SIGGRAPH 2023
LOS ANGELES+ 6-10 AUG

REVISITING CONTROLLED MIXTURE SAMPLING FOR RENDERING APPLICATIONS

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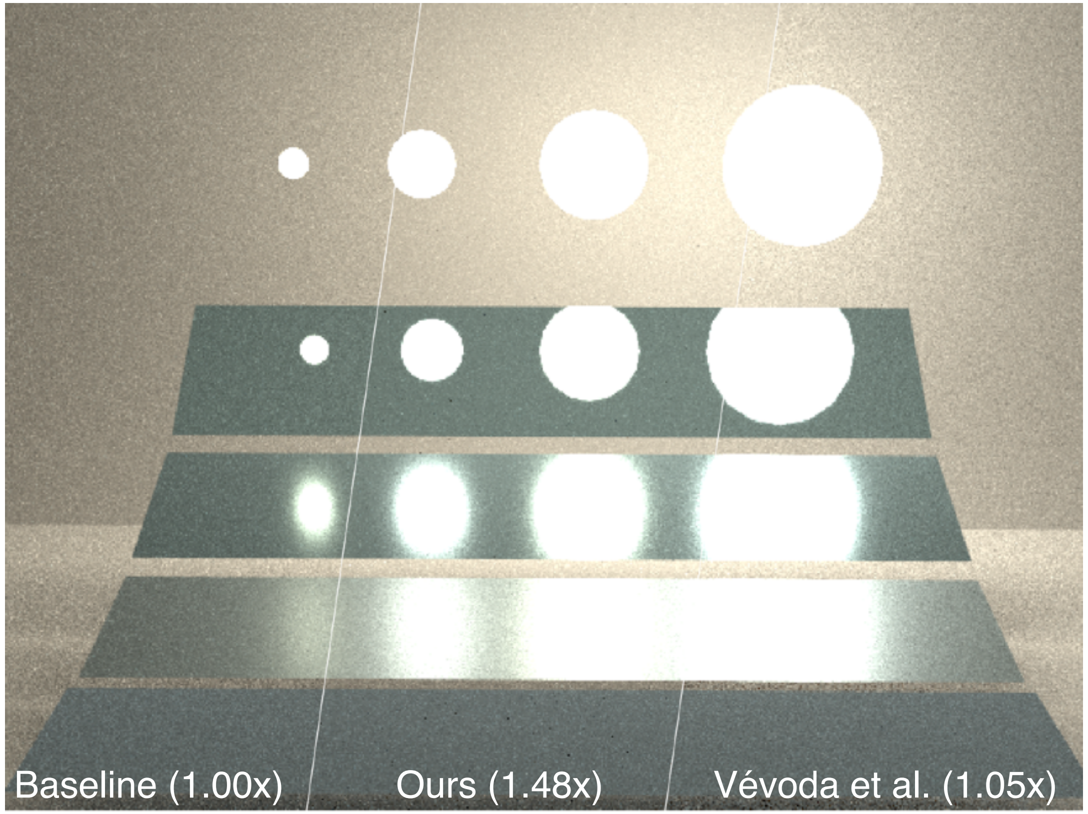
SIC Saarland Informatics Campus

UNIVERSITÄT DES SAARLANDES

TAKE-HOME MESSAGE

- How to use practical **control variate** to improve **Global Illumination**.

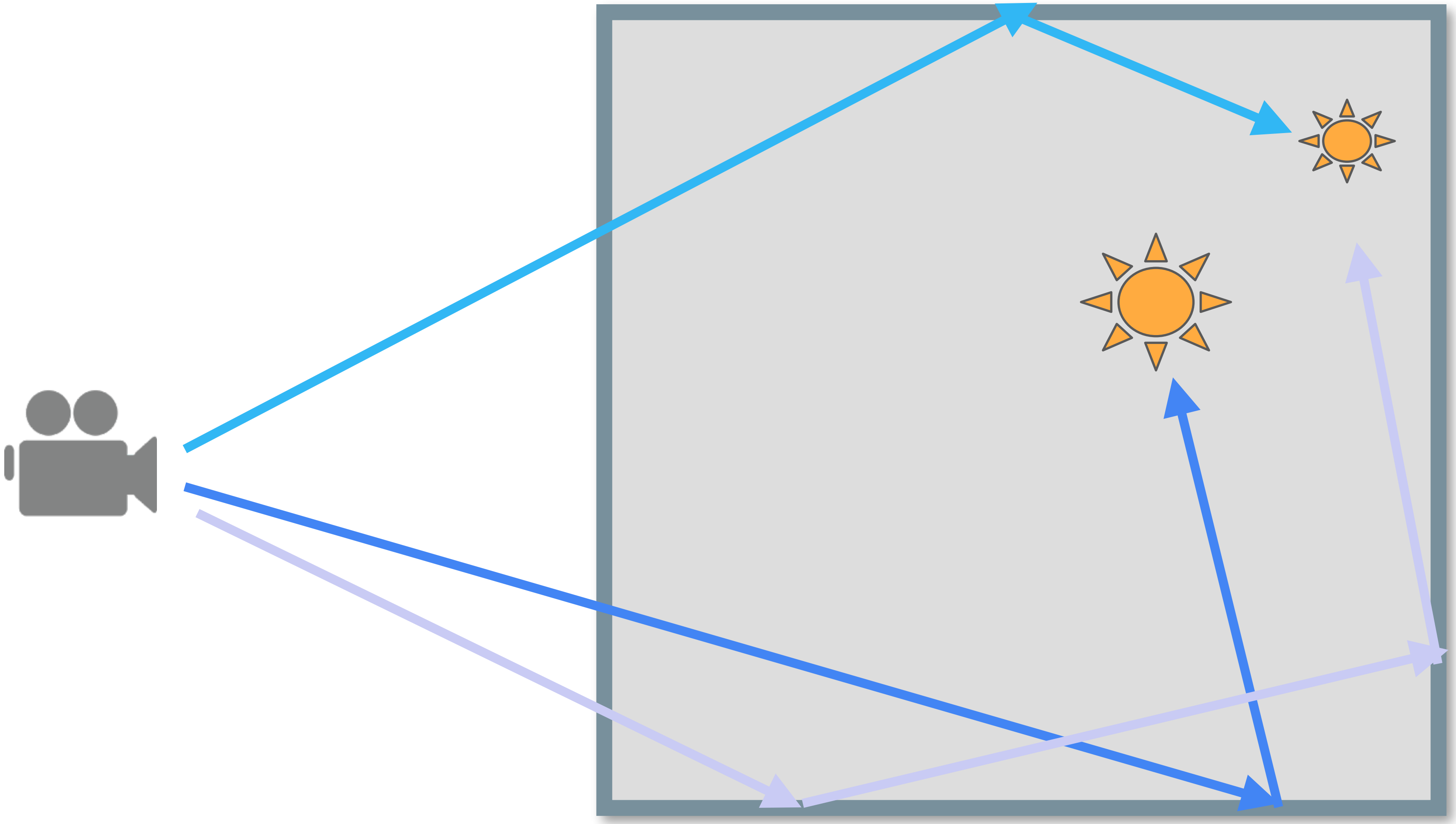
- Attention: Math ahead!



BACKGROUND

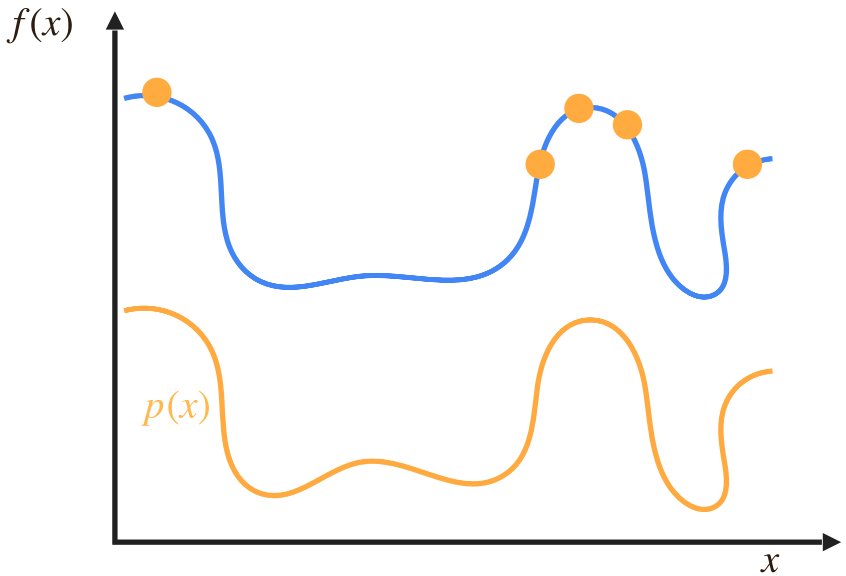
$$I_{pixel} = \int_X f(x)dx \qquad X = \text{paths}$$

