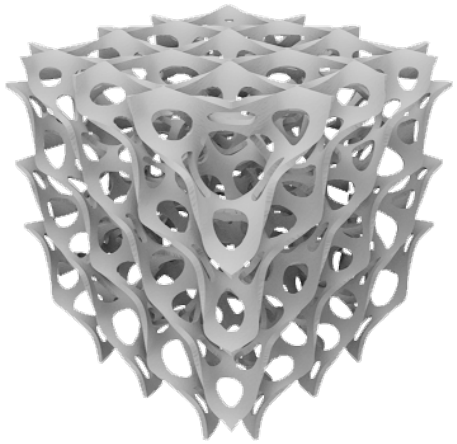
山东大学 计算机科学与技术学院
School of Computer Science and Technology, Shandong University

一种三周期极小曲面几何微结构的智能设计方法

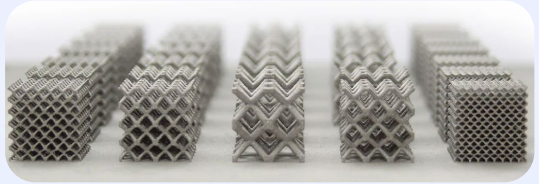
报告人：吕 琳
研究方向：计算机图形学、增材制造

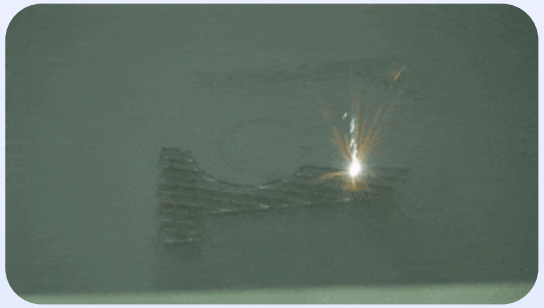


几何微结构

增材制造最大优势：

复杂结构的加工成型能力

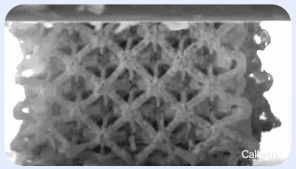


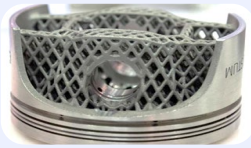


几何微结构

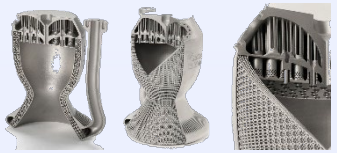
典型的复杂结构

- 轻量
- 减振
- 散热
- 吸能
- 降噪
- …



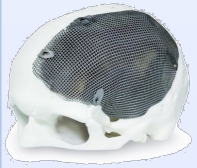


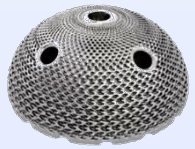
高性能活塞



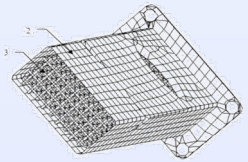
火箭发动机

应用于航空航天、交通运输、医疗植入体等重要领域，以提升关键零部件的综合物理性能





医疗植入体



汽车吸能盒

增材制造

专用软件

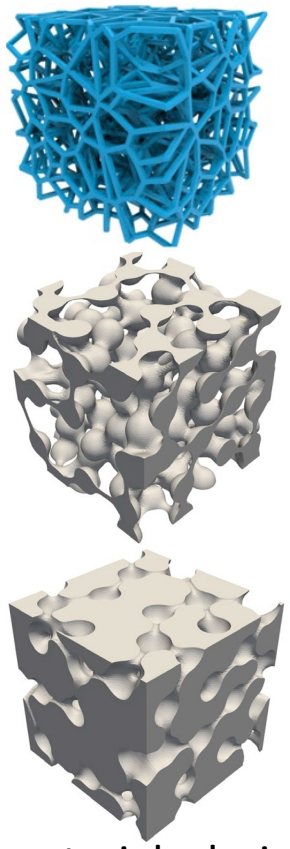
- 设计效率
- 产品功能
- 加工质量

基于微结构的设计制造专用软件将为智能制造基础软件体系补齐重要短板。

几何微结构 (超材料/晶格结构)

- **几何微结构** — 相对于模型整体而言尺寸非常小的几何结构，通常可以在小范围的几何空间内充分定义其形状，并采用周期性密铺等方式填充整体模型。(Gibson & Ashby, 1999)

Stochastic foams



Truss-based lattices

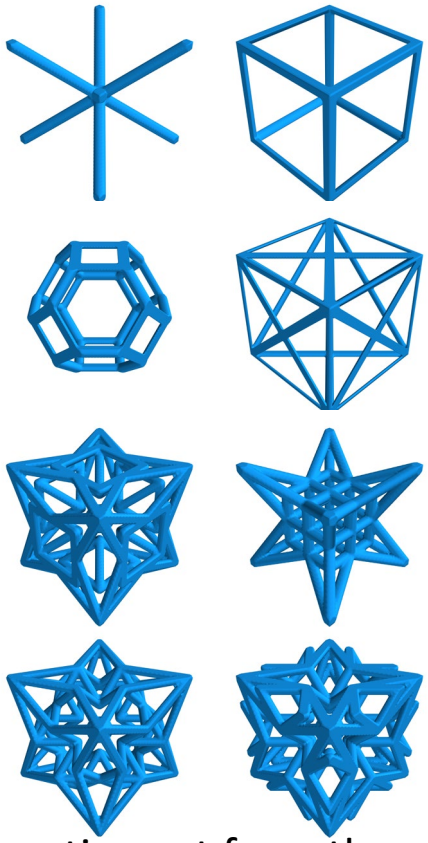
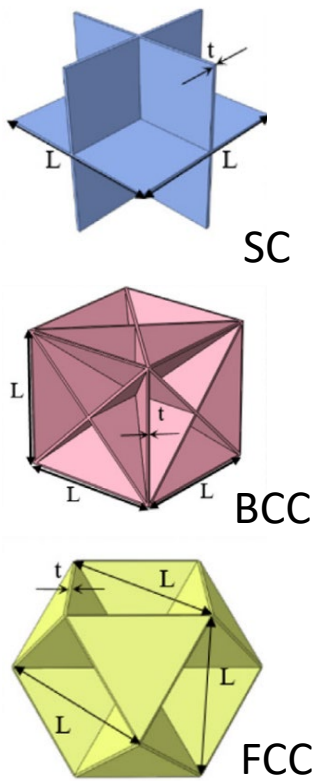
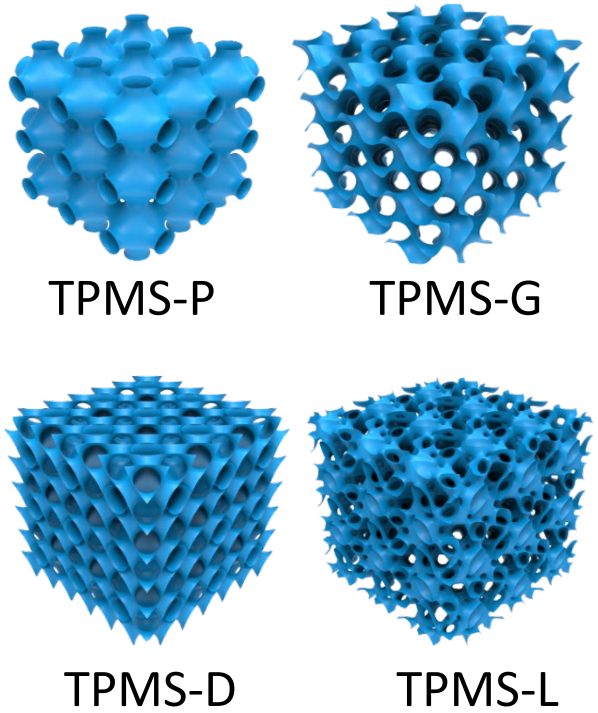


Plate-based lattices



Shellular lattices



- ✓ Metamaterials derive their properties not from the properties of the base materials, but from their newly designed structures.
- ✓ The precise shape, geometry, size, orientation and arrangement gives metamaterials smart properties that go beyond what is possible with conventional materials.

几何微结构

2023 年 5 月 CHINESE JOURNAL OF COMPUTERS May 2023

单材料几何微结构设计研究综述

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摘 要 增材制造(3D 打印)技术的快速发展促进了复杂几何结构在航空航天、辅助医疗、交通运输等工业生产中的应用.在自然界中广泛存在并具有优异物理性能的几何微结构可以通过 3D 打印的方式进行制备.如何设计几何微结构以达到特定物理性能目标的问题已成为计算机辅助设计、计算机辅助工程、机械工程与材料科学学科交叉的一项研究热点.本文对近年来面向单材料 3D 打印的几何微结构设计工作进行综述,系统地梳理了几何微结构的主要设计方法.首先,本文从几何约束和物理目标两个角度介绍了几何微结构的设计要求;随后将几何微结构单元的设计方法从优化方法、参数化方法和过程式方法三个类型分别进行了详细阐述与分析;此外,本文也对几何微结构单元在给定形状空间内的合成方法进行了总结.最后,本文对目前几何微结构设计研究中待解决的问题进行了讨论,并展望了可能的发展方向.

关键词 几何微结构;结构设计;结构优化;3D 打印
中图法分类号 TP391 **DOI 号** 10.11897/SP.J.1016.2023.00960

Recent Progress on Geometric Microstructure Design: A Survey

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Abstract Living creatures interact with the natural environment in the long process of evolution and have high adaptability to the environment. Through natural selection, various microstructures with complex shapes and excellent properties have been formed. These microstructures have a series of unique physical properties. In recent years, microstructures have also received increasing attention in industrial manufacturing, tissue engineering, and biomedicine. Inspired by microstructures in nature, bionics is based on the principles of engineering mechanics to simulate and optimize the geometrical expression, structural combination, and physical properties of objects' microstructures. The shape of microstructures fabricated based on traditional manufacturing methods is relatively simple, but the emergence of additive manufacturing (also known as 3D printing) has broken this limitation. The rapid development of additive manufacturing technology has promoted the application of complex geometric structures in industrial production such as aerospace, auxiliary medical care, and transportation. Geometric microstructures that are widespread in nature and have excellent physical properties can be prepared by 3D printing. The geometric

收稿日期:2021-12-09;在线发布日期:2022-10-09.本课题得到国家自然科学基金(61972232,62025207)资助.**田李昊**,博士研究生,中国计算机学会(CCF)学生会员,主要研究方向为计算机图形学、数字几何处理.E-mail:tianlihao0420@gmail.com.**吕 琳**(通信作者),博士,教授,博士生导师,中国计算机学会(CCF)高级会员,主要研究领域为计算机图形学、数字几何处理和计算制造.E-mail:llu@sdu.edu.cn.**彭 昊**,博士研究生,中国计算机学会(CCF)学生会员,主要研究领域为计算机图形学与计算制造.**刘培庆**,博士研究生,中国计算机学会(CCF)学生会员,主要研究领域为计算机图形学与计算制造.**闫 鑫**,博士,中国计算机学会(CCF)学生会员,主要研究领域为计算机图形学与计算制造.**刘利刚**,博士,教授,博士生导师,中国计算机学会(CCF)高级会员,主要研究领域为计算机图形学.**屠长河**,博士,教授,博士生导师,中国计算机学会(CCF)高级会员,主要研究领域为计算机图形学.

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三周期极小曲面结构设计及应用综述

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摘 要: 三周期极小曲面是一种隐函数表示的代数曲面,具有许多优良的性质.增材制造技术的快速发展极大地增强了复杂几何与拓扑结构的制造能力,三周期极小曲面作为几何建模工具越来越受到关注.首先,从数学表达、性质、应用以及几何设计方法等方面对三周期极小曲面的研究现状进行介绍;其次,对三周期极小曲面在力学、传热传质、组织工程和声学等方面的性能及相关应用进行总结,从几何建模方法的角度对三周期极小曲面进行分类梳理,将现有工作分为规则单元法、参数单元法、区域拼接法以及整体优化法 4 类,对各方法特点进行分析;最后,结合实际应用对该领域面临的挑战进行总结,并展望未来工作与发展趋势.

