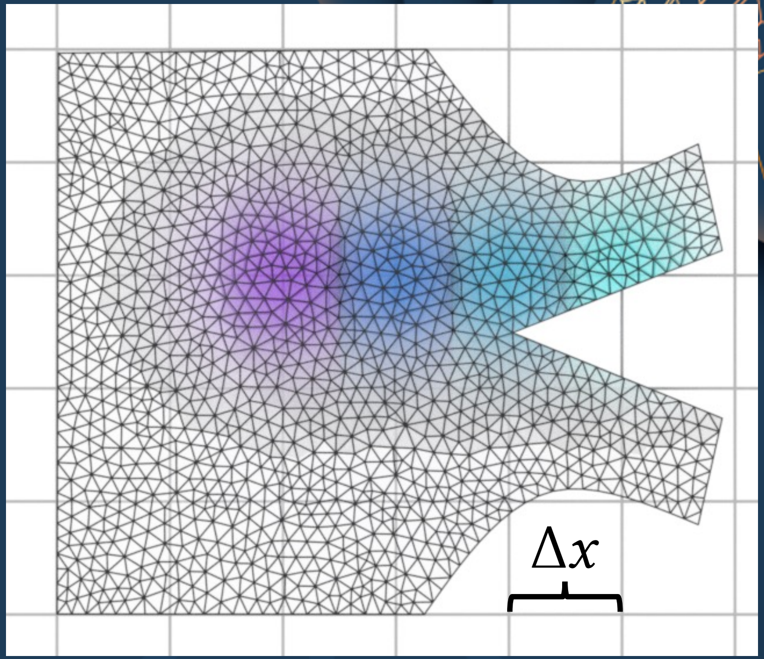
We only need a basis parameterized on 2D plane.

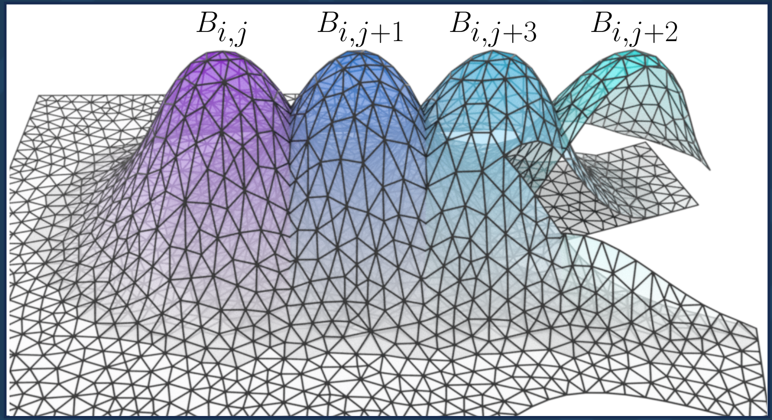
$$B_{ij}(X) = N(u/\Delta x - i)N(v/\Delta x - j)$$

$$N(x) = \begin{cases} \frac{3}{4} - x^2, & |x| < \frac{1}{2}, \\ \frac{1}{2}(\frac{3}{2} - |x|)^2, & \frac{1}{2} \le |x| < \frac{3}{2}, \\ 0, & \frac{3}{2} \le |x|. \end{cases}$$ Quadratic B-spline

$$\mathcal{P} = \mathcal{B} \otimes \mathcal{I}_3$$ Boost 1D basis to 3D basis

Partition of Unity is naturally satisfied.





Subspace PD Simulation

Partition of Unity is important

Ours

[Brandt et al. 2018]

Subspace-Preconditioned PD

Subspace cannot resolve high-frequency features, such as wrinkles.