OURS

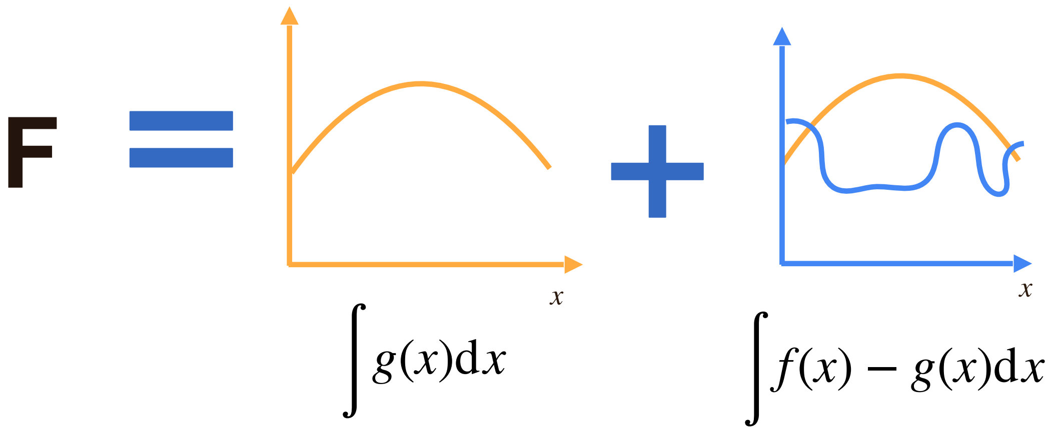


$$I_{pixel} = \int_X f(x)dx$$ X = paths

BACKGROUND

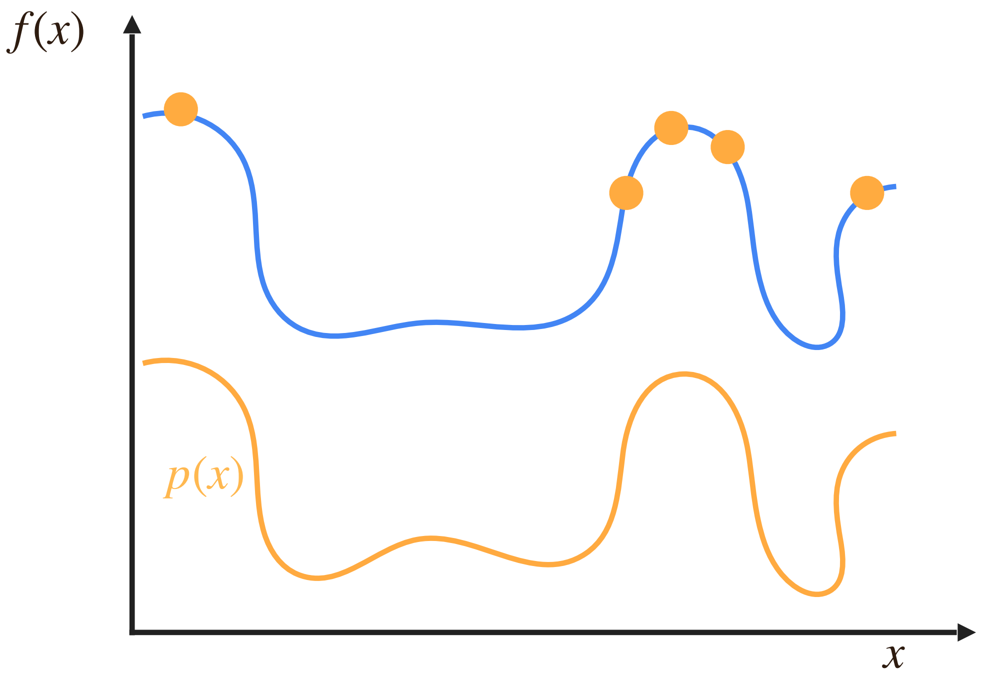


Importance Sampling



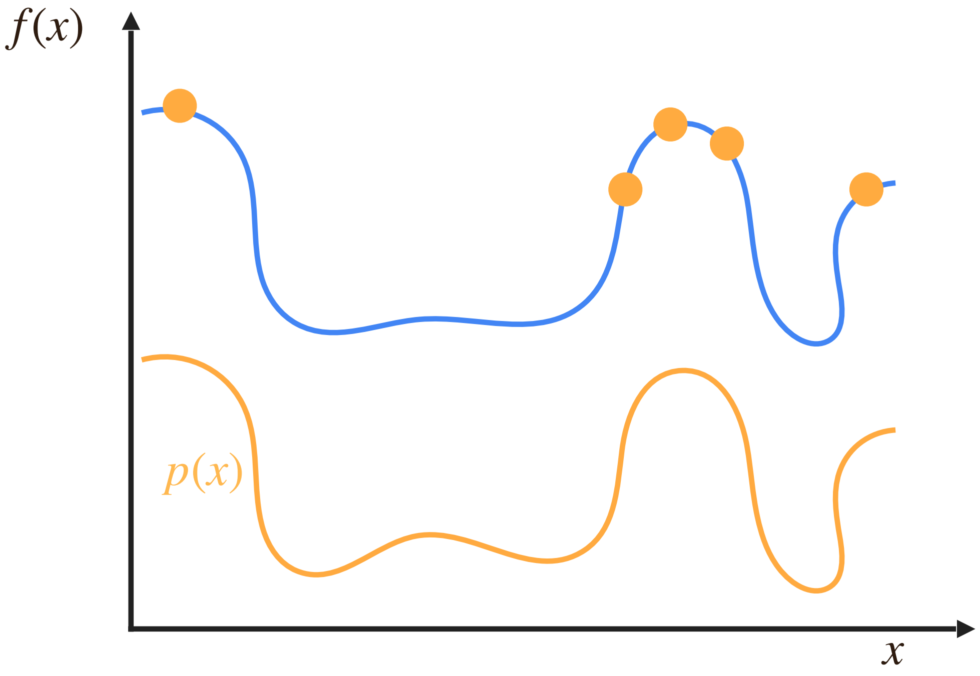
Control Variate

BACKGROUND



Importance Sampling

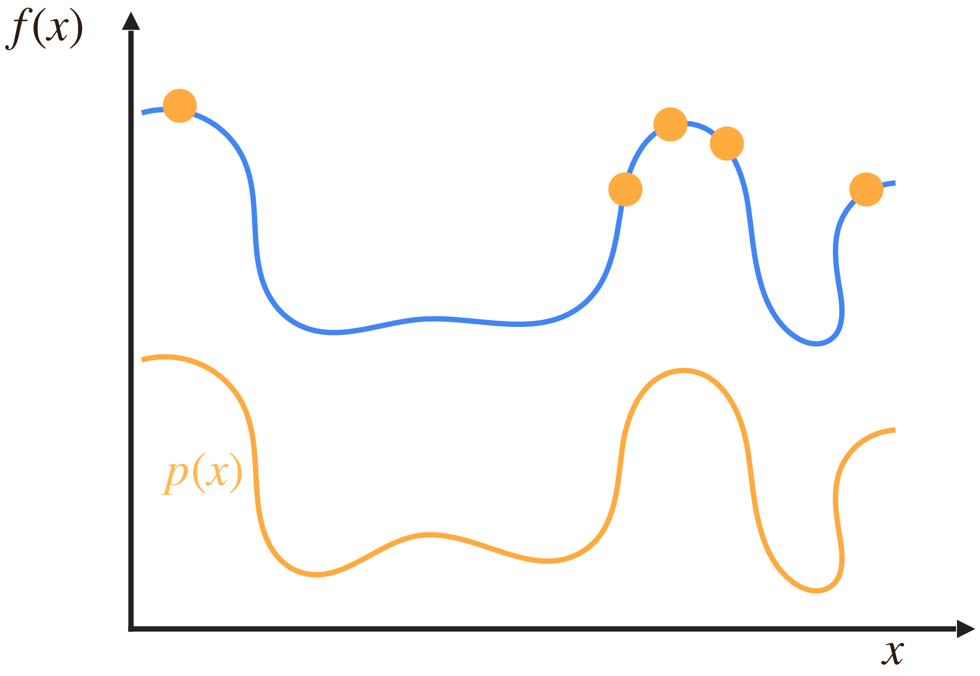
IMPORTANCE SAMPLING



$$F = \int_X f(x)\mathrm{d}x$$

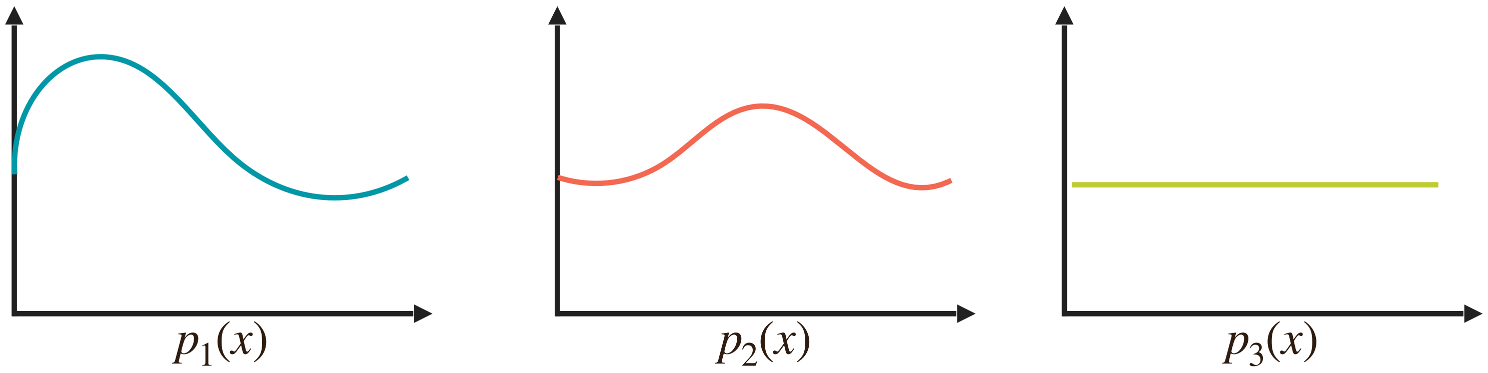
$$\approx \langle F \rangle = \frac{1}{n}\sum_{i=1}^{n} \frac{f\left(x_i\right)}{p\left(x_i\right)}$$

IMPORTANCE SAMPLING



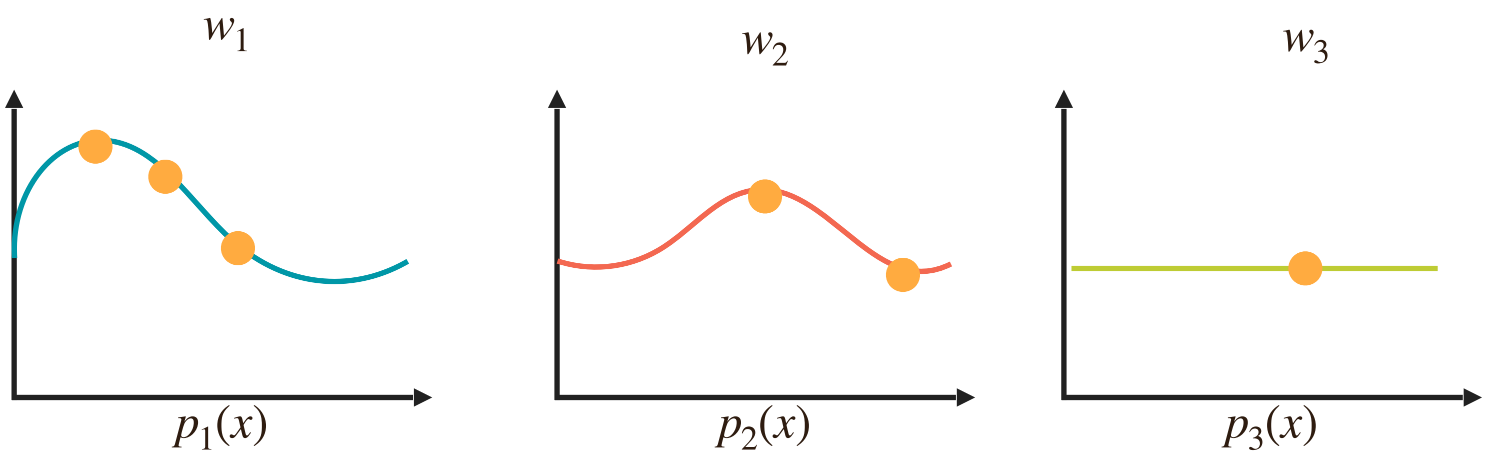
HARD!