关键词: 三周期极小曲面; 几何建模; 仿生结构; 隐式曲面; 增材制造
中图法分类号: TP391.41 **DOI:** 10.3724/SP.J.1089.2023.19359

Design and Applications of Triply Periodic Minimal Surfaces: A Survey

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Abstract: Triply Periodic Minimal Surface is a kind of algebraic surface expressed by implicit function. The rapid development of additive manufacturing technology has greatly enhanced the manufacturing capabilities of complex geometries and topological structures. Triply Periodic Minimal Surface, as a powerful geometric modeling tool, has received more and more attention. Firstly, this paper introduces the current research status of Triply Periodic Minimal Surface in terms of mathematical expression, properties, applications, and geometric design methods. Secondly, the mechanical property, heat and mass transfer, tissue engineering and acoustics performance are summarized. The existing works related to Triply Periodic Minimal Surface are classified and combed from the perspective of geometric modeling methods. They are grouped into four categories: regular unit method, parametric unit method, region splicing method and overall optimization method. The characteristics of each method are analyzed. Finally, in accordance with practical applications, we summarized challenges and prospected future works in this field.

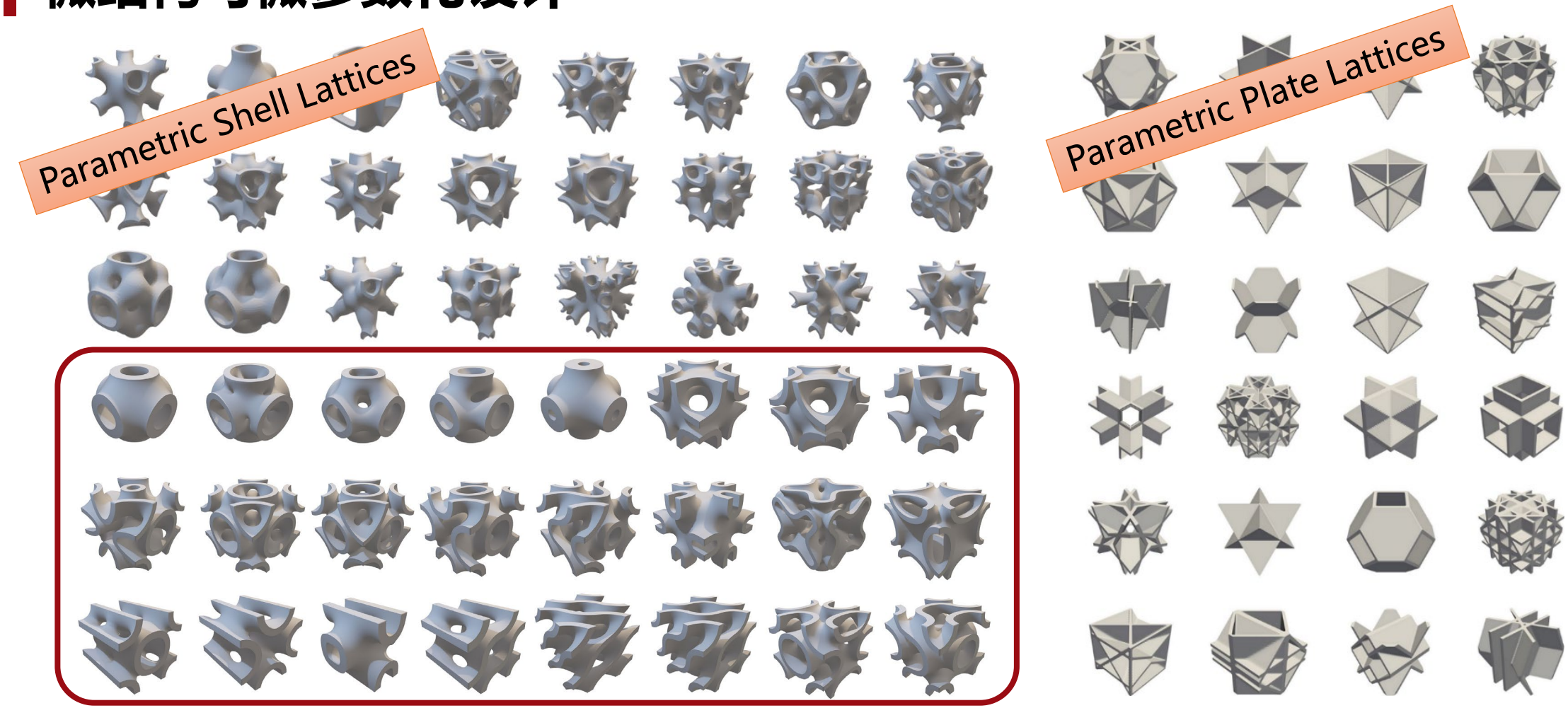
Key words: triply periodic minimal surfaces; geometric modeling; bionic structure; implicit surface; additive manufacturing

仿生学是指通过分析自然界中各种生物体在结构和功能等方面的特性并加以借鉴,从而为设计和制造新的产品、设备等提供新思路和方法的新兴交叉学科.仿生结构[1]已被广泛应用于新材料研

收稿日期: 2021-09-14; 修回日期: 2021-10-31. **基金项目:** 国家自然科学基金(61972232).**闫鑫**(1991—), 男, 博士研究生, 主要研究方向为计算机图形学; **田李昊**(1996—), 男, 博士研究生, 主要研究方向为计算机图形学; **彭昊**(1994—), 男, 博士研究生, 主要研究方向为计算机图形学; **吕琳**(1981—), 女, 博士, 教授, 博士生导师, CCF 高级会员, 论文通信作者, 主要研究方向为计算机图形学、计算几何、数字制造等.

- 田李昊，**吕琳***，彭昊，刘培庆，闫鑫，刘利刚，屠长河. **单材料几何微结构设计研究综述**[J]，计算机学报，2023, 46(05): 960-986.
- 闫鑫, 田李昊, 彭昊, **吕琳***. **三周期极小曲面结构设计及应用综述**[J]. 计算机辅助设计与图形学学报, 2023, 35(3): 329-340.

微结构可微参数化设计

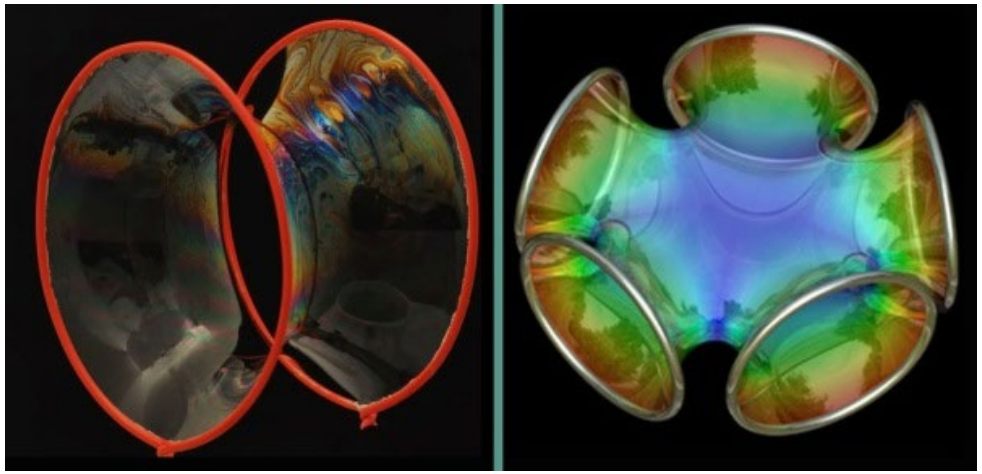


- Peiqing Liu, Bingteng Sun, Jikai Liu, **Lin Lu***, Parametric Shell Lattice with Tailored Mechanical Properties, *Additive Manufacturing*, Volume 60, Part B, December 2022, 103258.
- Bingteng Sun, Xin Yan, Peiqinge Liu, Yang Xia, **Lin Lu***, Parametric Plate Lattices: Modeling and Optimization of Plate Lattices with Superior Mechanical Properties, *Additive Manufacturing*, May 2023, 103626.
- Yonglai Xu, Hao Pan, Ruonan Wang, Qiang Du, **Lin Lu***, New families of Triply Periodic Minimal Surface-like Shell Lattices, Additive Manufacturing, September 2023, 103779.

极小曲面（Minimal Surfaces)

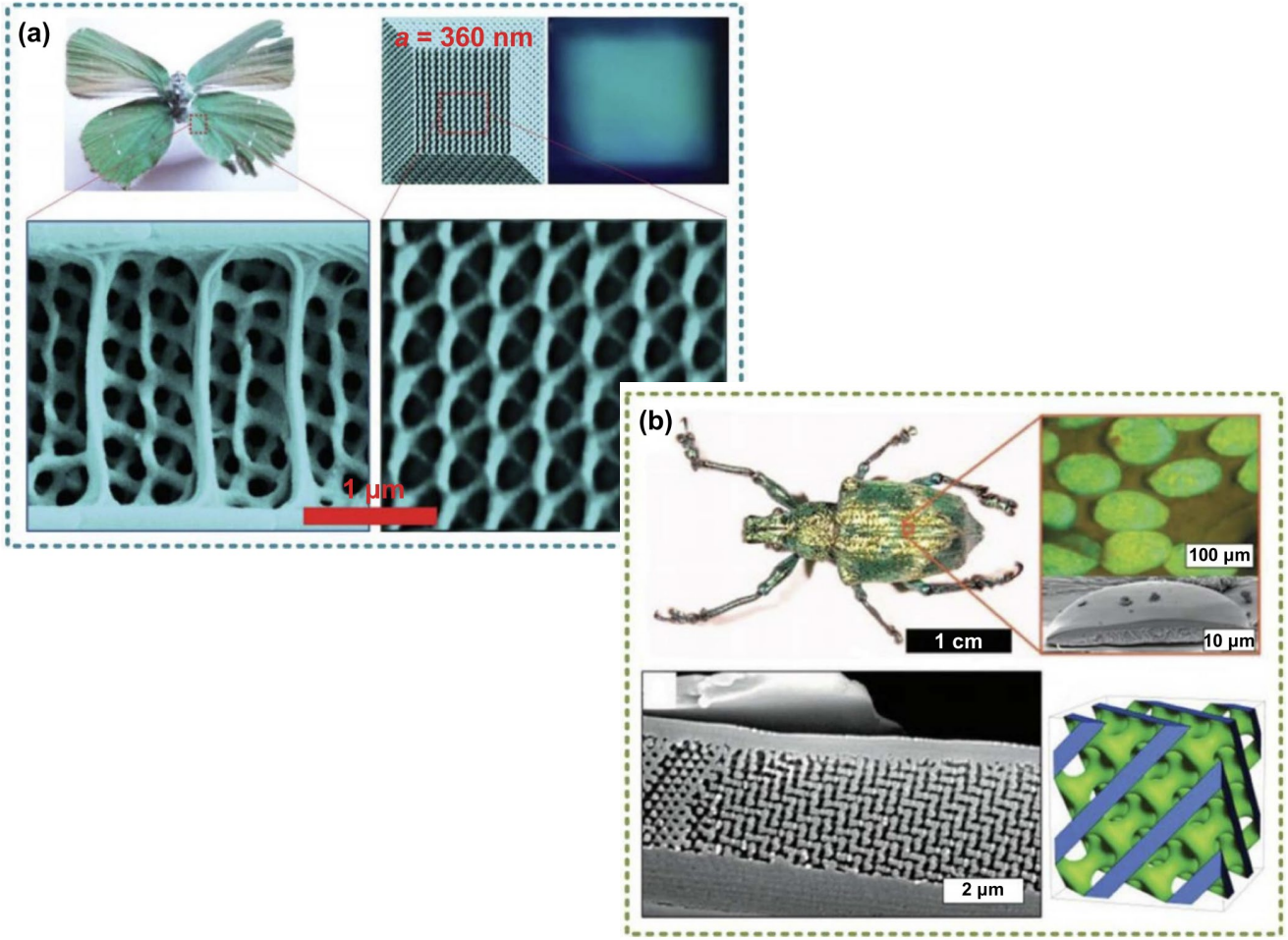
极小曲面是微分几何中的一个重要概念，它指在某个几何背景中，曲面在局部范围内表现出**表面积局部极小**的性质。具体而言，对于曲面上的任意一点，其**平均曲率为零。**

Minimal surfaces in nature.