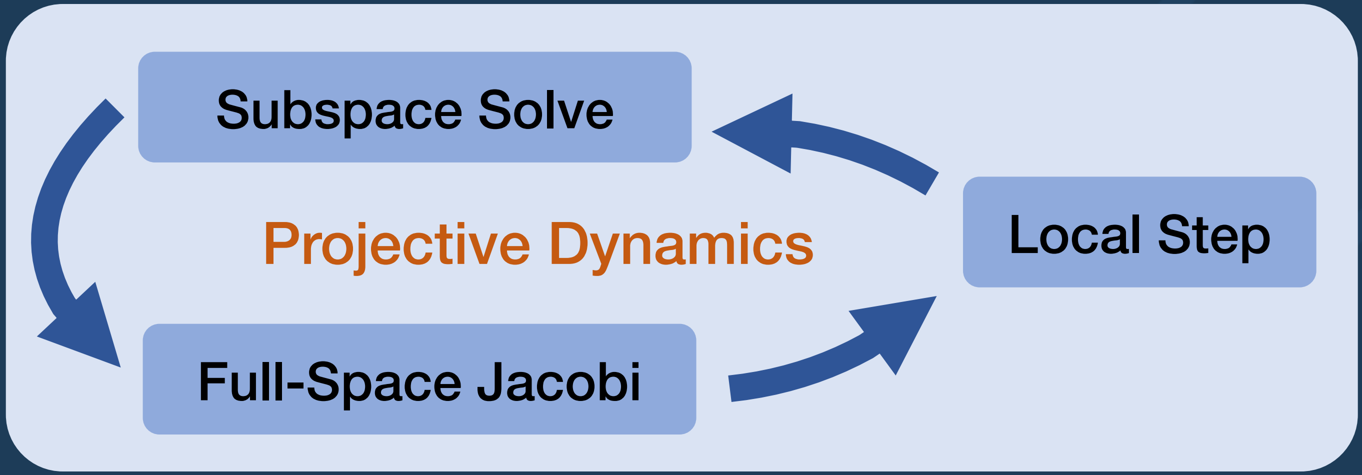
Need full-order relaxation to reduce high-frequency residual

Original global step $$\min_{\boldsymbol{x}^k} \frac{1}{2h^2}\|\boldsymbol{x}^k - \tilde{\boldsymbol{x}}\|_M^2 + \frac{E_{\text{mem}}}{2}\sum_t \|\boldsymbol{F}_t^k - \boldsymbol{R}_t^k\|^2 + \frac{E_{\text{bend}}}{2}\sum_e \|\boldsymbol{x}^k\|_{\boldsymbol{Q}_e}^2$$

Step I: Subspace Solve $$\mathcal{P}^T \boldsymbol{H} \mathcal{P} \boldsymbol{y}^k = \mathcal{P}^T \boldsymbol{b}^{k-1} \qquad \boldsymbol{u}^{k,0} = \mathcal{P}\boldsymbol{y}^k$$

Step II: Jacobi Relaxation $$\boldsymbol{u}^{k,j} = \boldsymbol{u}^{k,j-1} + \omega\, \text{diag}(\boldsymbol{H})^{-1}(\boldsymbol{b}^{k-1} - \boldsymbol{H}\boldsymbol{u}^{k,j-1})$$ For j = 1, 2, 3, …, N

$$\boldsymbol{u}^{k,N} = \boldsymbol{x}^k - \boldsymbol{x}^{k-1}$$



Subspace-Preconditioned PD Simulation

Ours	[Wang 2015]
3.0s/frame	26.2s/frame

Time-Splitting for Contact

Contact is ubiquitous in cloth motions

Codimensional Incremental Potential Contact (CIPC) [Li et al. 2021]:

$$M\frac{\boldsymbol{x}-\tilde{\boldsymbol{x}}}{h} = h(\underbrace{-\nabla\Psi^E(\boldsymbol{x})}_{\text{Stretching + Bending}} - \underbrace{\nabla B(\boldsymbol{x})}_{\text{Contact barrier energy}})$$

Stretching + Bending

Contact barrier energy
Guarantee penetration-free by working with CCD

Splitting with Quadratic Proxy [Xie et al. 2023]:

Elasticity Step

$$M\frac{\hat{\boldsymbol{x}}-\tilde{\boldsymbol{x}}}{h} = h\left(-\nabla\Psi^E(\hat{\boldsymbol{x}}) - \underbrace{\left(\nabla^2 B(\boldsymbol{x}^*)(\hat{\boldsymbol{x}}-\boldsymbol{x}^*) + \nabla B(\boldsymbol{x}^*)\right)}_{\text{Quadratic Proxy}}\right)$$