IMPORTANCE SAMPLING

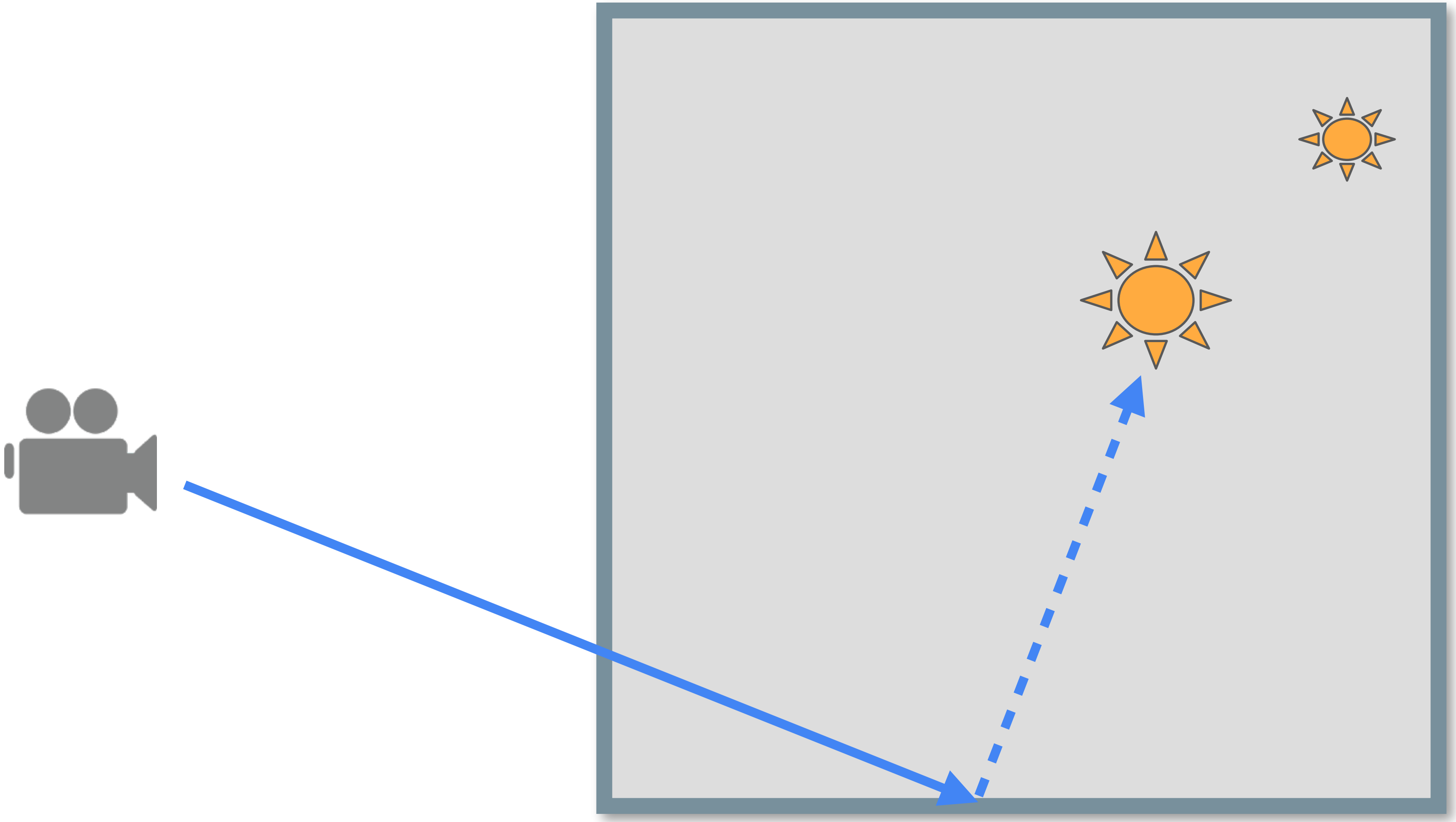


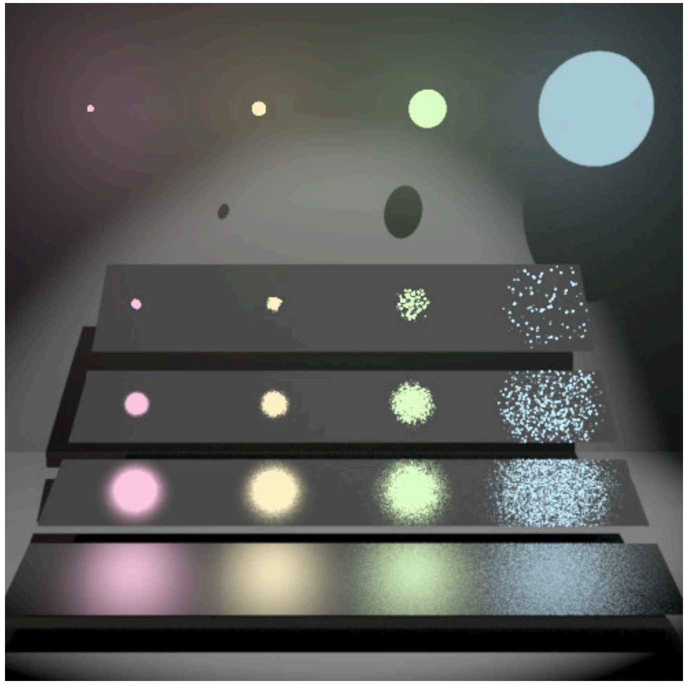
MULTIPLE IMPORTANCE SAMPLING

$$\langle F \rangle_{\mathrm{MIS}} = \sum_{k} \sum_{i=1}^{n_k} \frac{w_k\left(x_{i,k}\right) f\left(x_{i,k}\right)}{n_k p_k\left(x_{i,k}\right)}.$$



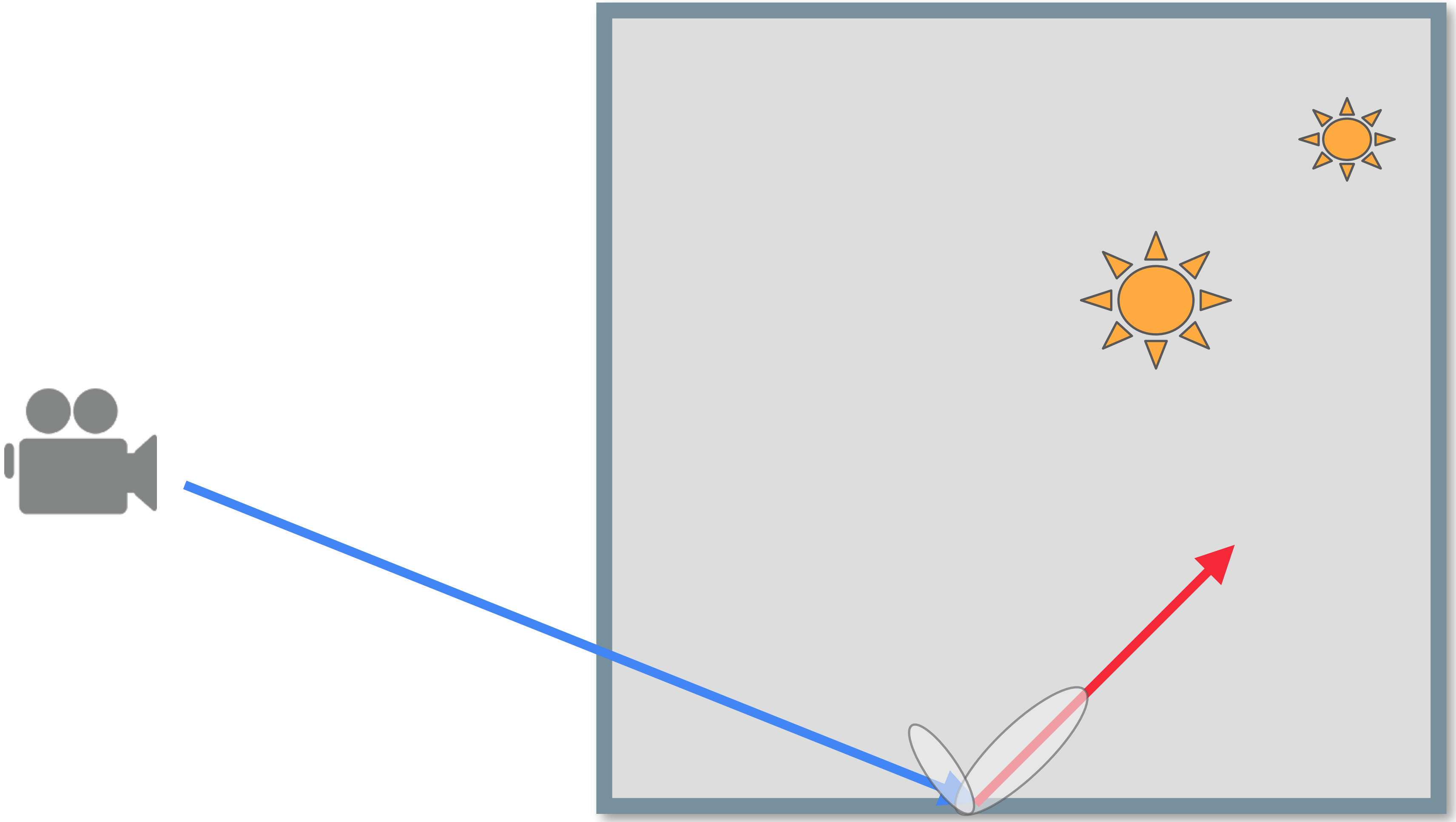
MULTIPLE IMPORTANCE SAMPLING

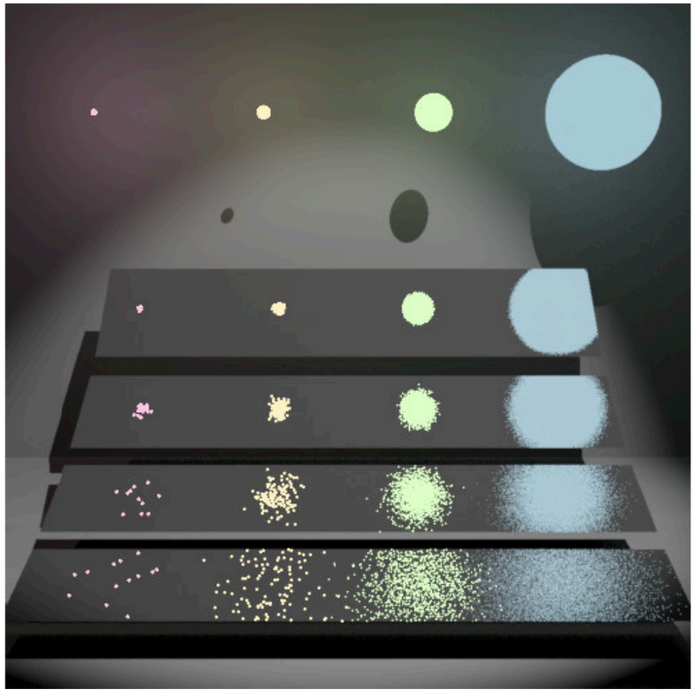




light source sampling
[Veach 1997]