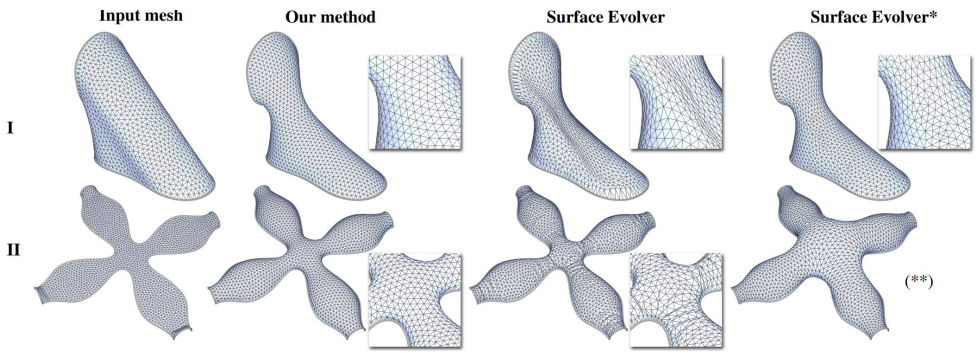


Soap films are minimal surfaces.

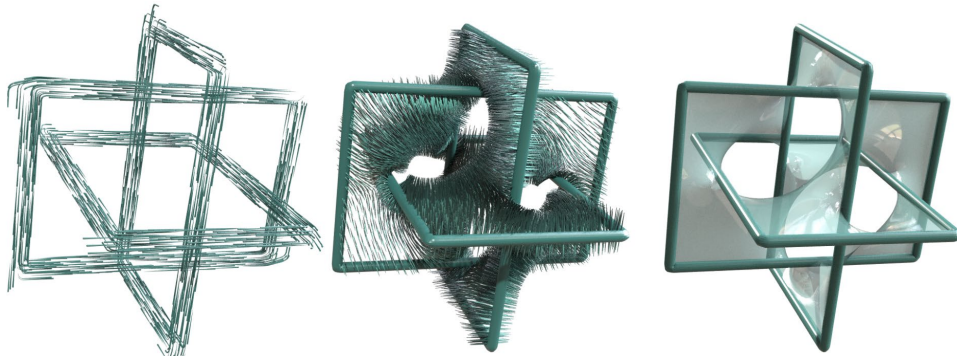
Joseph Plateau proved that a soap film minimizes area among nearby surface.



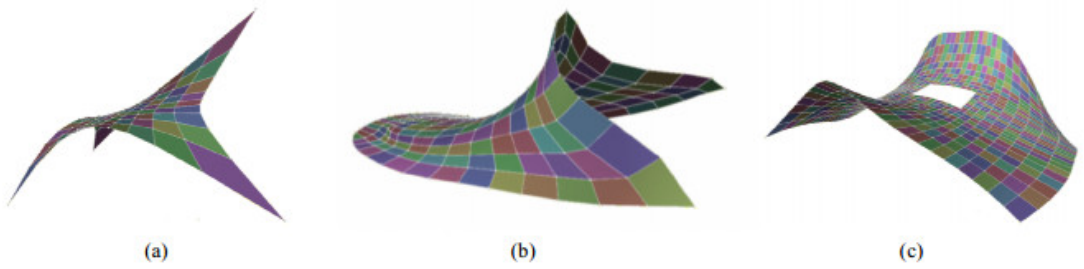
极小曲面的计算



[Pan et al. ACM TOG 2012]



[Wang et al. ACM TOG 2021]

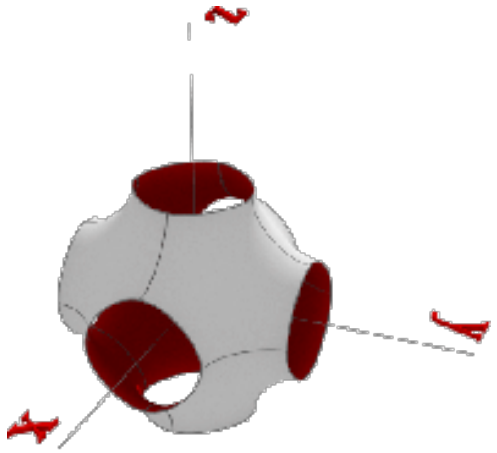


[Xu et al. Journal of Software 2016]

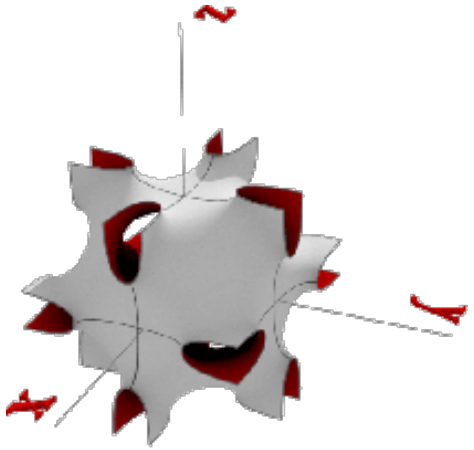
[Palmer et al. CVPR 2022]

三周期极小曲面 (Triply Periodic Minimal Surface, TPMS)

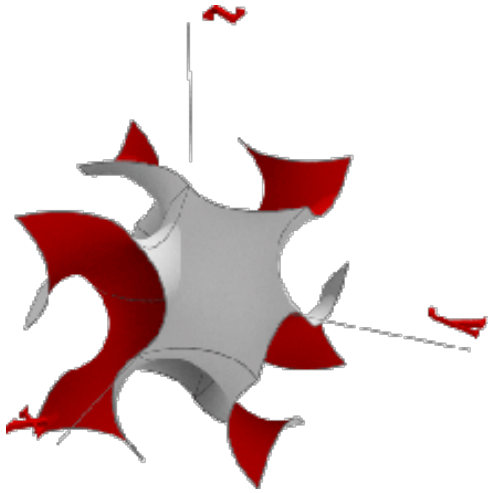
TPMS — 在空间中三个维度方向周期性重复的一类极小曲面。



Schwartz_P surface



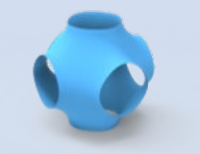
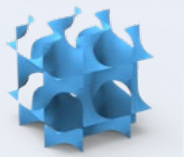
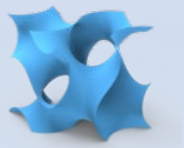
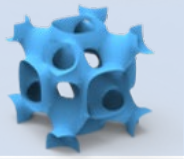
Neovius surface



Gyroid surface

三周期极小曲面 (Triply Periodic Minimal Surface, TPMS)

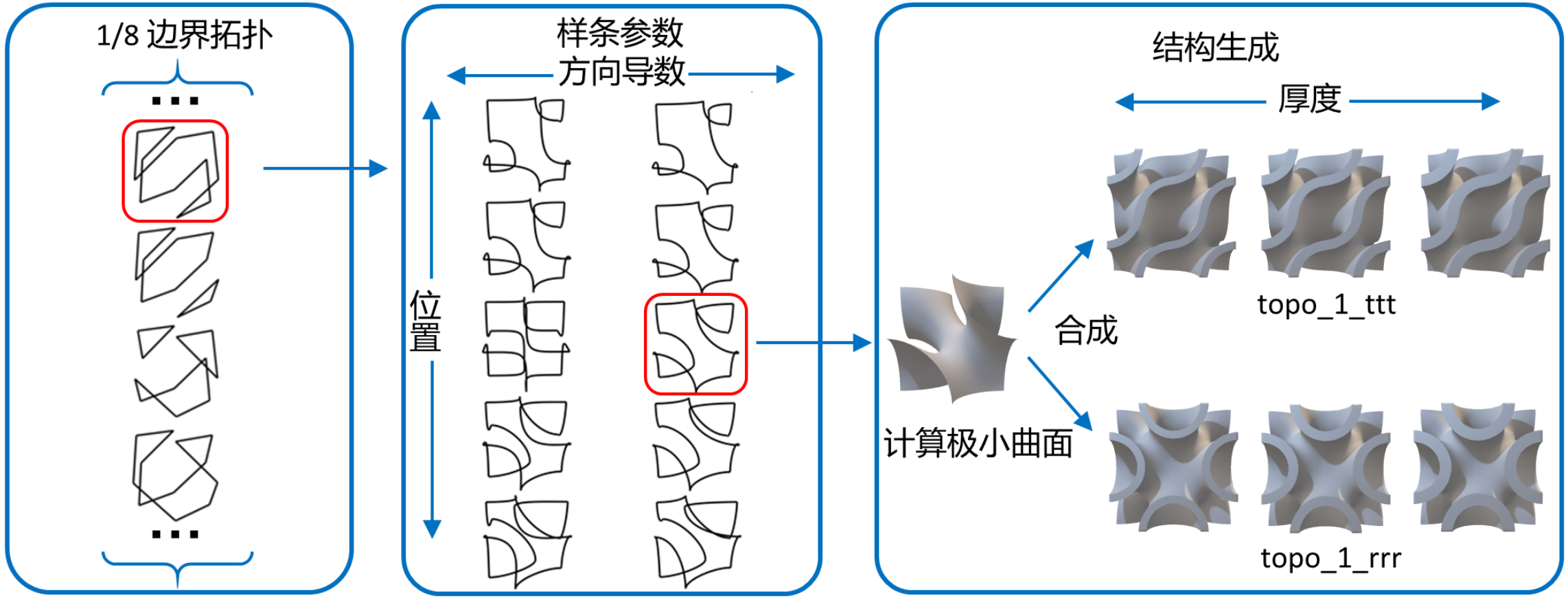
TPMS最常用的定义方式为傅里叶级数形式： $\Psi(\boldsymbol{r}) = \sum_k F(\boldsymbol{k})\cos[2\pi\boldsymbol{k}\cdot\boldsymbol{r} - \alpha(\boldsymbol{k})] = C$

TPMS类型	函数表达	单元结构
P	$\phi(\mathbf{r}) = \cos(X) + \cos(Y) + \cos(Z) = C$	
D	$\phi(\mathbf{r}) = \cos(X)\cos(Y)\cos(Z) - \sin(X)\sin(Y)\sin(Z) = C$	
G	$\phi(\mathbf{r}) = \sin(X)\cos(Y)+\sin(Z)\cos(X)+\sin(Y)\cos(Z) = C$	
I-WP	$\phi(\mathbf{r}) = 2[\cos(X)\cos(Y) + \cos(Y)\cos(Z) + \cos(Z)\cos(X)] - [\cos(2X) + \cos(2Y) + \cos(2Z)] = C$	
F-RD	$\phi(\mathbf{r}) = 4\cos(X)\cos(Y)\cos(Z) - [\cos(2X)\cos(2Y) + \cos(2X)\cos(2Z) + \cos(2Y)\cos(2Z)] = C$	
Tubular P	$\phi(\mathbf{r}) = 10[\cos(X) + \cos(Y) + \cos(Z)] - 5.1[\cos(X)\cos(Y) + \cos(Y)\cos(Z) + \cos(Z)\cos(X)] - 14.6 = C$	