Quadratic Proxy

Penetration Correction Step

$$M\frac{\boldsymbol{x}^{n+1}-\hat{\boldsymbol{x}}}{h} = -h\nabla B(\boldsymbol{x}^{n+1})$$

$$\Rightarrow \quad \min_{\boldsymbol{x}} \frac{1}{2h^2}\|\boldsymbol{x}-\hat{\boldsymbol{x}}\|_{\boldsymbol{M}}^2 + B(\boldsymbol{x})$$

Optimized by Newton PCG

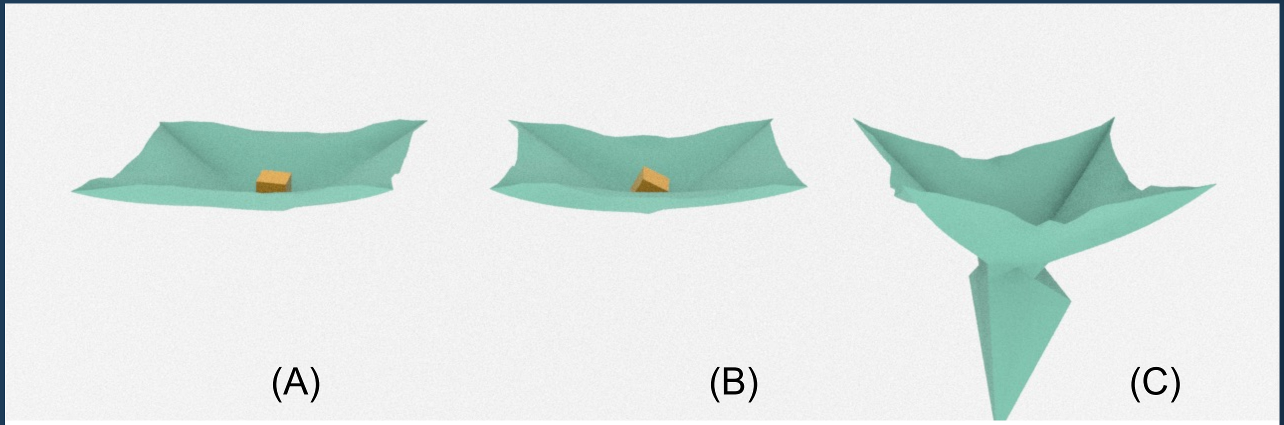
Splitting with Quadratic Proxy [Xie et al. 2023]

Momentum Equation

$$M(v^{n+1} - v^n) = h(f^{E,n+1} + f^{C,n+1})$$

Naive Splitting

$$M(\tilde{v} - v^n) = hf^{E,n+1}$$
$$M(v^{n+1} - \tilde{v}) = hf^{C,n+1}$$



Splitting with Contact Proxy

$$M(\tilde{v} - v^n) = h(f^{E,n+1} + \frac{1}{2}\tilde{f}^C)$$
$$M(v^{n+1} - \tilde{v}) = h(f^{C,n+1} - \frac{1}{2}\tilde{f}^C) \approx \frac{1}{2}hf^{C,n+1}$$

Global Step with Contact Proxy

Recall: Without contact, the global step is a linear system $\boldsymbol{H}\boldsymbol{u}^k = \boldsymbol{b}^{k-1}$

Goal: $(\boldsymbol{H} + \nabla^2 B(\boldsymbol{x}^*))\boldsymbol{u}^k = \hat{\boldsymbol{b}}^{k-1} = \boldsymbol{b}^{k-1} - \left(\nabla^2 B(\boldsymbol{x}^*)(\boldsymbol{x}^{k-1} - \boldsymbol{x}^*) + \nabla B(\boldsymbol{x}^*)\right)$

Residual w.o. contact

Subspace Solve: $(\mathcal{P}^T \boldsymbol{H} \mathcal{P} + \mathcal{P}^T \nabla^2 B(\boldsymbol{x}^*)\mathcal{P})\boldsymbol{y}^k = \mathcal{P}^T \hat{\boldsymbol{b}}^{k-1}$

Prefectorized — Constant only in the same time step

BFGS

(avoid refactorization)