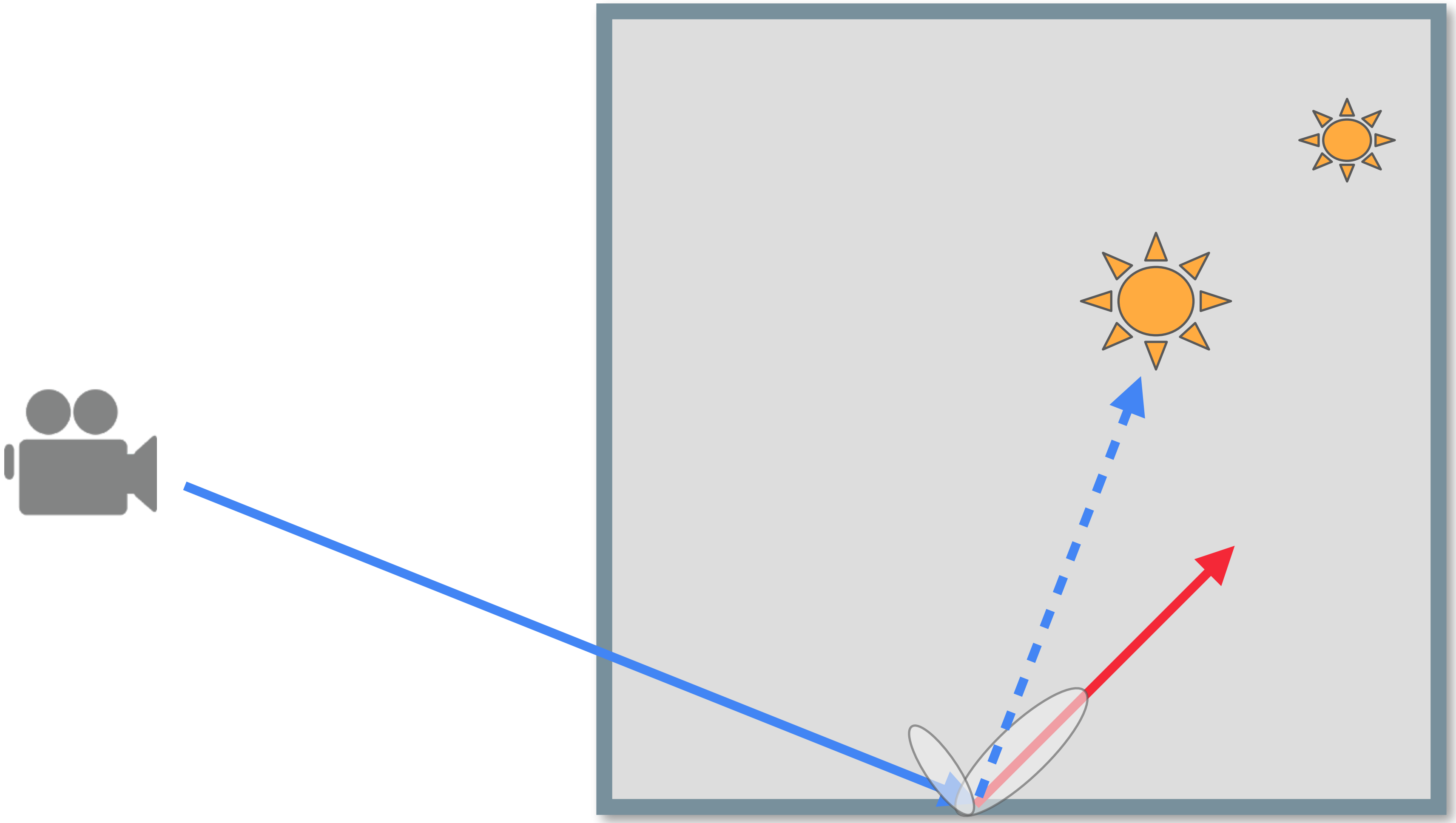
MULTIPLE IMPORTANCE SAMPLING

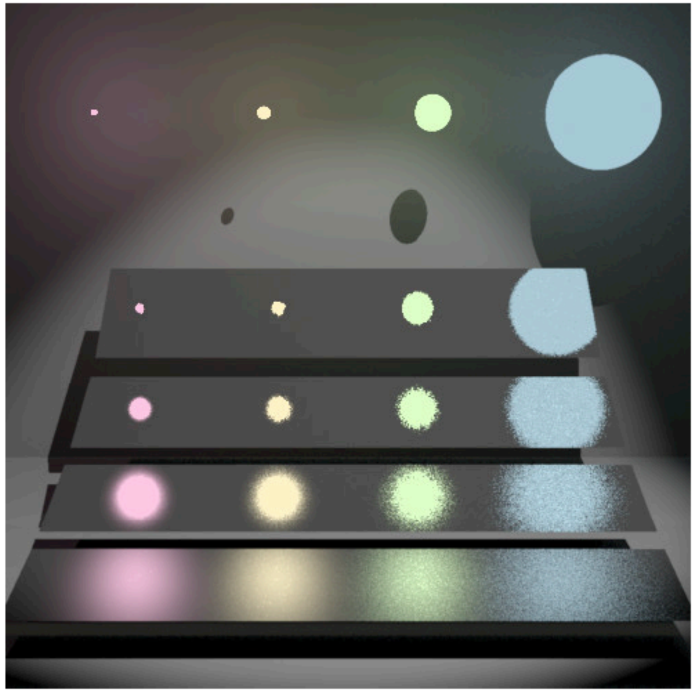




BSDF sampling
[Veach 1997]

MULTIPLE IMPORTANCE SAMPLING

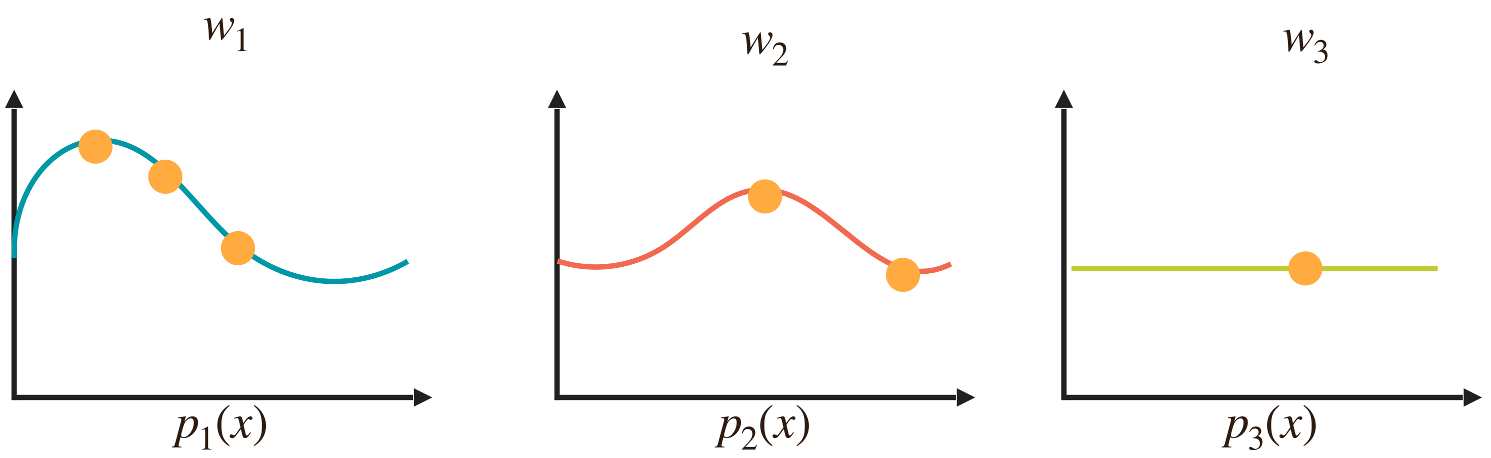




MIS weighted
[Veach 1997]

MULTIPLE IMPORTANCE SAMPLING

$$\langle F \rangle_{\text{MIS}} = \sum_{k} \sum_{i=1}^{n_k} \frac{w_k(x_{i,k}) f(x_{i,k})}{n_k p_k(x_{i,k})}.$$



MULTIPLE IMPORTANCE SAMPLING

$$\langle F \rangle_{\mathrm{MIS}} = \sum_{k} \sum_{i=1}^{n_k} \frac{w_k\left(x_{i,k}\right) f\left(x_{i,k}\right)}{n_k p_k\left(x_{i,k}\right)}.$$

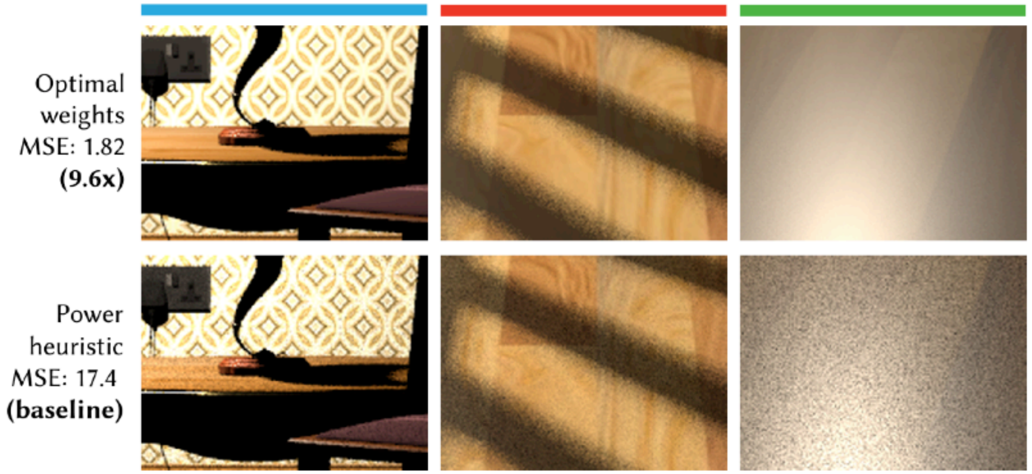
- **Different weighting functions**
  - Provably good: Balance, Power, Maximum [Veach and Guibas 1995]

$$\hat{w}_i(x) = \frac{n_i p_i(x)}{\sum_k n_k p_k(x)}.$$

MULTIPLE IMPORTANCE SAMPLING

$$\langle F \rangle_{\mathrm{MIS}} = \sum_{k} \sum_{i=1}^{n_k} \frac{w_k(x_{i,k}) f(x_{i,k})}{n_k p_k(x_{i,k})} .$$

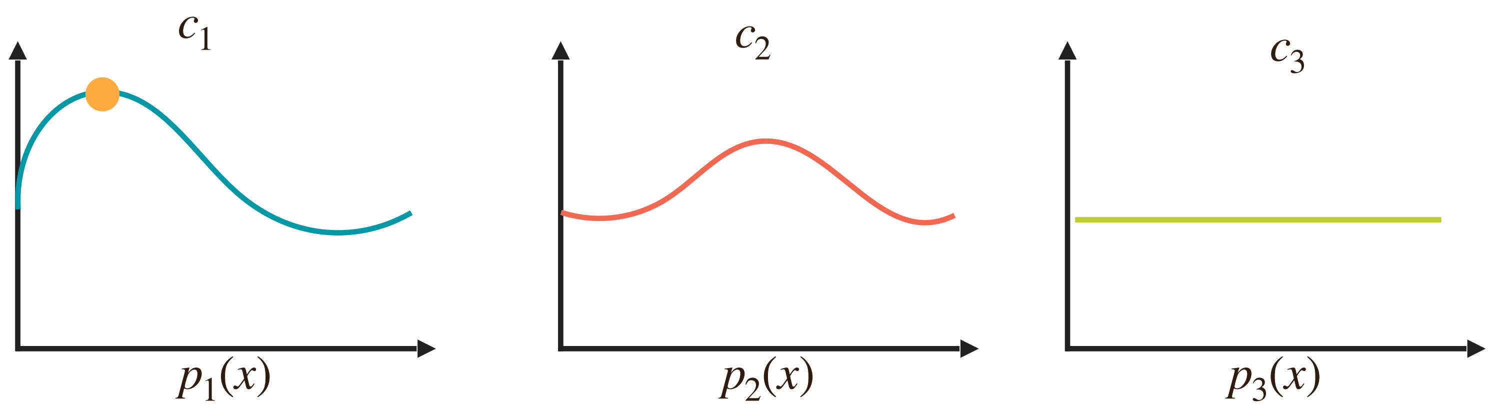
- **Different weighting functions**
  - Provably good: Balance, Power, Maximum [Veach
  - Optimal: Optimal MIS [Kondapaneni et al. 2019]



[Kondapaneni et al. 2019] Fig. 1(c)

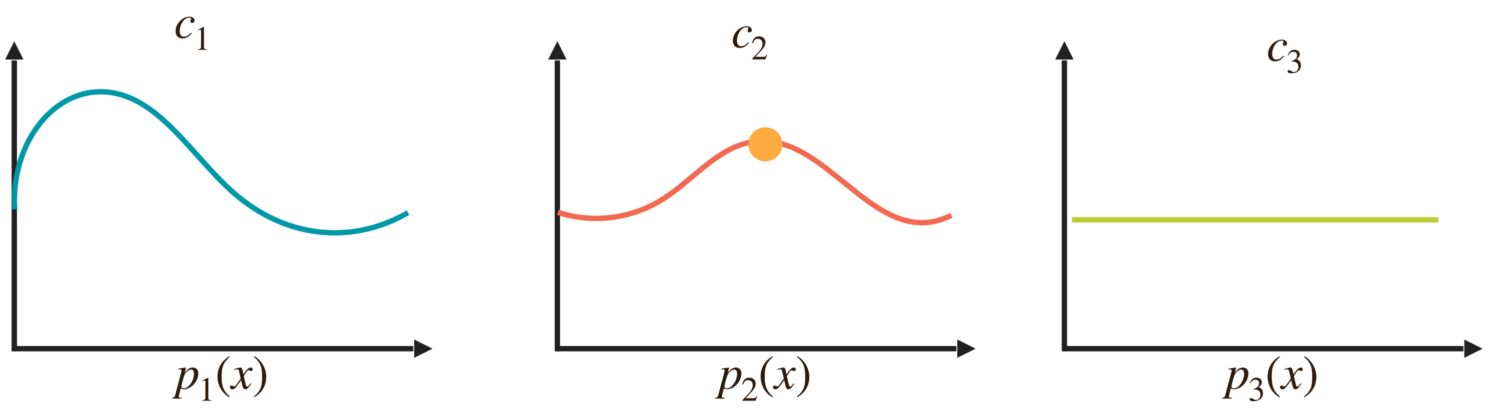
MIXTURE SAMPLING

$$\langle F \rangle_{\text{mix}} = \frac{1}{n} \sum_{i=1}^{n} \frac{f(x_i)}{\sum_k c_k p_k(x_i)}$$