常见三周期极小曲面表达式

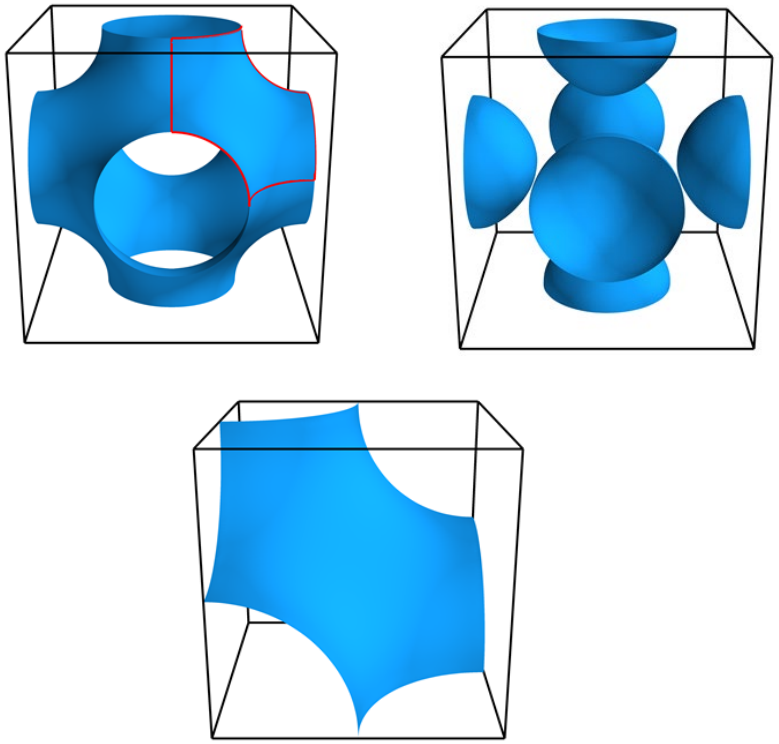
$X = 2\pi x,\ Y = 2\pi y,\ Z = 2\pi z$

New Families of TPMS—方法流程

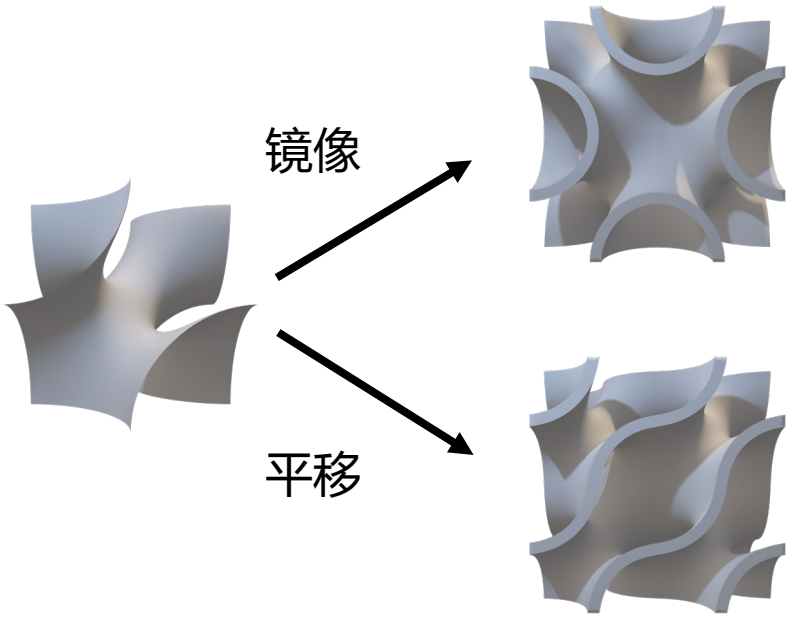


- Yonglai Xu, Hao Pan, Ruonan Wang, Qiang Du, Lin Lu*, New families of Triply Periodic Minimal Surface-like Shell Lattices, *Additive Manufacturing*, September 2023, 103779.

基本建模单元

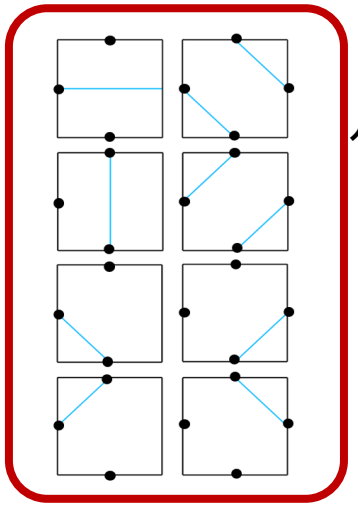


以1/8立方体为基本单元



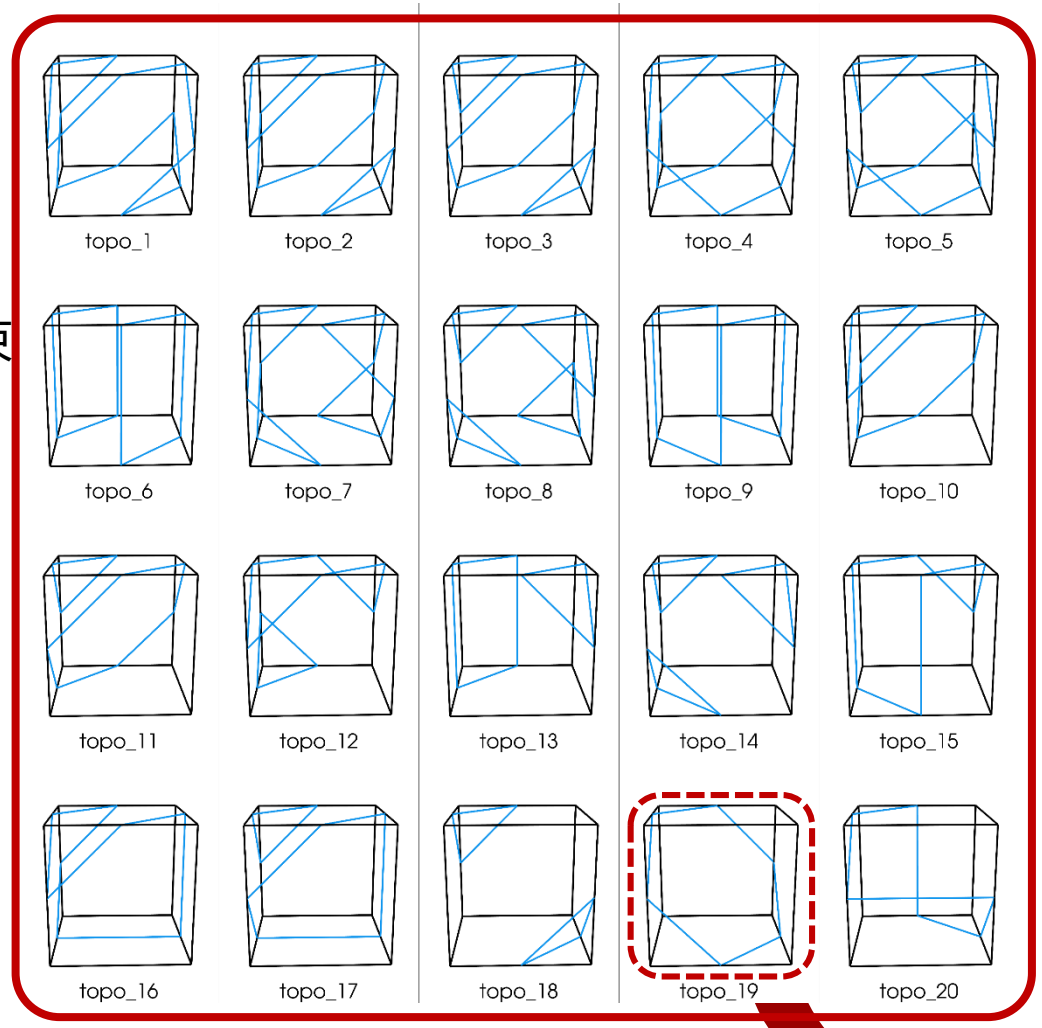
从1/8结构到单胞结构

边界拓扑设计及其参数化



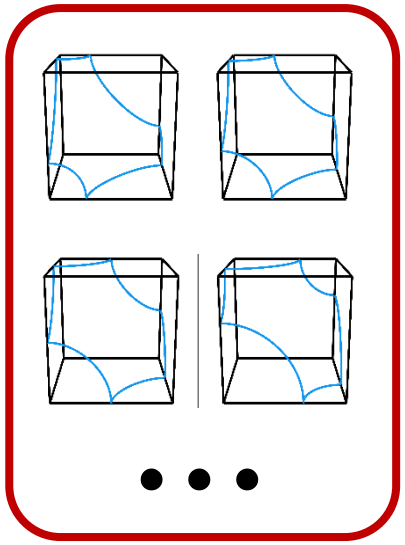
单独一个面上可能的连接方式

几何与制造约束
- 闭合
- 无交叉



满足约束的边界拓扑

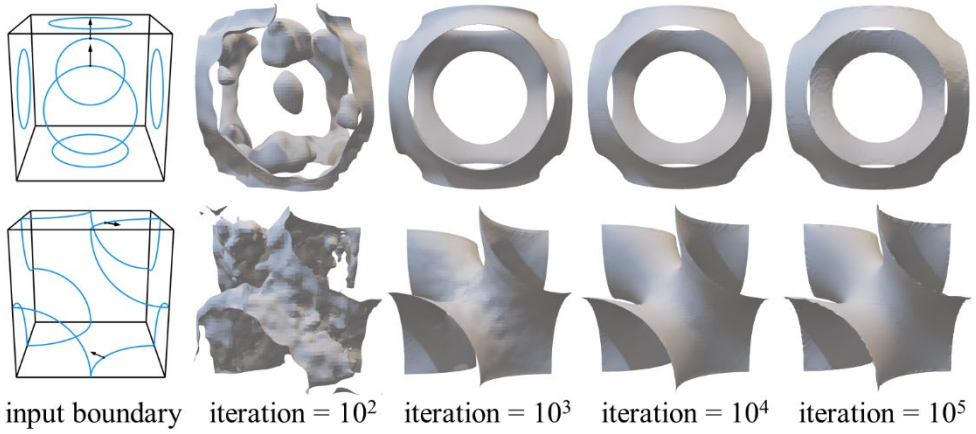
样条参数化
- 位置
- 方向导数



最终边界

计算极小曲面

- 用神经网络隐式表示曲面，不需要构造初始拓扑结构
- 可微性，易于加入边界连续性约束

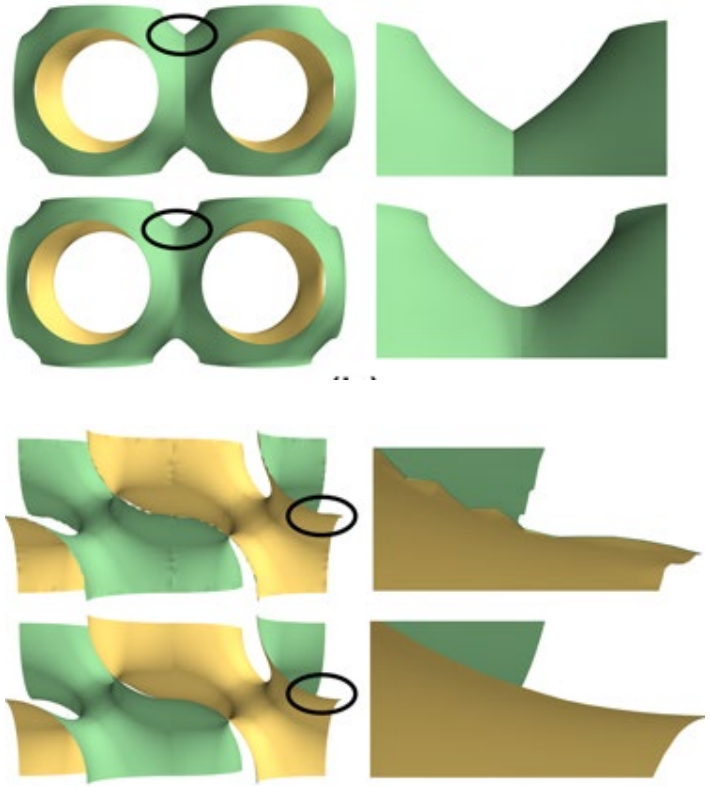


$$\arg\min_{\theta} E_{X\sim U}\left[\left|\alpha_{\Gamma}^{\#}(x) + \nabla_x f_\theta(x)\right|\right]$$

[DeepCurrents: Learning Implicit Representations of Shapes with Boundaries，Palmer et al. CVPR 2022]

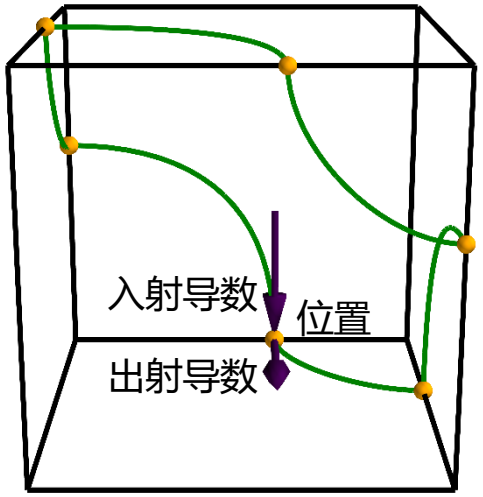
+ 边界C^1约束：

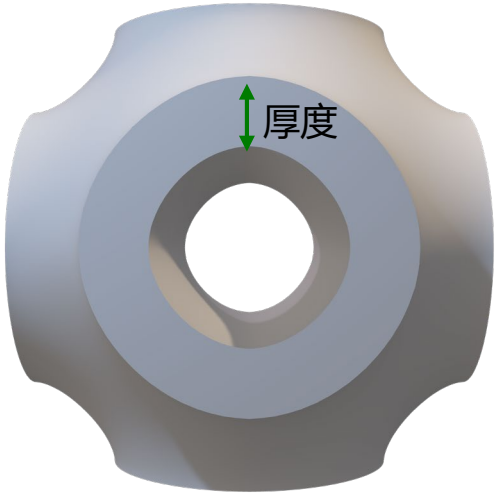
$$\sum_{p_{bdry}\in\Gamma_t}\left|\nabla_{p_{bdry}} f_\theta(p_{bdry}) \pm \nabla_{p'_{bdry}} f_\theta(p'_{bdry})\right|$$