Initial Hessian: $\mathcal{P}^T \boldsymbol{H} \mathcal{P}$

Low-rank updates to approximate: $\mathcal{P}^T \nabla^2 B(\boldsymbol{x}^*)\mathcal{P}$

Shared low-rank updates across all global steps

Jacobi Relaxation: $\boldsymbol{H} + \nabla^2 B(\boldsymbol{x}^*)$

Full Algorithm

Algorithm 1 Timestepping of subspace-preconditioned PD

if it is the first time step **then**
  Construct subspace basis sparse matrix $\mathcal{P}$. ▷ Sec. 4.2
  Factorize the reduced-order global matrix $\mathcal{P}^\top \boldsymbol{H}\mathcal{P}$. ▷ Sec. 4.3
end if
Update predictive position $\tilde{\boldsymbol{x}}$.
Run a reduced-order global step w.o. contact for an initial guess.
Construct quadratic barrier proxy at current state $\boldsymbol{x}^*$.
Initialize subspace BFGS history.
while not converged **do** ▷ Sec. 4.4.1
  Run 2 iterations of subspace BFGS and update the history.
  Run 5 fullspace Jacobi iterations.
  Run PD local projections in parallel.
end while
Run penetration correction step. ▷ Sec. 4.4.2

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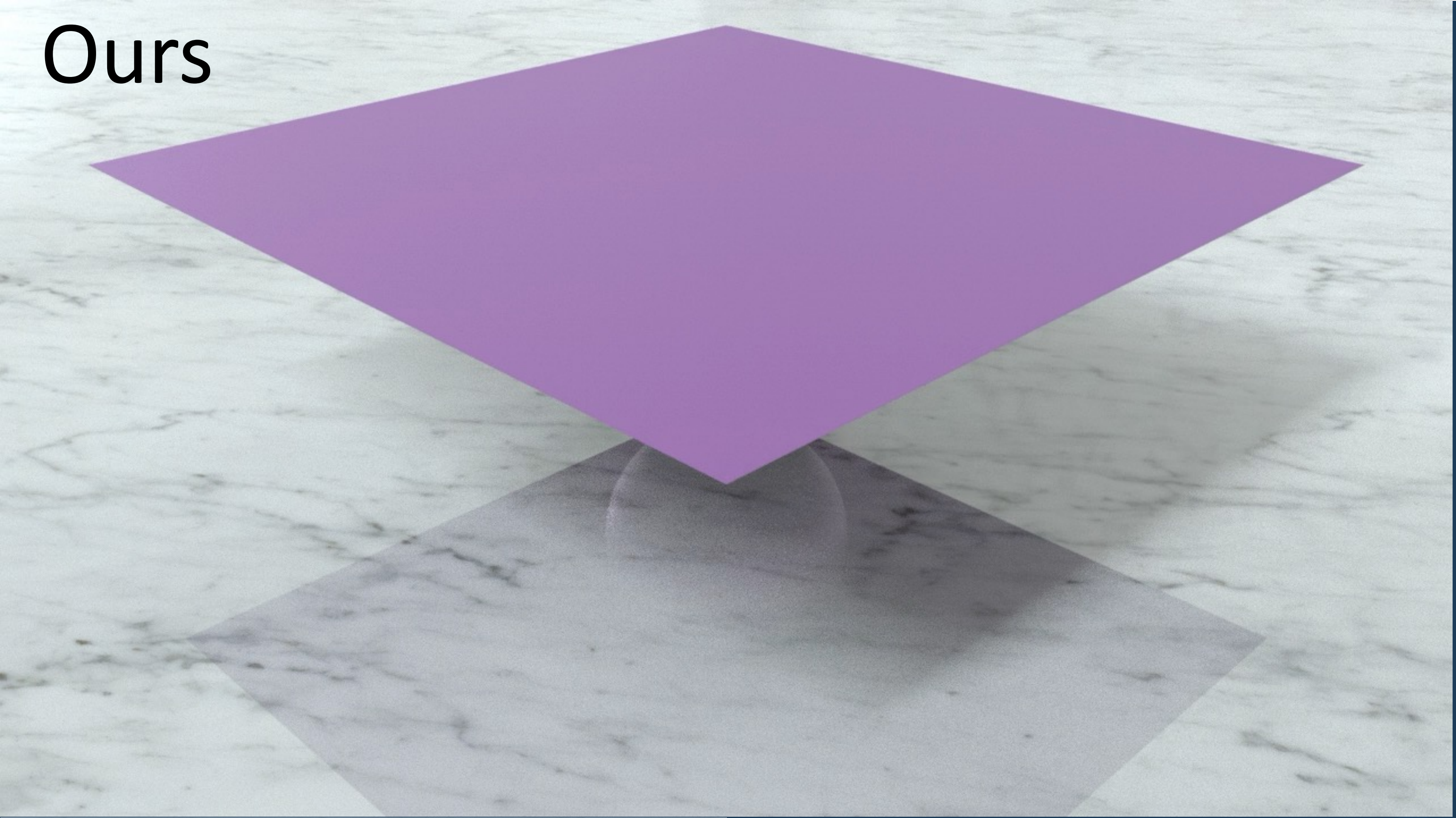
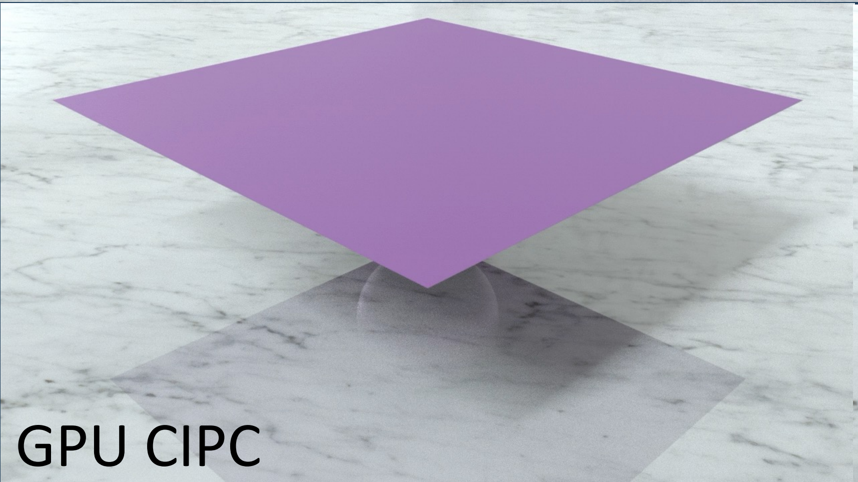
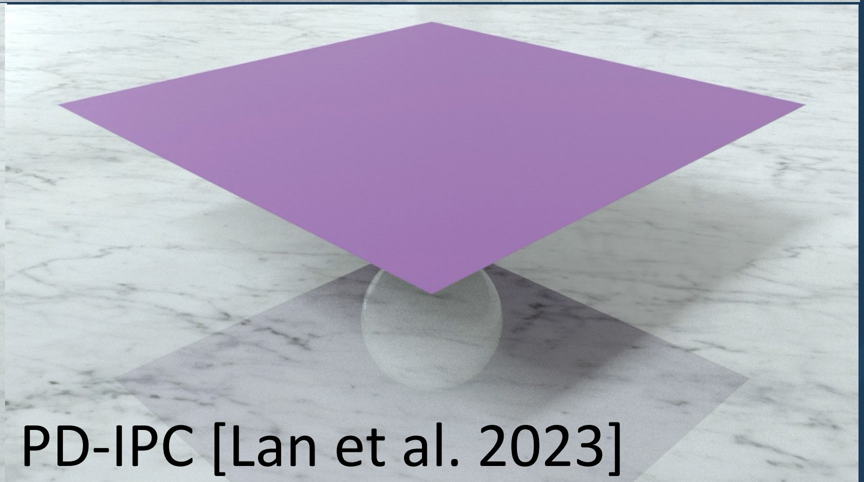
Comparisons



Ours



GPU CIPC



PD-IPC [Lan et al. 2023]