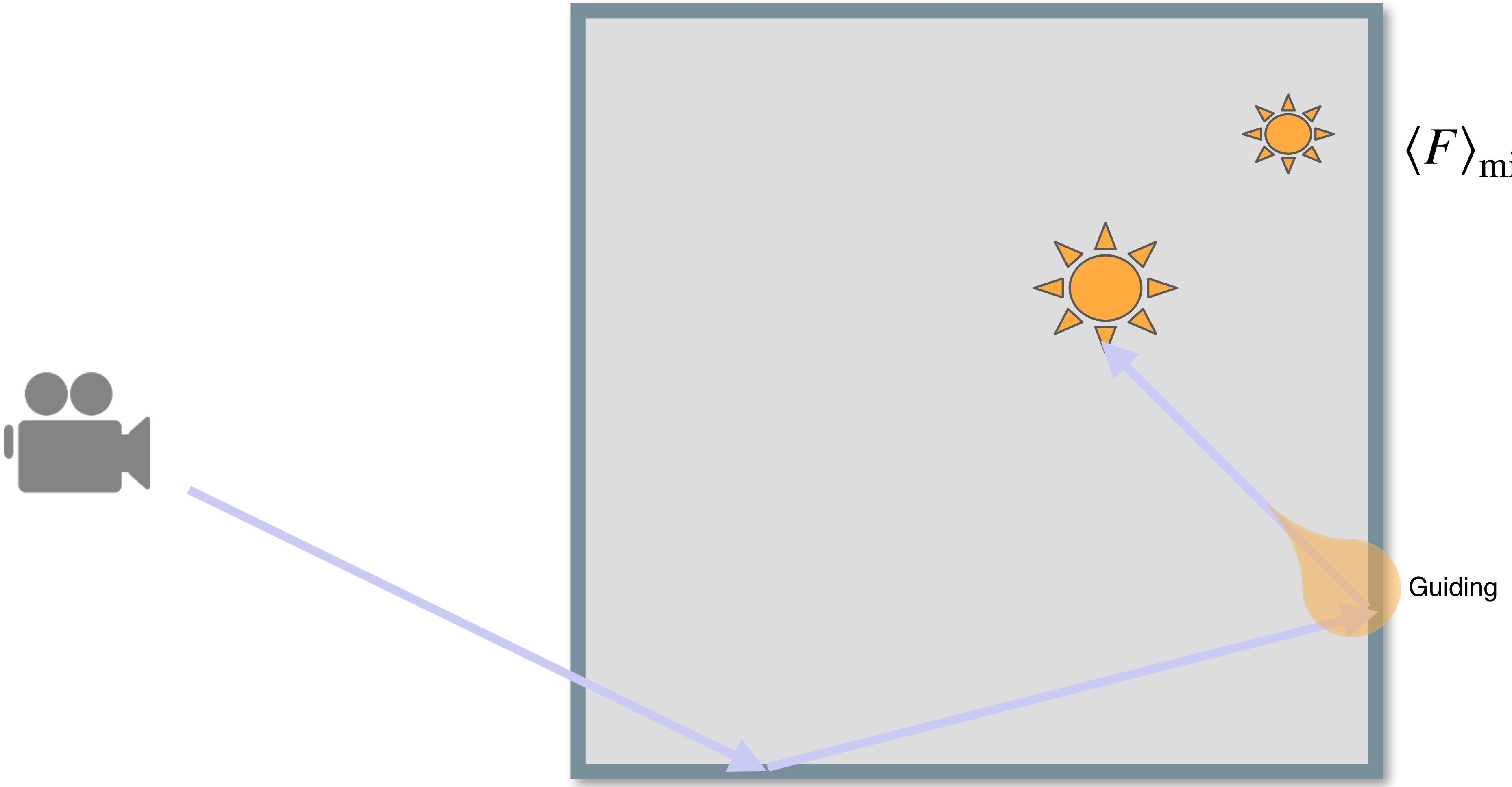


MIXTURE SAMPLING

$$\langle F \rangle_{\text{mix}} = \frac{1}{n} \sum_{i=1}^{n} \frac{f(x_i)}{\sum_k c_k p_k(x_i)}$$

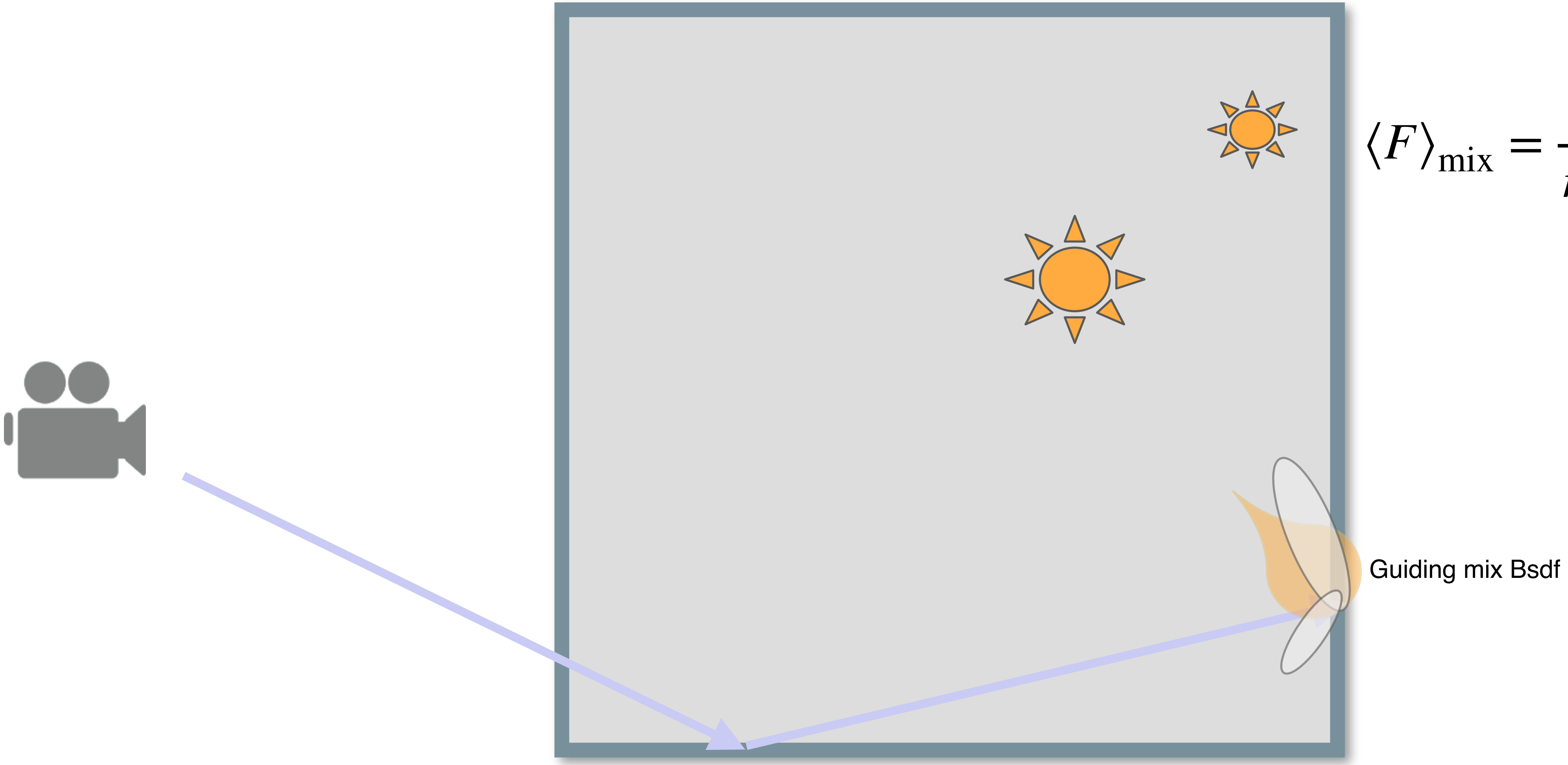


MIXTURE SAMPLING



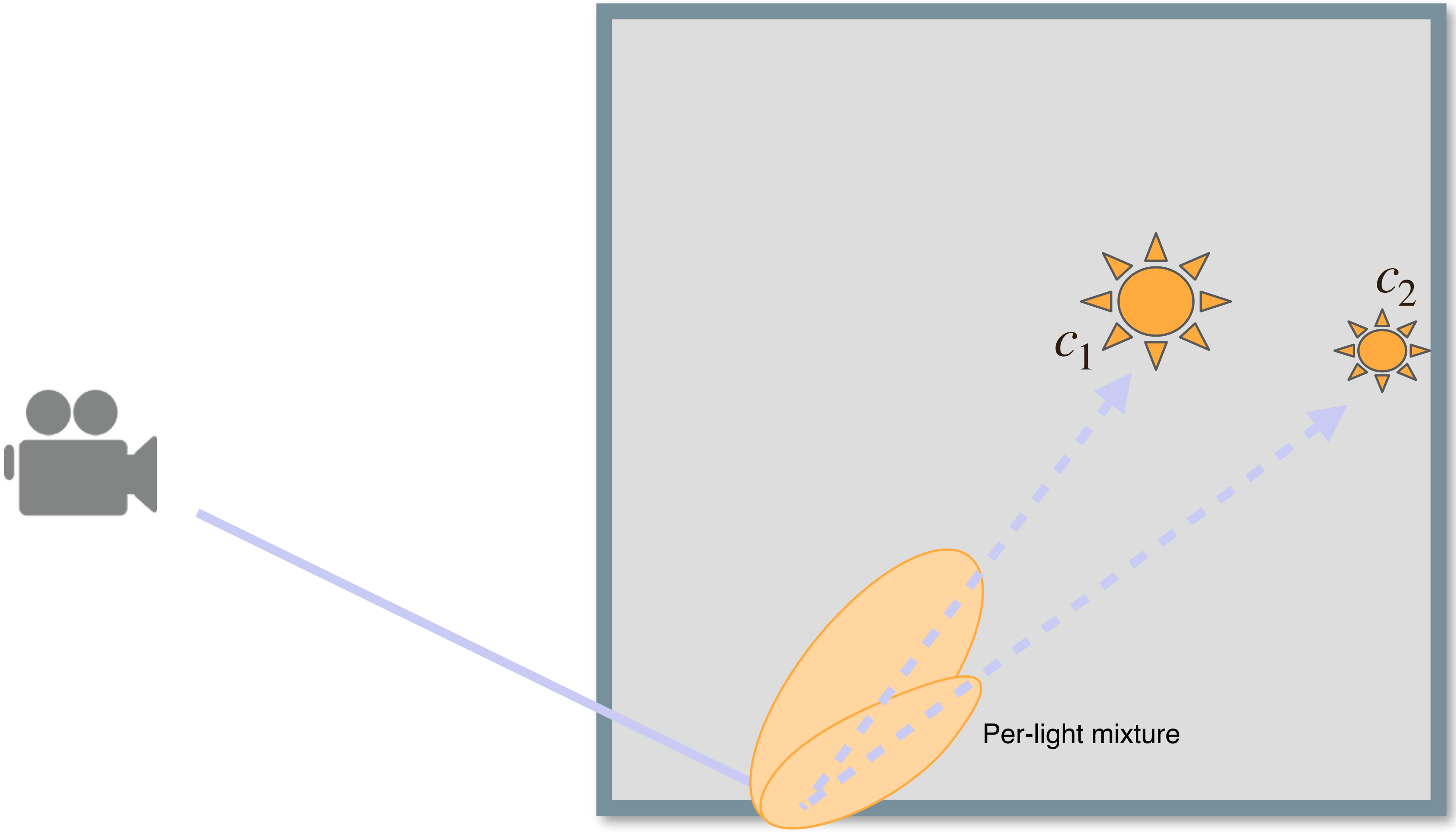
$$\langle F\rangle_{\text{mix}} = \frac{1}{n}\sum_{i=1}^{n} \frac{f(x_i)}{\sum_k c_k p_k(x_i)}$$