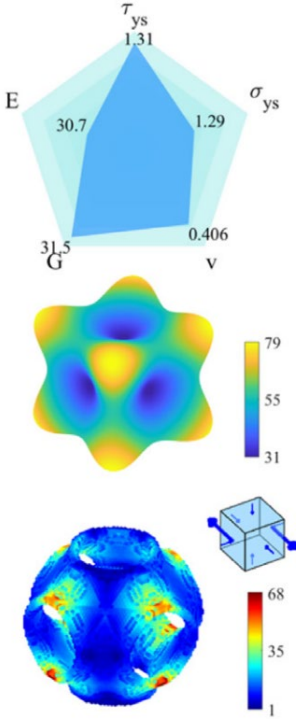


参数化模型

拓扑类型: topo_19_rrr



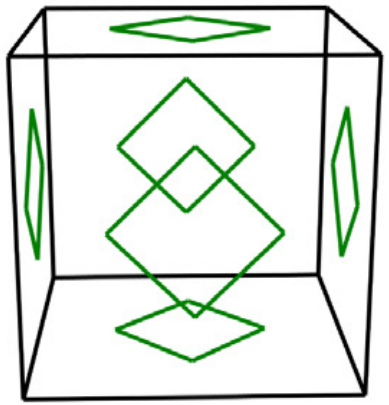


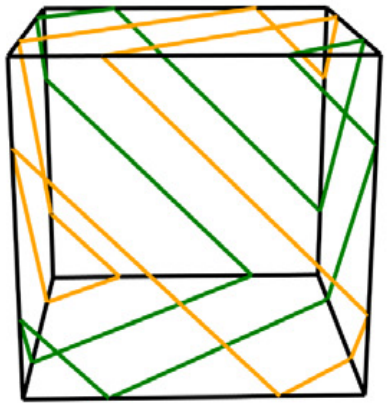


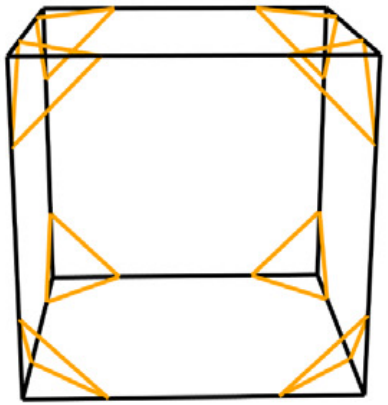
均质化力学性质分析

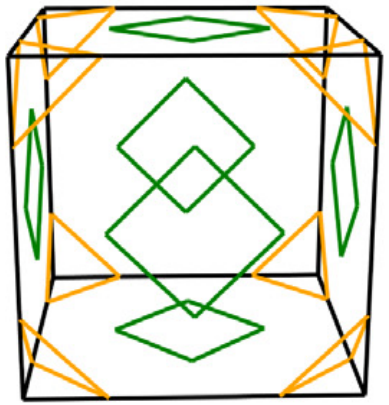
{ topo_19_rrr, thickness, p_1, p_2, p_3, p_4, p_5, p_6, d_1_in, d_1_out, d_2_in, d_2_out, d_3_in, d_3_out, d_4_in, d_4_out, d_5_in, d_5_out, d_6_in, d_6_out }

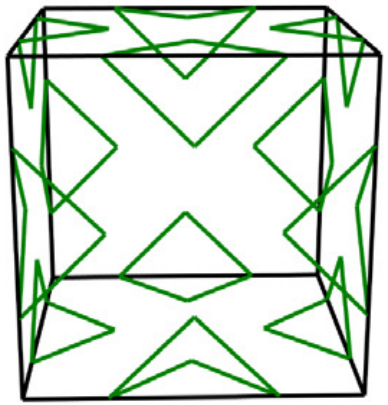
结果展示

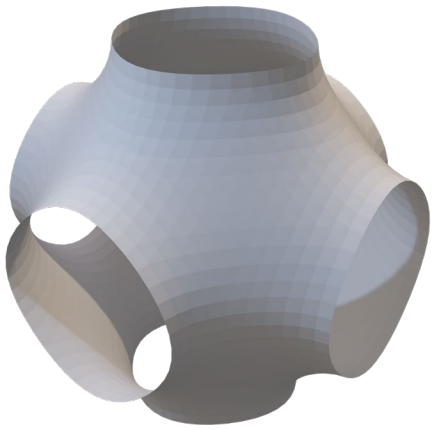




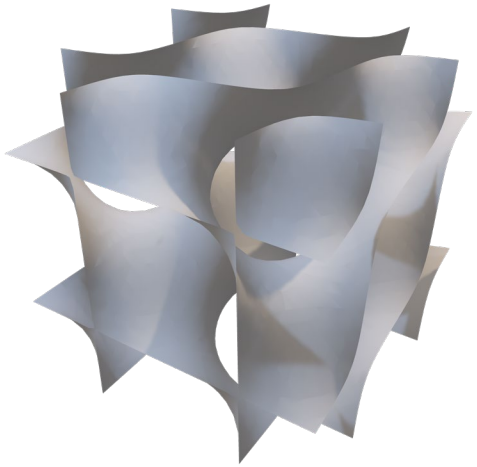




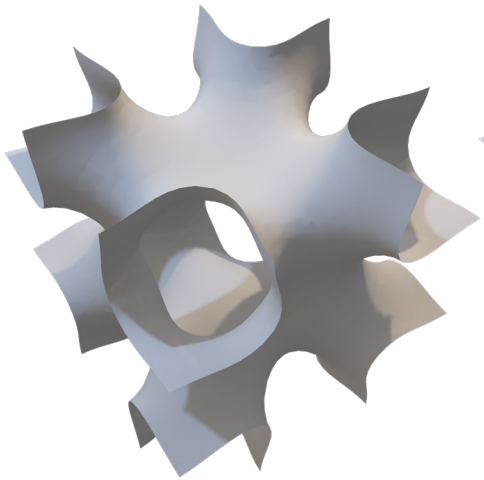




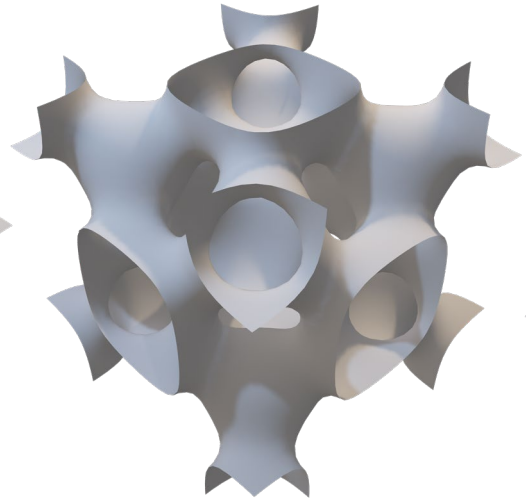
Schwarz Primitive



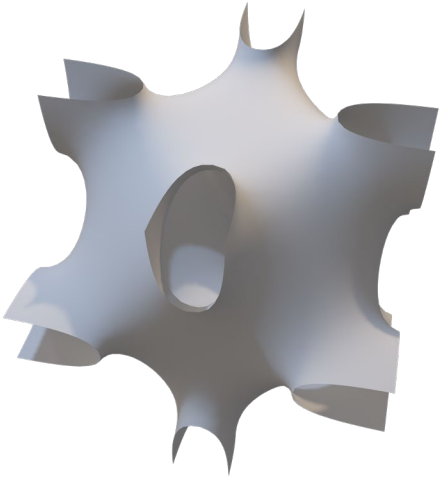
Schwarz Diamond



Schoen’s I-WP

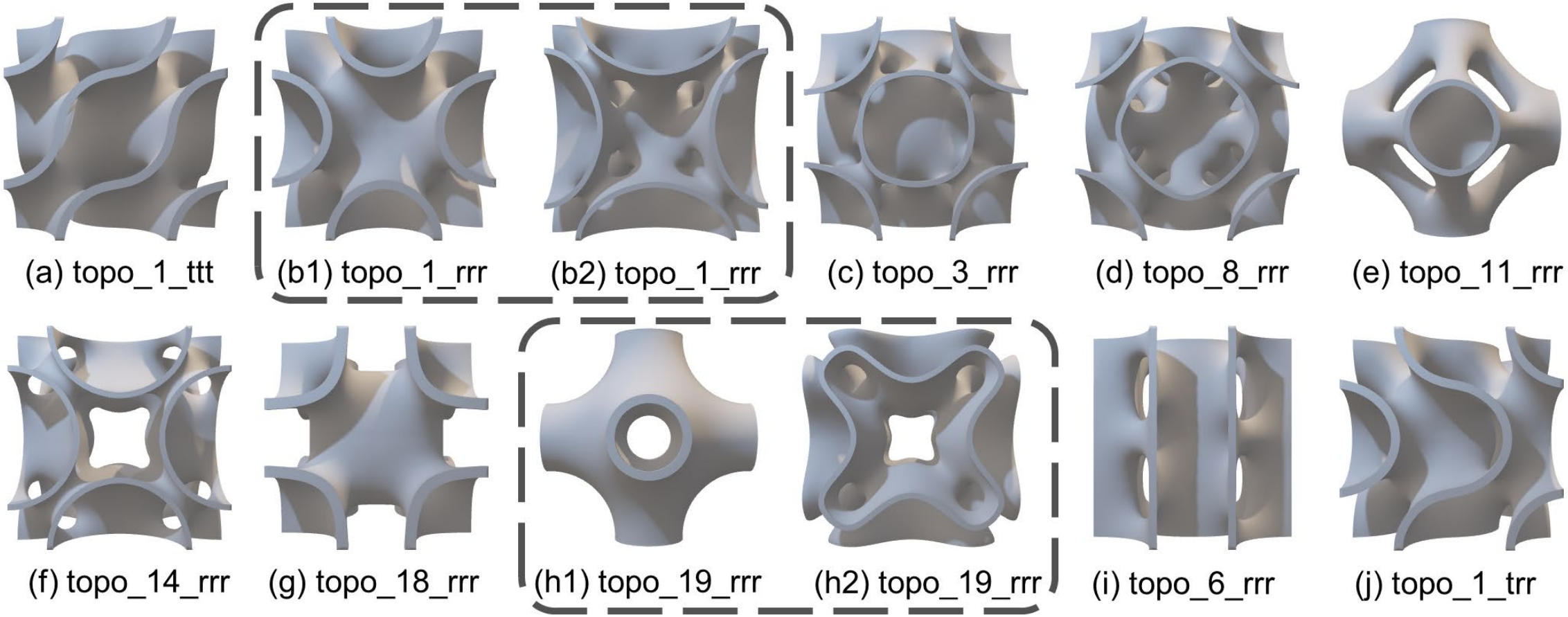


Schoen’s F-RD



Neovius

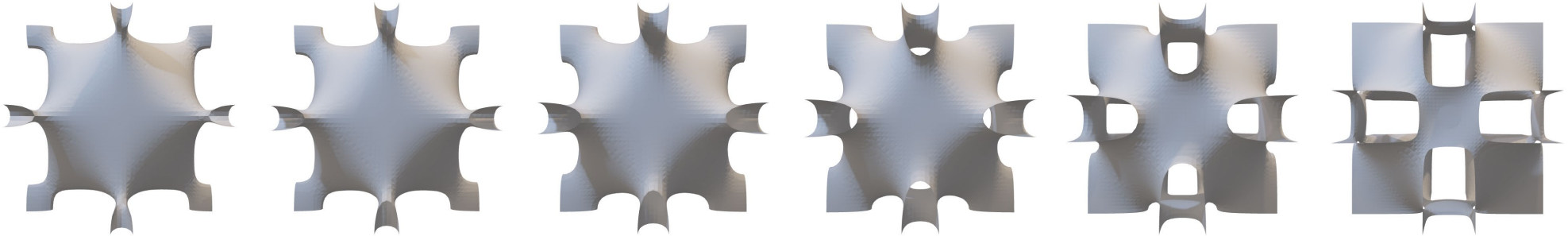
结果展示


(a) topo_1_ttt
(b1) topo_1_rrr
(b2) topo_1_rrr
(c) topo_3_rrr
(d) topo_8_rrr
(e) topo_11_rrr
(f) topo_14_rrr
(g) topo_18_rrr
(h1) topo_19_rrr
(h2) topo_19_rrr
(i) topo_6_rrr
(j) topo_1_trr