#V: 252K

#Basis: 8427

Ours: 27s/frame

GPU CIPC: 241s/frame

PD-IPC: 49s/frame

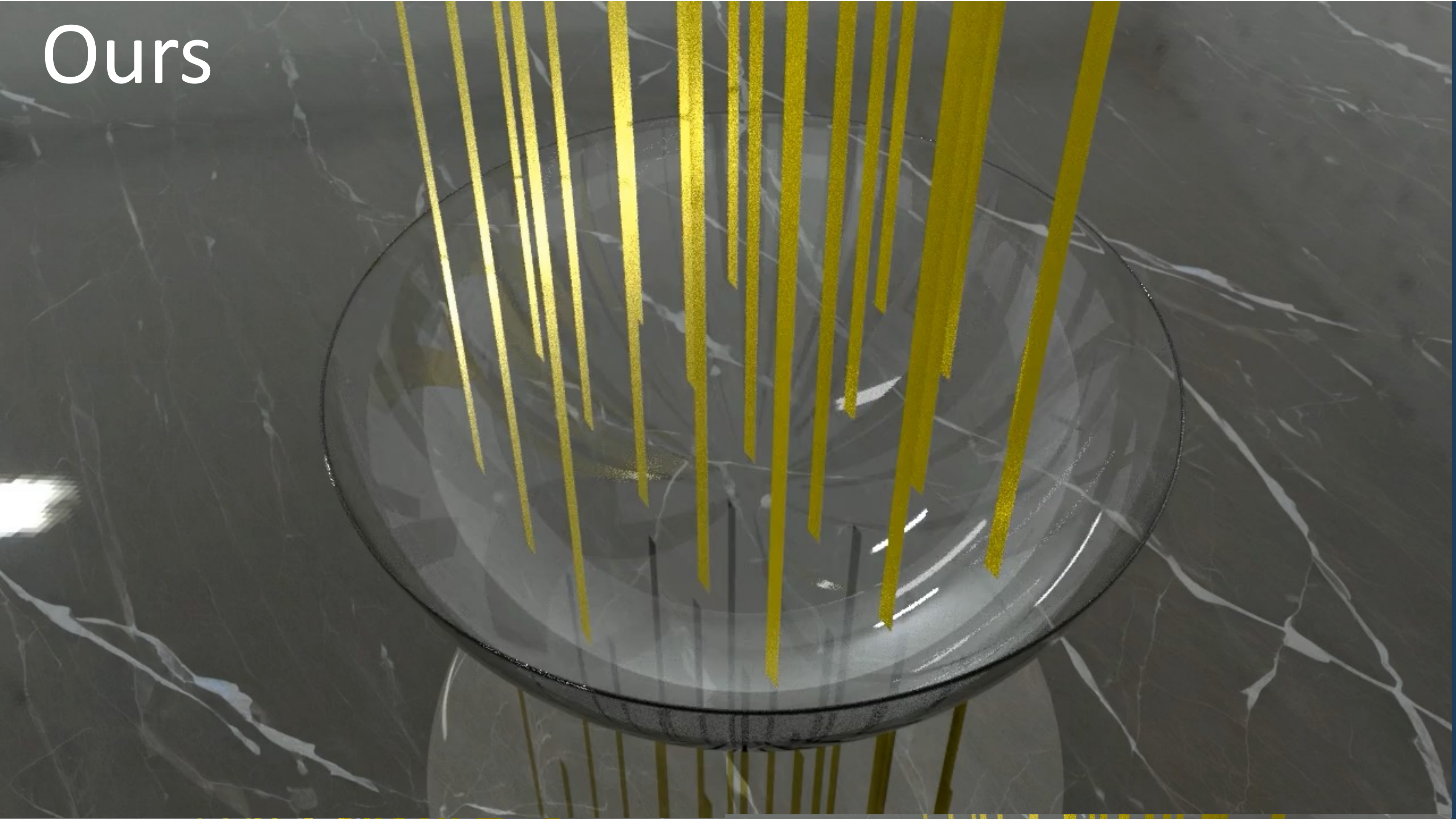
#V: 104K

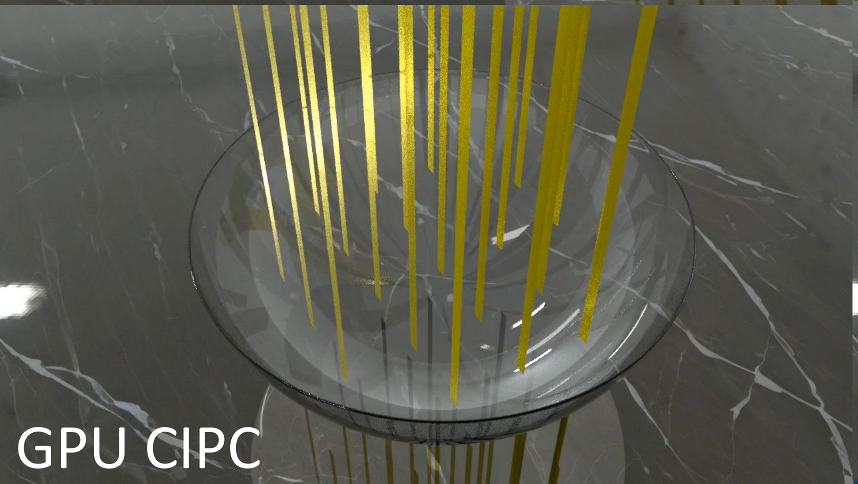
#Basis: 7665

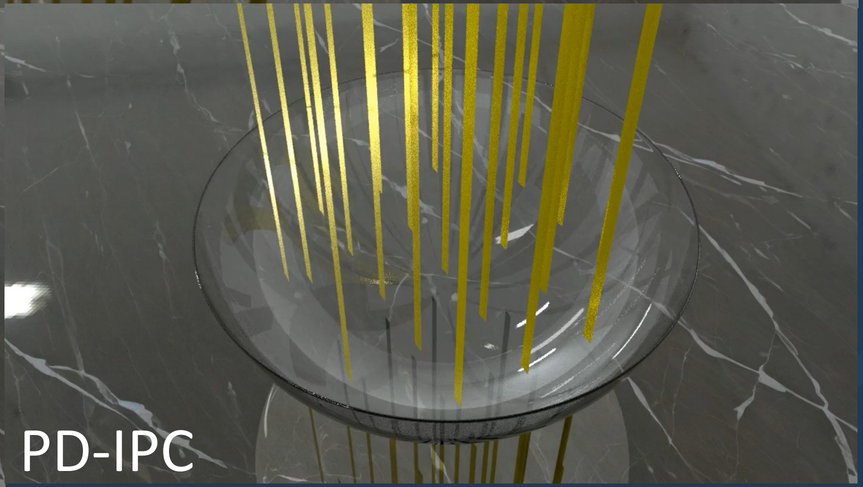
Ours: 10s/frame

GPU CIPC: 90s/frame

PD-IPC: 18s/frame







#V: 285K

#Basis: 7119

Ours: 46s/frame

GPU CIPC: 1033s/frame

PD-IPC: 134s/frame

Ours

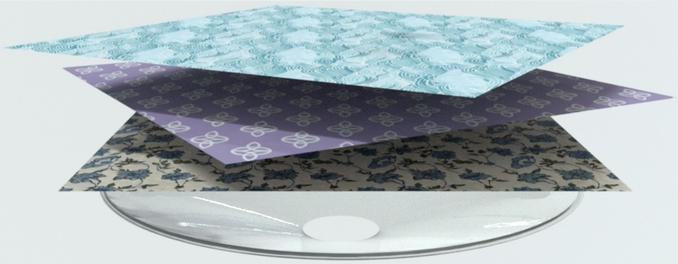
#V: 232K

#Basis: 9432

Ours: 23s/frame

GPU CIPC: 362s/frame

PD-IPC: 253s/frame



GPU CIPC

PD-IPC


Ours
Ours: 23s/frame
GPU CIPC: 150s/frame
GPU CIPC

Conclusion

Conclusion

- An efficient GPU cloth simulation method based on the projective dynamics (PD) framework.
- Combination of subspace integration and full-space relaxation effectively reduces both high-frequency and low-frequency residuals.
- Seamlessly integrate with IPC to ensure penetration-free in a time-splitting manner.
- Significant performance improvements over existing GPU solvers for high-resolution cloth simulation.

- Under high speed, the time splitting error can lead to damping effects.
  - ➢ Adaptive substepping
- Newton's method in the penetration correction step may have overshooting problems.
  - ➢ A dedicated solver for penetration correction

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Thank you!