MIXTURE SAMPLING



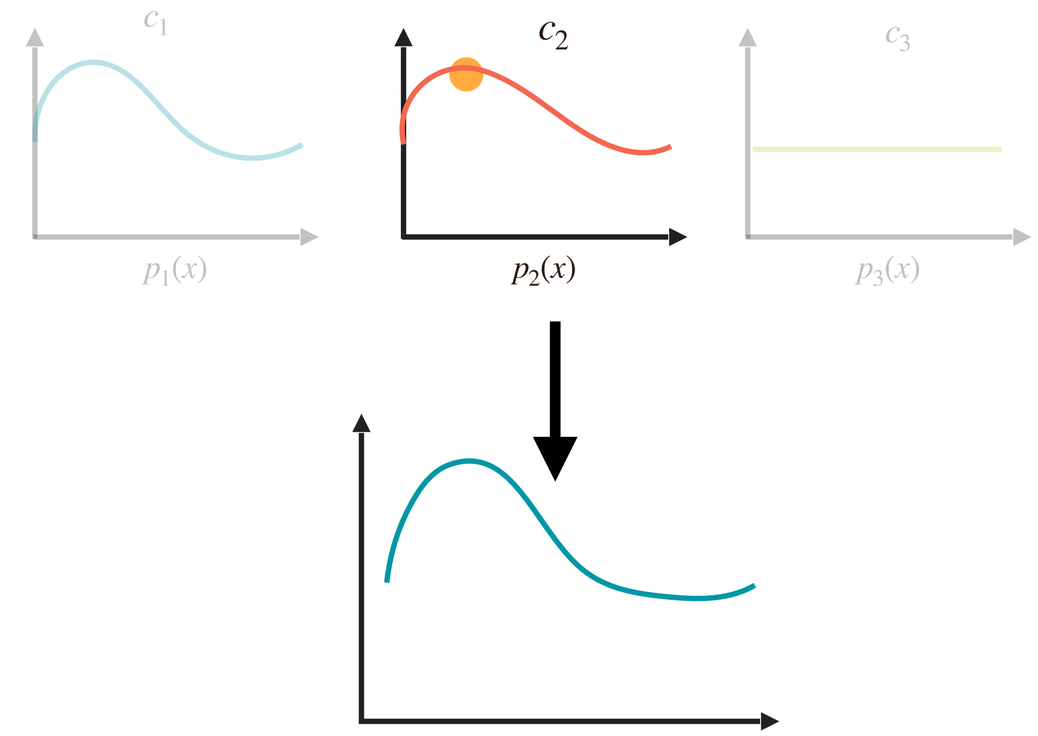
$$\langle F \rangle_{\text{mix}} = \frac{1}{n} \sum_{i=1}^{n} \frac{f(x_i)}{\sum_k c_k p_k(x_i)}$$

MIXTURE SAMPLING

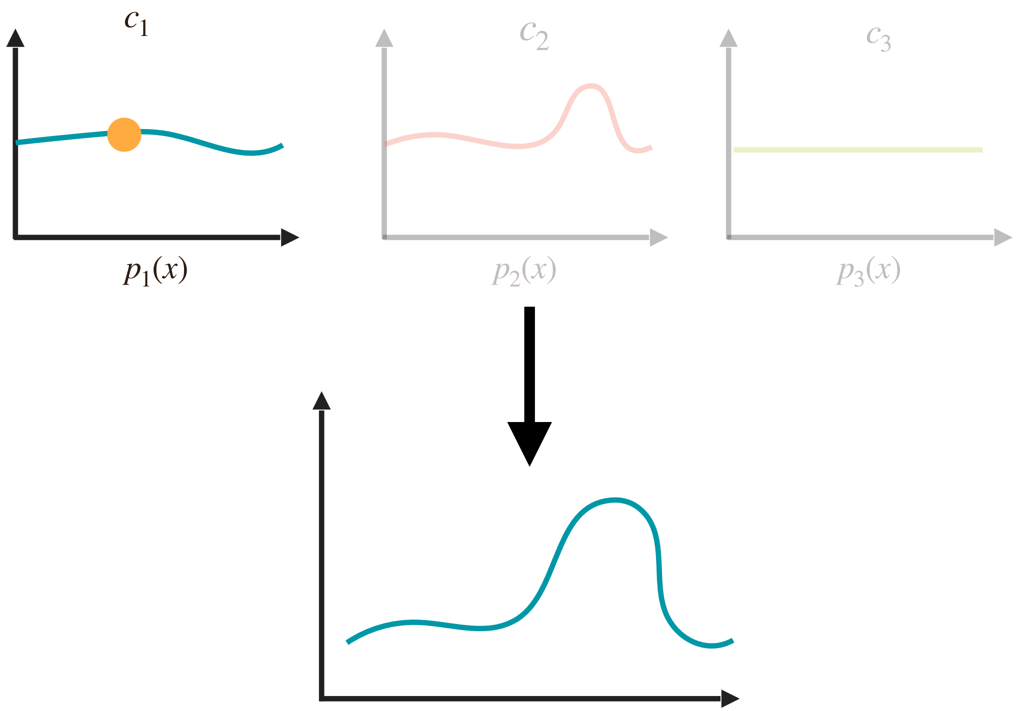


$$\langle F \rangle_{\text{mix}} = \frac{1}{n} \sum_{i=1}^{n} \frac{f(x_i)}{\sum_k c_k p_k(x_i)}$$