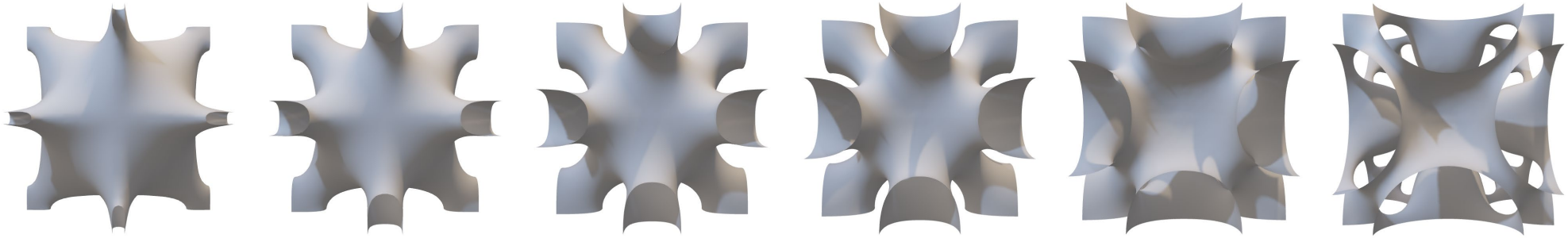
结果分析

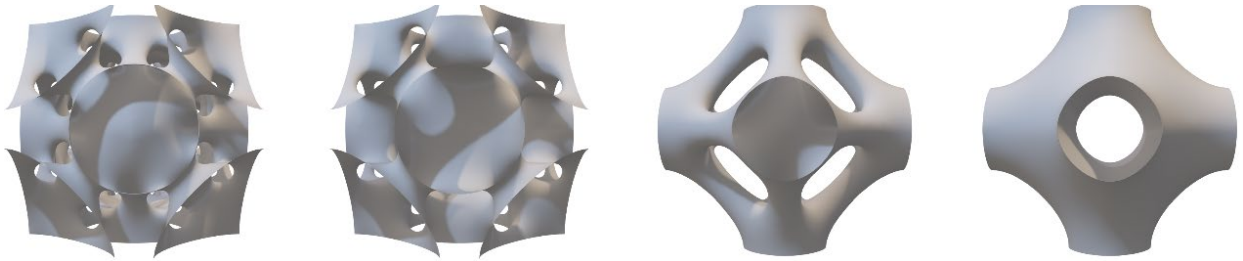
- 更丰富的参数来控制曲面
- 相同外部边界可以产生不同内部拓扑的曲面

N类型曲面:



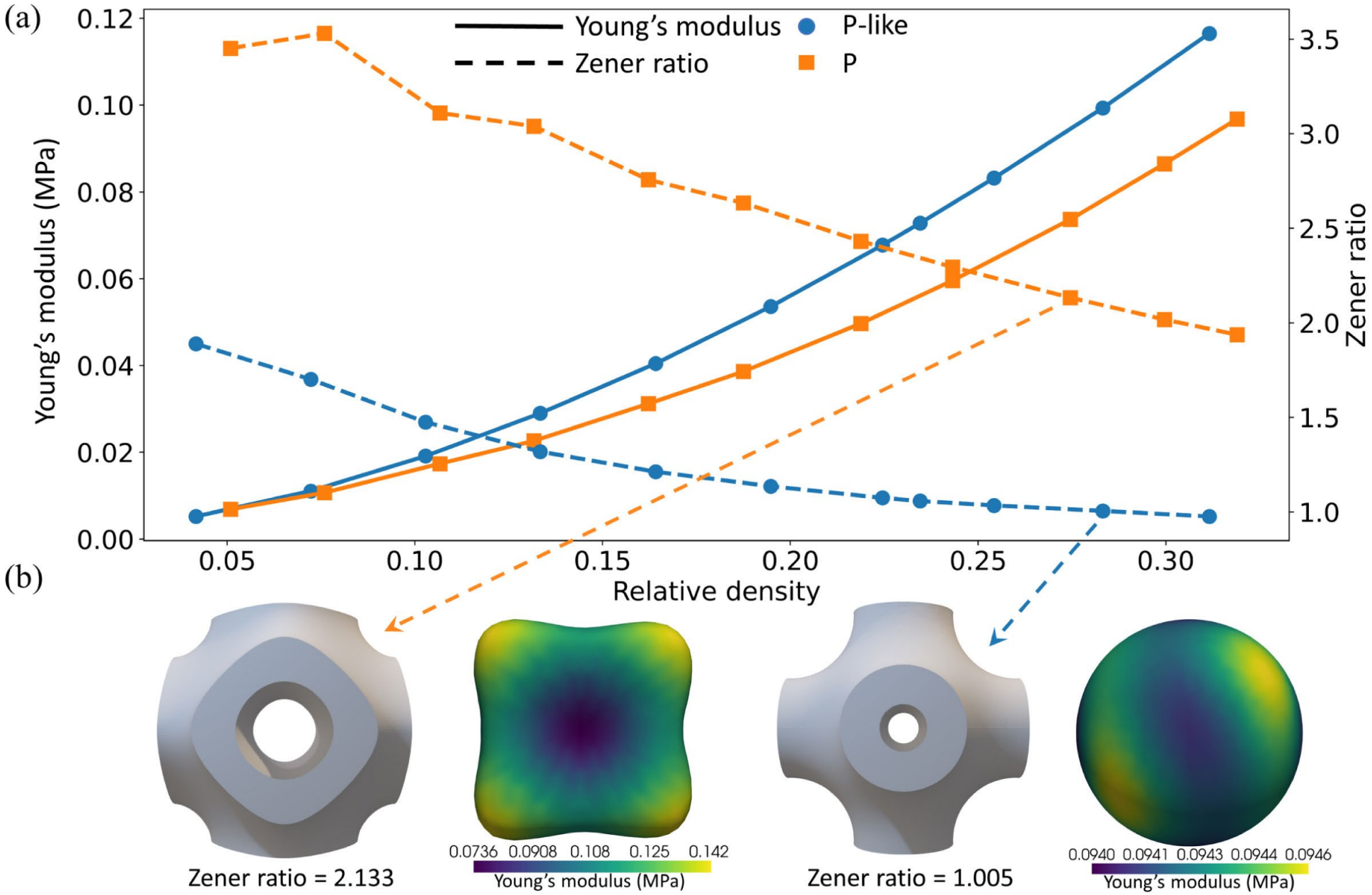
N_like类型曲面:

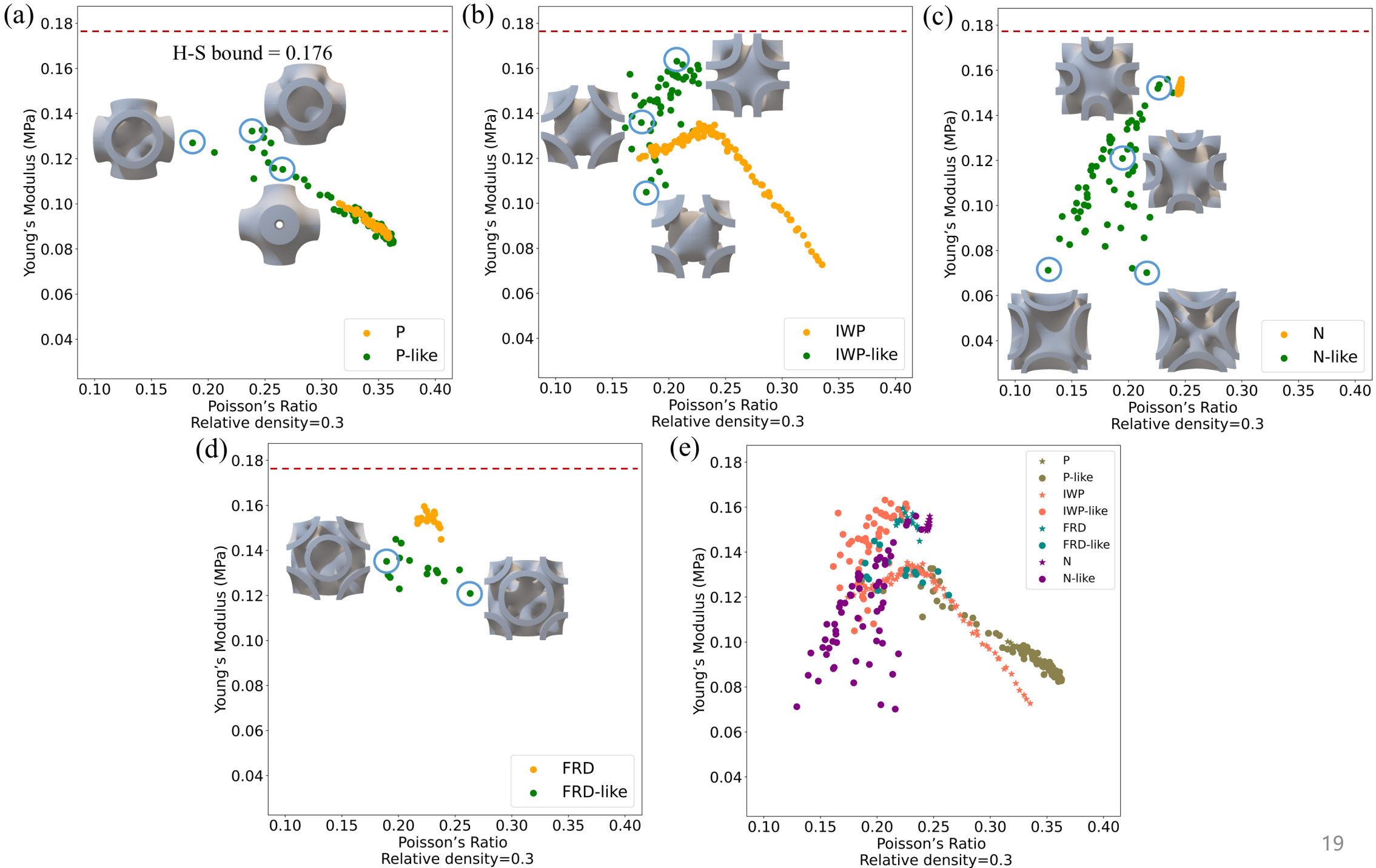




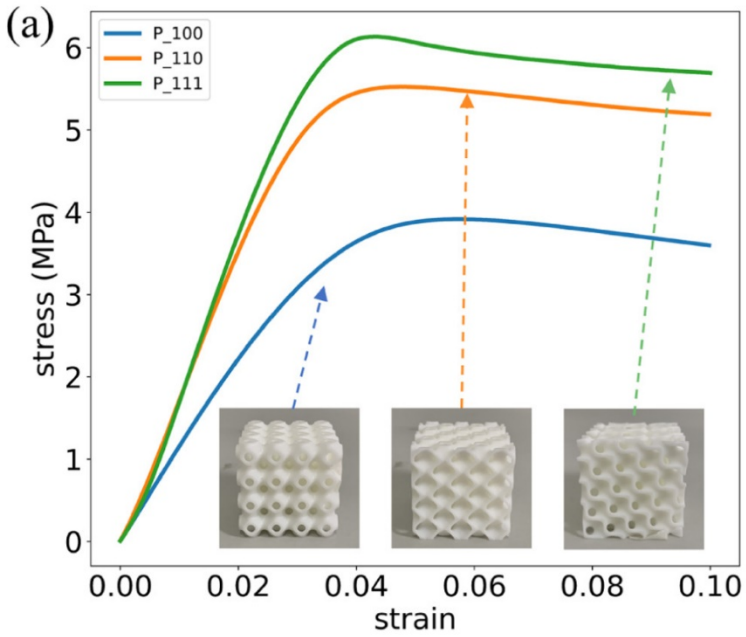
topo_3_rrr　topo_8_rrr　topo_11_rrr　topo_19_rrr

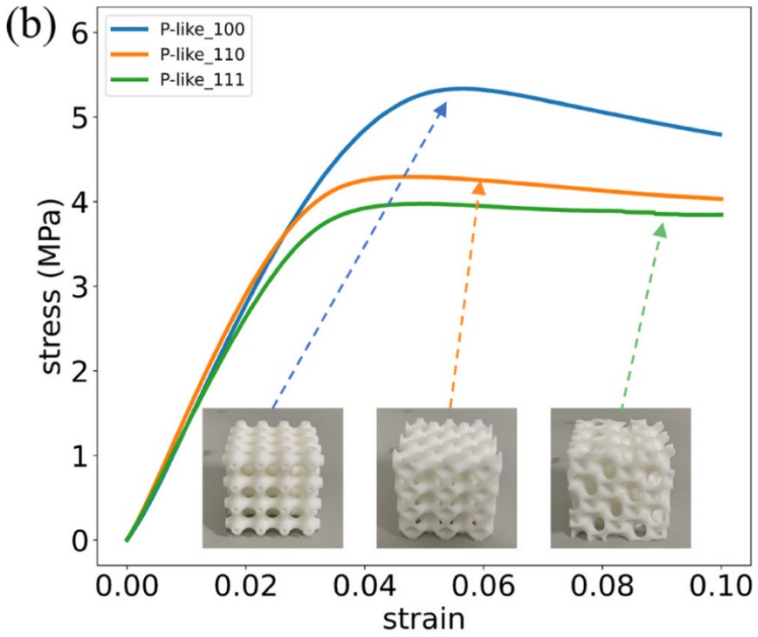
力学性质

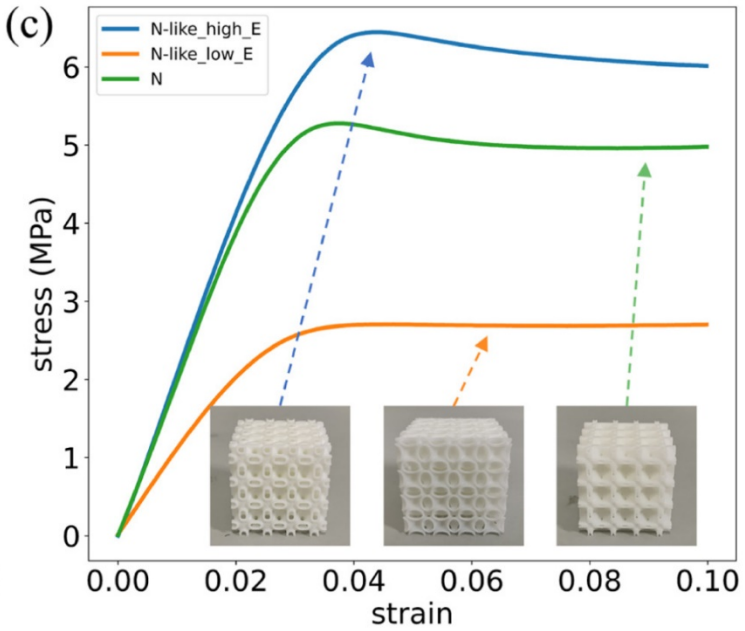

(a)
Young's modulus
Zener ratio
P-like
P
Young's modulus (MPa)
Zener ratio
Relative density
0.00
0.02
0.04
0.06
0.08
0.10
0.12
1.0
1.5
2.0
2.5
3.0
3.5
0.05
0.10
0.15
0.20
0.25
0.30
(b)
Zener ratio = 2.133
0.0736 0.0908 0.108 0.125 0.142
Young's modulus (MPa)
Zener ratio = 1.005
0.0940 0.0941 0.0943 0.0944 0.0946
Young's modulus (MPa)

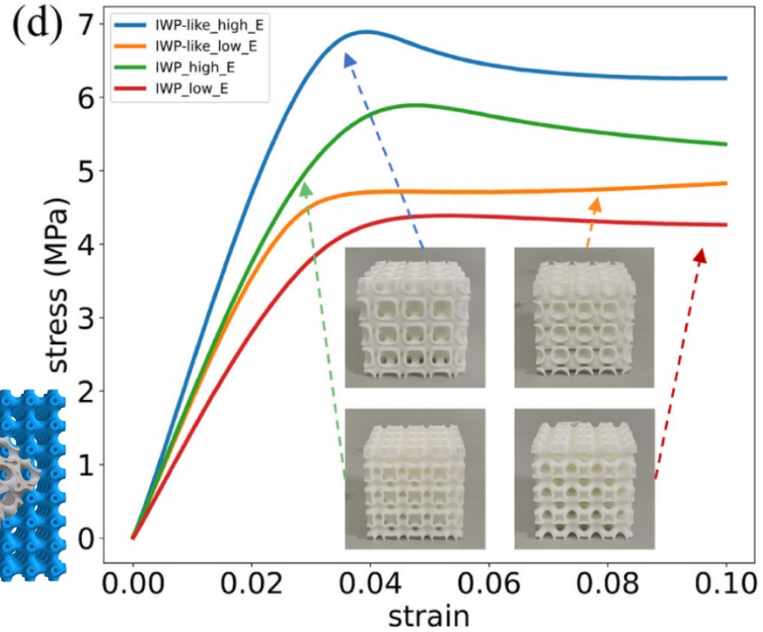

(a)
H-S bound = 0.176
Young's Modulus (MPa)
Poisson's Ratio
Relative density=0.3
P
P-like
(b)
IWP
IWP-like
(c)
N
N-like
(d)
FRD
FRD-like
(e)
P
P-like
IWP
IWP-like
FRD
FRD-like
N
N-like

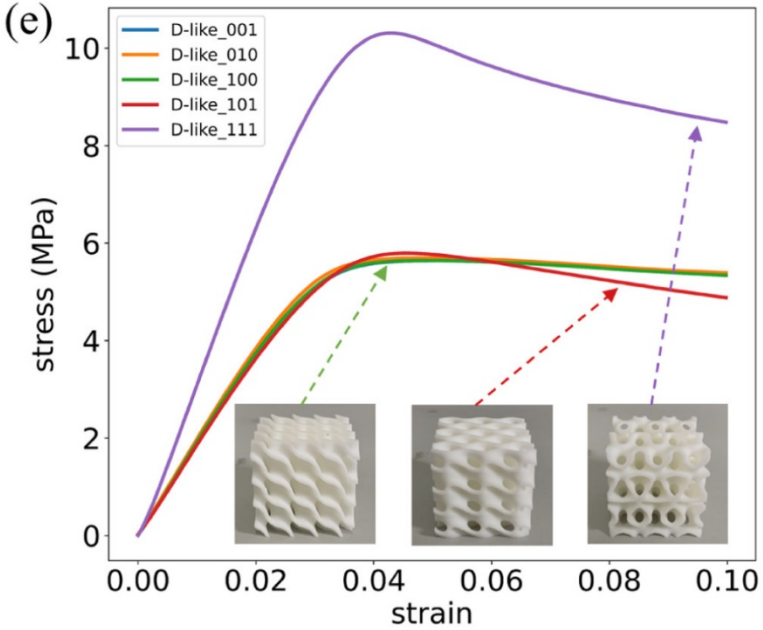
力学性质

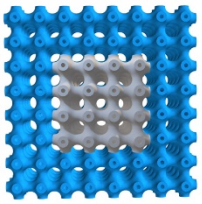




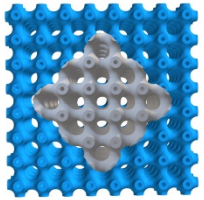




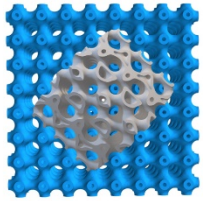




[100]

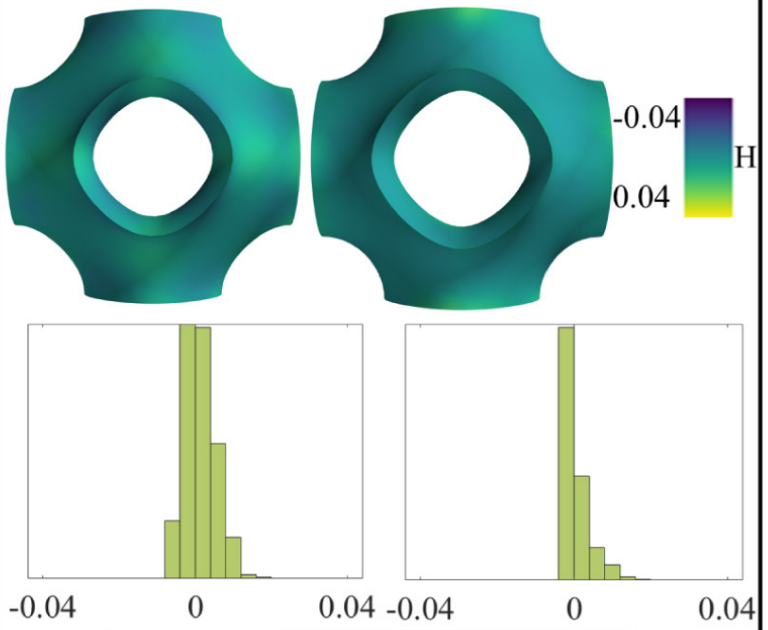
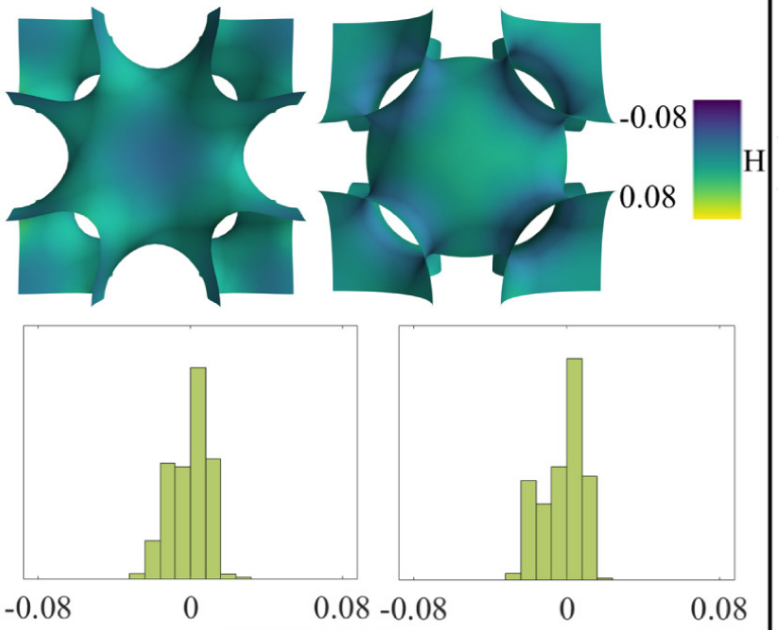
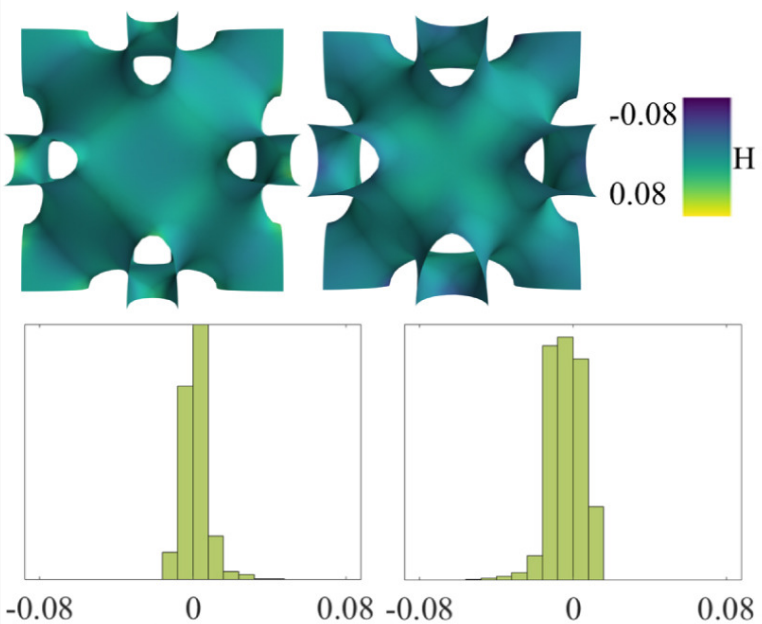


[110]