MIS + MIXTURE

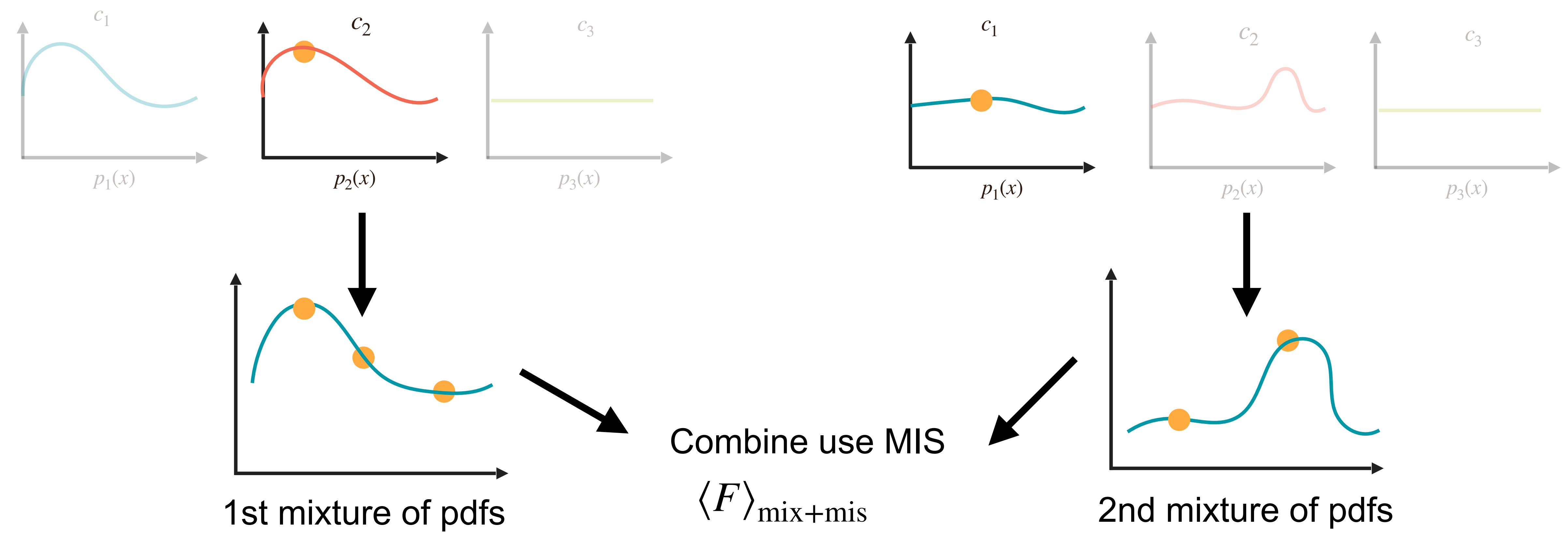


1st mixture of pdfs

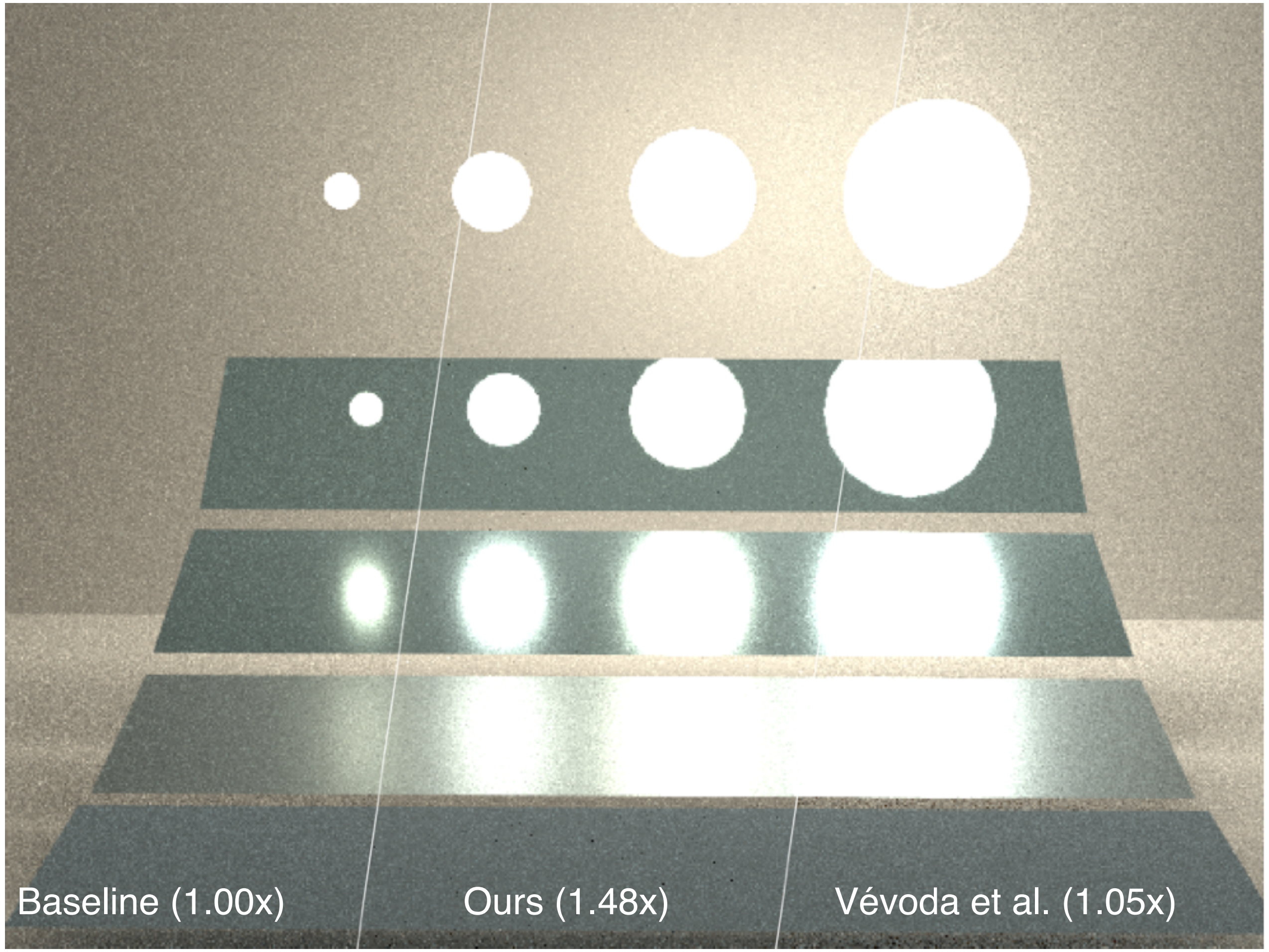


2nd mixture of pdfs

MIS + MIXTURE


c_1
c_2
c_3
$p_1(x)$
$p_2(x)$
$p_3(x)$
c_1
c_2
c_3
$p_1(x)$
$p_2(x)$
$p_3(x)$
1st mixture of pdfs
Combine use MIS
$\langle F \rangle_{\text{mix+mis}}$
2nd mixture of pdfs

OUR CONTRIBUTION



OUR CONTRIBUTION

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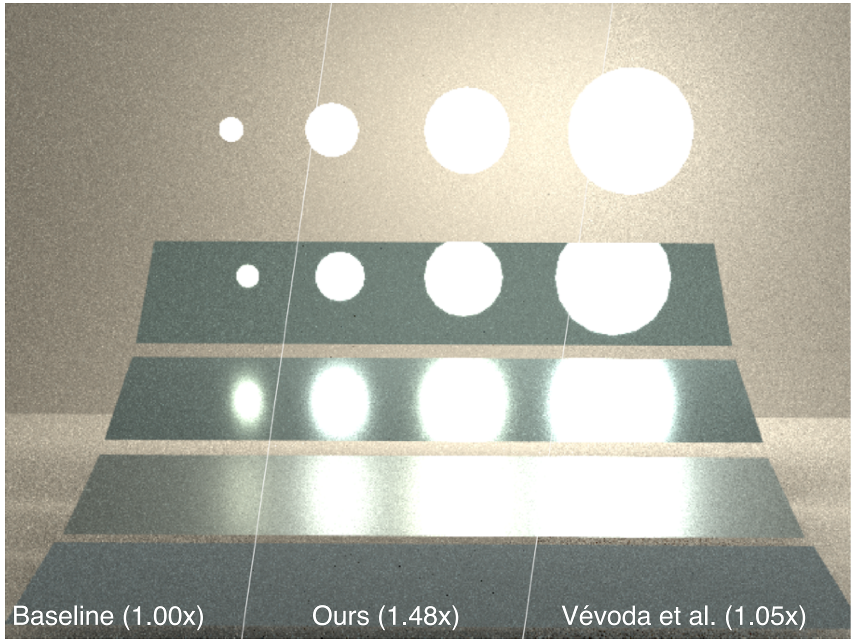
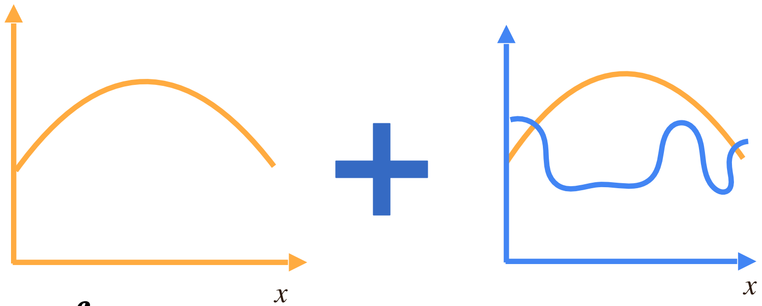
- **Generic theory**
  - Benefits from Optimal MIS and optimised mixture sampling
  - Applicable to any MIS/mixture sampling technique
  - Optimise one CV for multiple integrals => Global Illumination
- **Practical use case**
  - Multi-light & Bsdf sampling



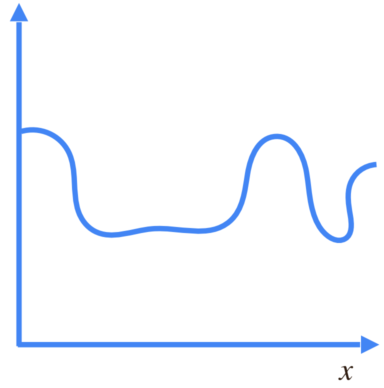
$$F = \underbrace{\int g(x)\mathrm{d}x}_{} + \int f(x) - g(x)\mathrm{d}x$$



Control Variate

CONTROL VARIATE

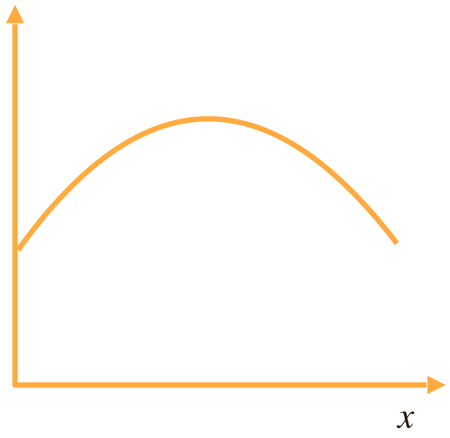
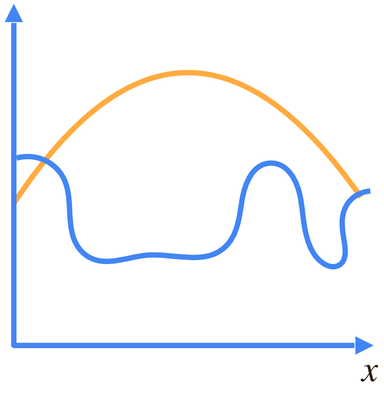
F =



$$\int f(x)\mathrm{d}x$$

CONTROL VARIATE

Low variance = Analytical + $f(x) \approx g(x)$

F =  + 

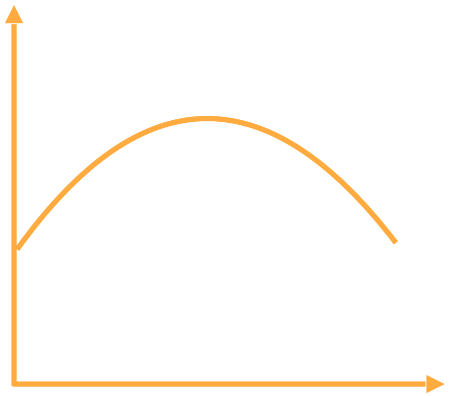
$$\int g(x)\mathrm{d}x$$

$$\int f(x) - g(x)\mathrm{d}x$$

CONTROL VARIATE

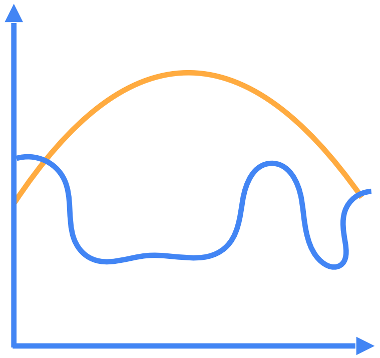
$$\langle F\rangle_{\mathrm{CV}} = G + \frac{f(x)-g(x)}{p(x)}$$

F =



$$\int g(x)\mathrm{d}x$$

+



$$\int f(x)-g(x)\mathrm{d}x$$

CONTROL VARIATE

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$$\langle F \rangle_{\mathrm{CV}} = G + \frac{f(x) - g(x)}{p(x)}$$

(p_i integrates to 1)

Owen and Zhou 2000
Mixture Sampling

$$\langle F \rangle_{\mathrm{mixCV}} = \sum_{i}^{N} \alpha_i + \frac{f(x) - \sum_k \alpha_k p_k(x)}{\sum_k c_k p_k(x)}$$

CONTROL VARIATE

$$\langle F \rangle_{\mathrm{CV}} = G + \frac{f(x) - g(x)}{p(x)}$$

(p_i integrates to 1)

Owen and Zhou 2000
Mixture Sampling

$$g(x) = \sum_k \alpha_k p_k(x)$$

CONTROL VARIATE

$$\langle F \rangle_{\mathrm{CV}} = G + \frac{f(x) - g(x)}{p(x)}$$

(p_i integrates to 1)

Owen and Zhou 2000
Mixture Sampling

$$\langle F \rangle_{\mathrm{mixCV}} = \sum_{i}^{N} \alpha_i + \frac{f(x) - \sum_k \alpha_k p_k(x)}{\sum_k c_k p_k(x)}$$

Kondapaneni et al. 2019
Optimal MIS

$$\langle F \rangle^{o} = \sum_{i=1}^{N} \alpha_i + \sum_{i=1}^{N} \sum_{j=1}^{n_i} \left(\frac{f(x_{ij}) - \sum_k \alpha_k p_k(x_{ij})}{\sum_k n_k p_k(x_{ij})} \right)$$

OUR THEORY

(p_i integrates to 1)

Owen and Zhou 2000
Mixture Sampling

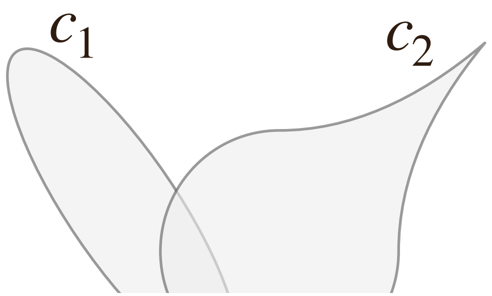
$$\langle F \rangle_{\text{mixCV}} = \sum_{i}^{N} \alpha_i + \frac{f(x) - \sum_k \alpha_k p_k(x)}{\sum_k c_k p_k(x)}$$

Kondapaneni et al. 2019
Optimal MIS

$$\langle F \rangle^{o} = \sum_{i=1}^{N} \alpha_i + \sum_{i=1}^{N} \sum_{j=1}^{n_i} \left(\frac{f(x_{ij}) - \sum_k \alpha_k p_k(x_{ij})}{\sum_k n_k p_k(x_{ij})} \right)$$