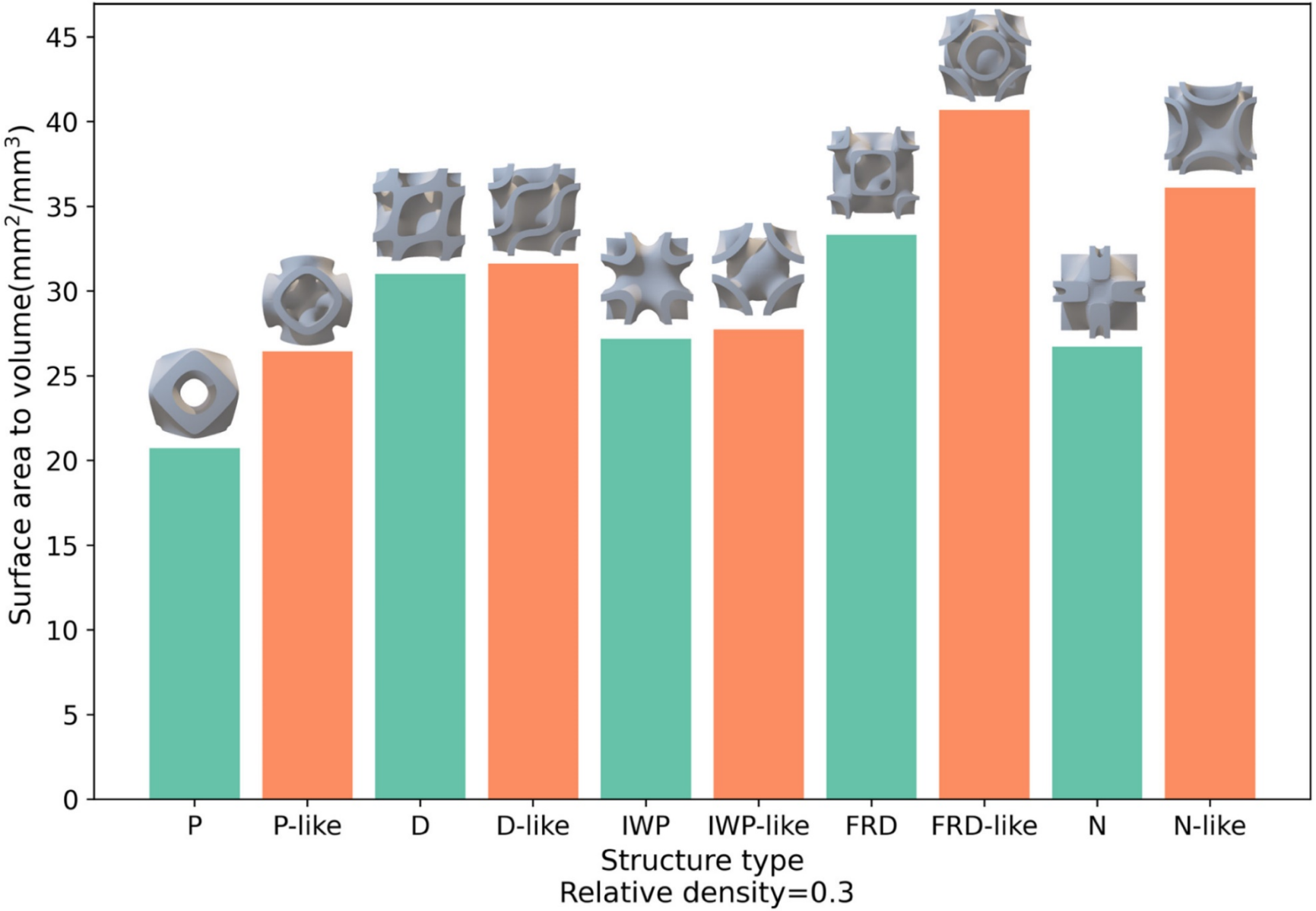


[111]

几何性质—平均曲率

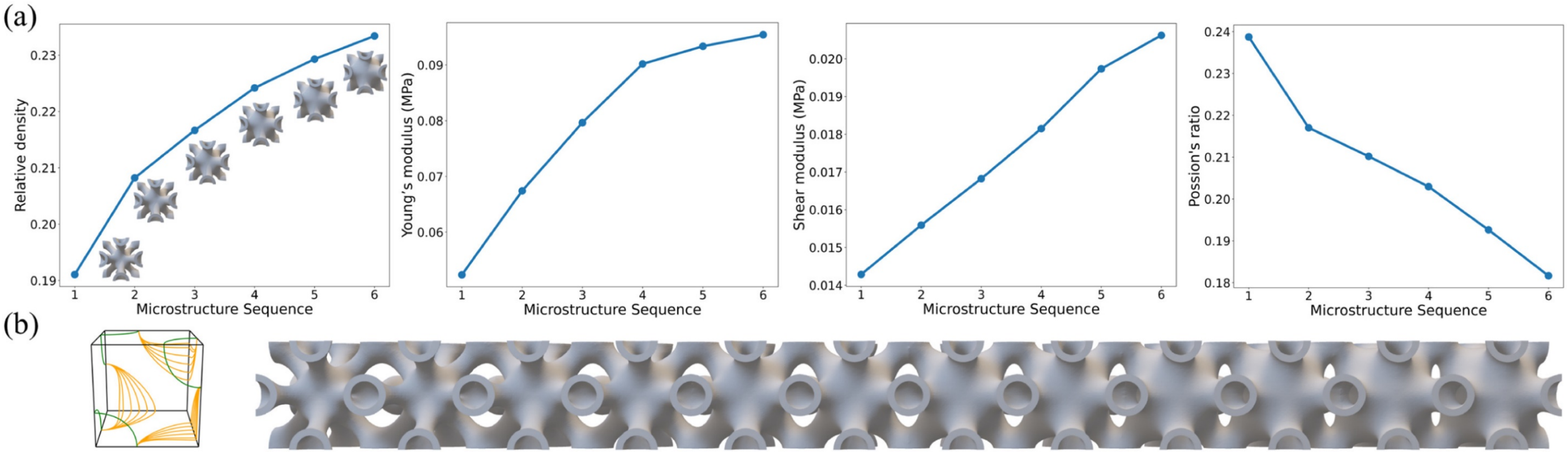
Type	P	P-like	IWP	IWP-like	N	N-like
Mean curvature						
Surface area(cm^2)	2.35	2.33	3.55	3.56	3.53	3.55
Vertices number	3192	3186	4822	4912	4678	4753
Faces number	6085	6042	9171	9307	8900	9037

几何性质—比表面积

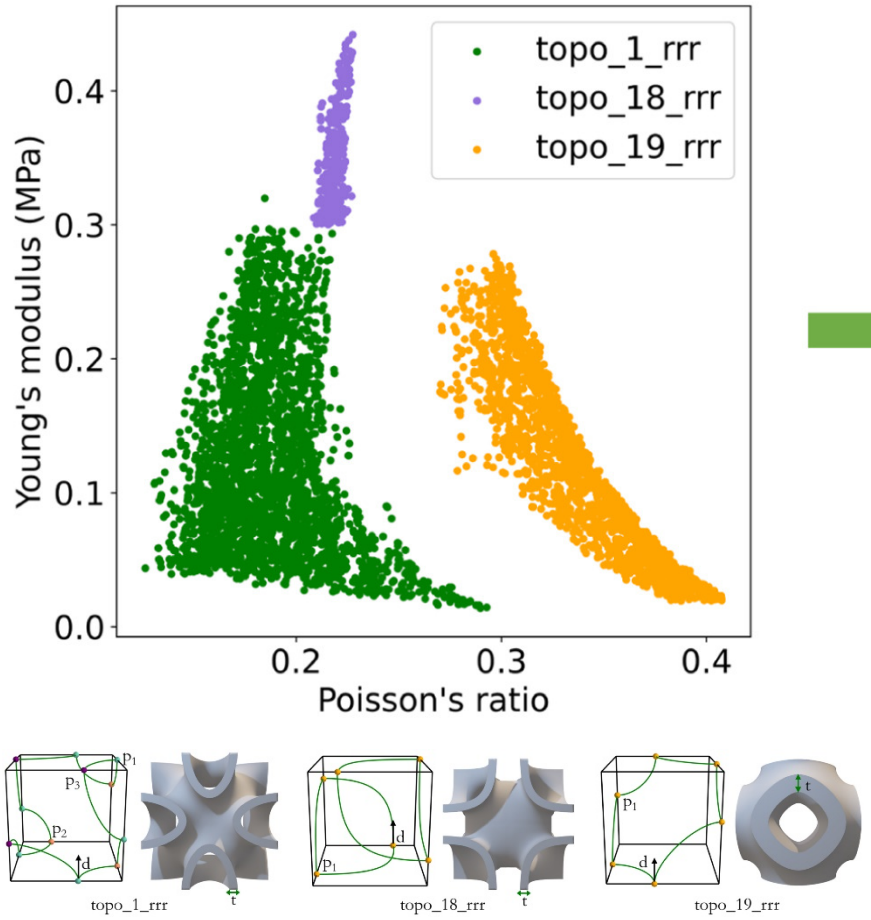

Surface area to volume(mm²/mm³)
0
5
10
15
20
25
30
35
40
45
P
P-like
D
D-like
IWP
IWP-like
FRD
FRD-like
N
N-like
Structure type
Relative density=0.3

应用：功能梯度结构设计

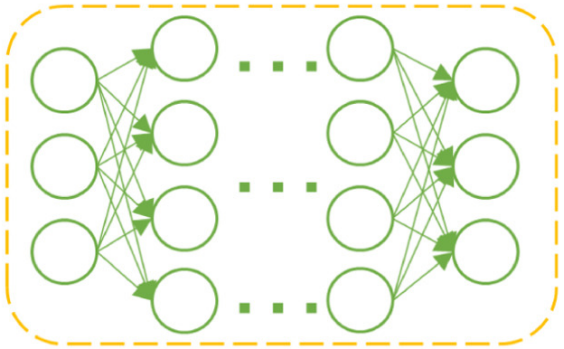
- 构建保证几何与物理性能连续性的梯度功能微结构族
- 具体方法为固定外边界参数，只修改内部边界参数



应用：逆均质化



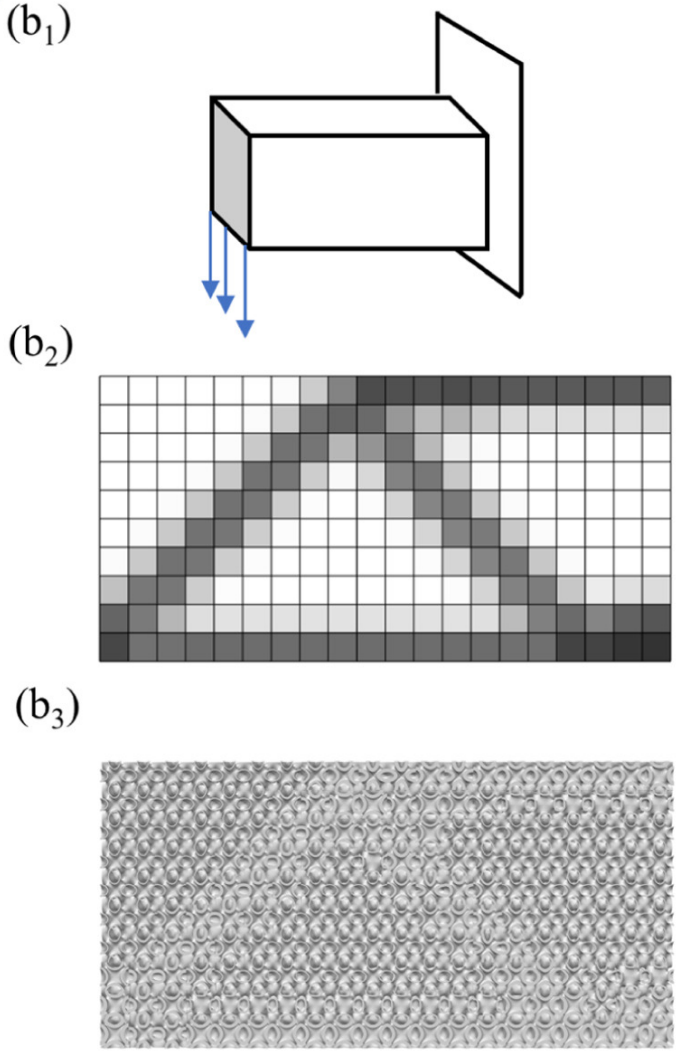
Input: Young's modulus
Poisson's ratio



Inverse homogenization network

Output: Topology type
Model thickness
Boundary parameters

(a) Training



(b) Example

Thank you!

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Geometric Modeling and Processing 2024

GMP 2024

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