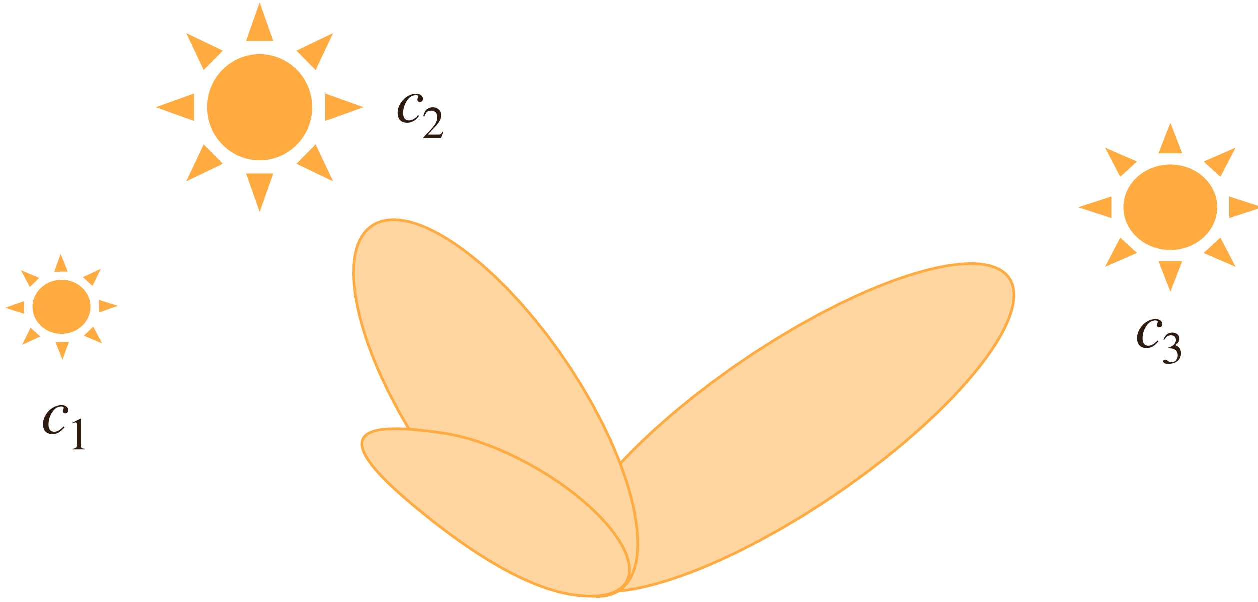
Same time!

OUR EXAMPLE APPLICATION



BSDF Mixture

OUR EXAMPLE APPLICATION



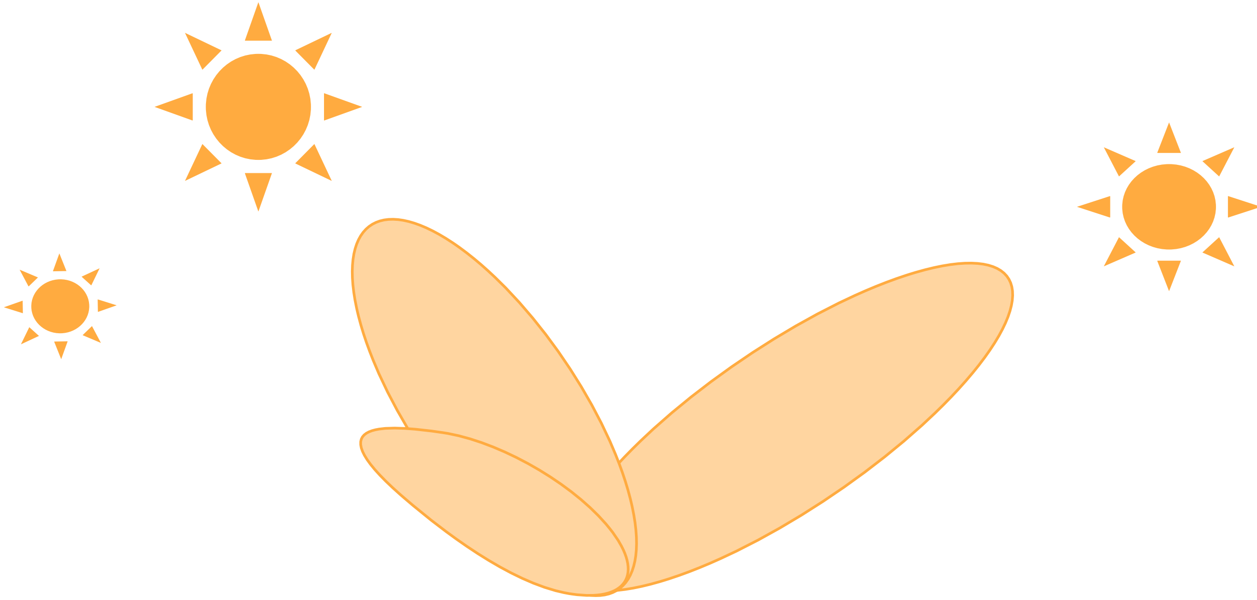
Per-light Mixture

OUR EXAMPLE APPLICATION

BSDF Mixture

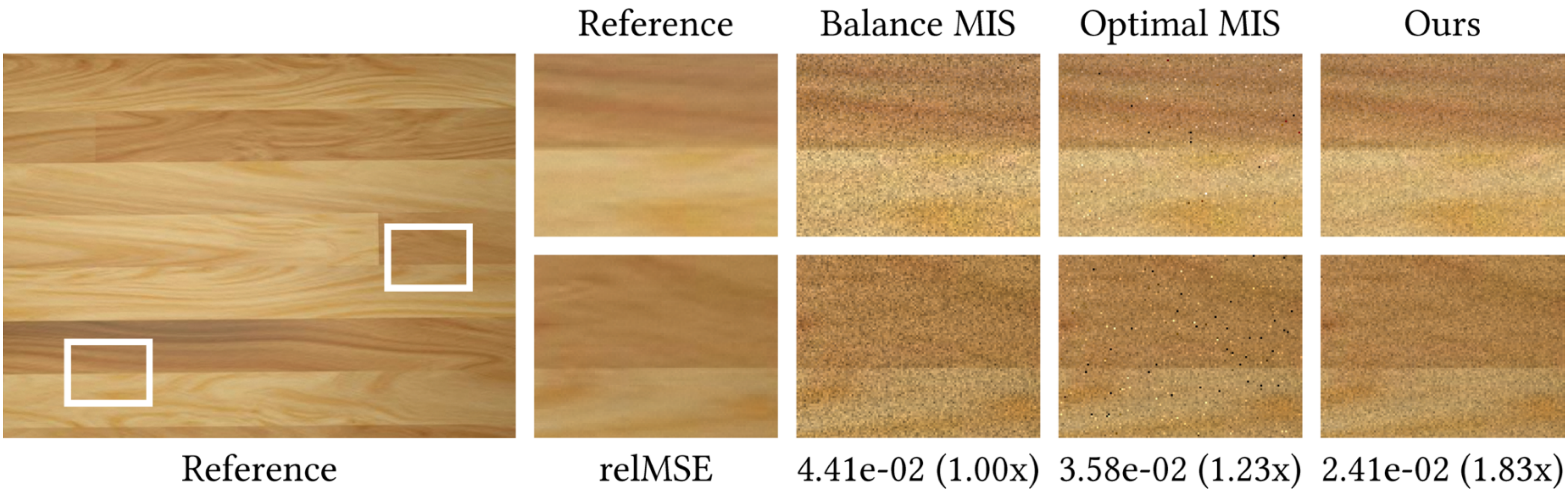
+

MIS



Per-light Mixture

DEFENSIVE SAMPLING



PREVIOUS WORK

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No high dimension!

EUROGRAPHICS 2006 / E. Gröller and L. Szirmay-Kalos
(Guest Editors)

Optimizing Control Variate Estimators

Shaohua Fan[1], Stephen Chenney[2], Bo Hu[1], Kam-Wah Tsui[1],

[1]University of Wisconsin – Madison
[2]Emergent Game Technologies

Abstract

We present the Optimizing Control Variate (OCV) estimator, a new estimator fo upon a deterministic sampling framework, OCV allows multiple importance sai in one algorithm. Its optimizing nature addresses a major problem with contr ing: users supply a generic correlated function which is optimized for each esti tuned one that must work well everywhere. We demonstrate OCV with both direc examples, showing improvements in image error of over 35% in some cases, for

Categories and Subject Descriptors (according to ACM CCS): I.3.7 [Compute Graphics and Realism Color, shading, shadowing, and texture G.3 [Probability ar rithms

Keywords: *direct lighting, deterministic mixture sampling, control variates*

Optimal Multiple Importance Sampling

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PETR VÉVODA*, Charles University, Prague and Render Legion, a. s.
PASCAL GRITTMANN, Saarland University
TOMÁŠ SKŘIVAN, IST Austria
PHILIPP SLUSALLEK, Saarland University and DFKI
JAROSLAV KŘIVÁNEK, Charles University, Prague and Render Legic

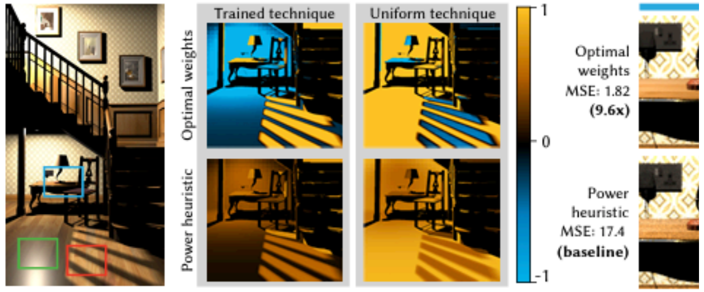


Fig. 1. Equal-sample comparison (20 per technique per pixel) of direct illumination estim (*Trained* and *Uniform*, see Sec. 8.2 for details) with our optimal weights (top row) and the per-pixel average MIS weight values as determined by the two weighting strategies. Unlike a can have *negative* values, which provides additional opportunity for variance reduction, lead power heuristic in this scene.

Regression-based Monte Carlo Integration

CORENTIN SALAÜN, Max-Planck-Institut für Informatik, Germany
ADRIEN GRUSON, McGill University & École de Technologie Supérieure, Canada
BINH-SON HUA, VinAI Research, Vietnam
TOSHIYA HACHISUKA, University of Waterloo, Canada
GURPRIT SINGH, Max-Planck-Institut für Informatik, Germany

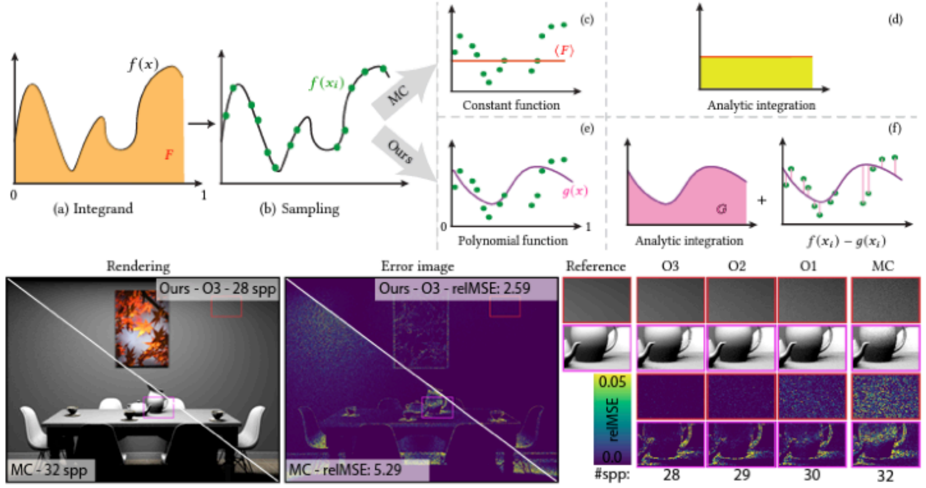
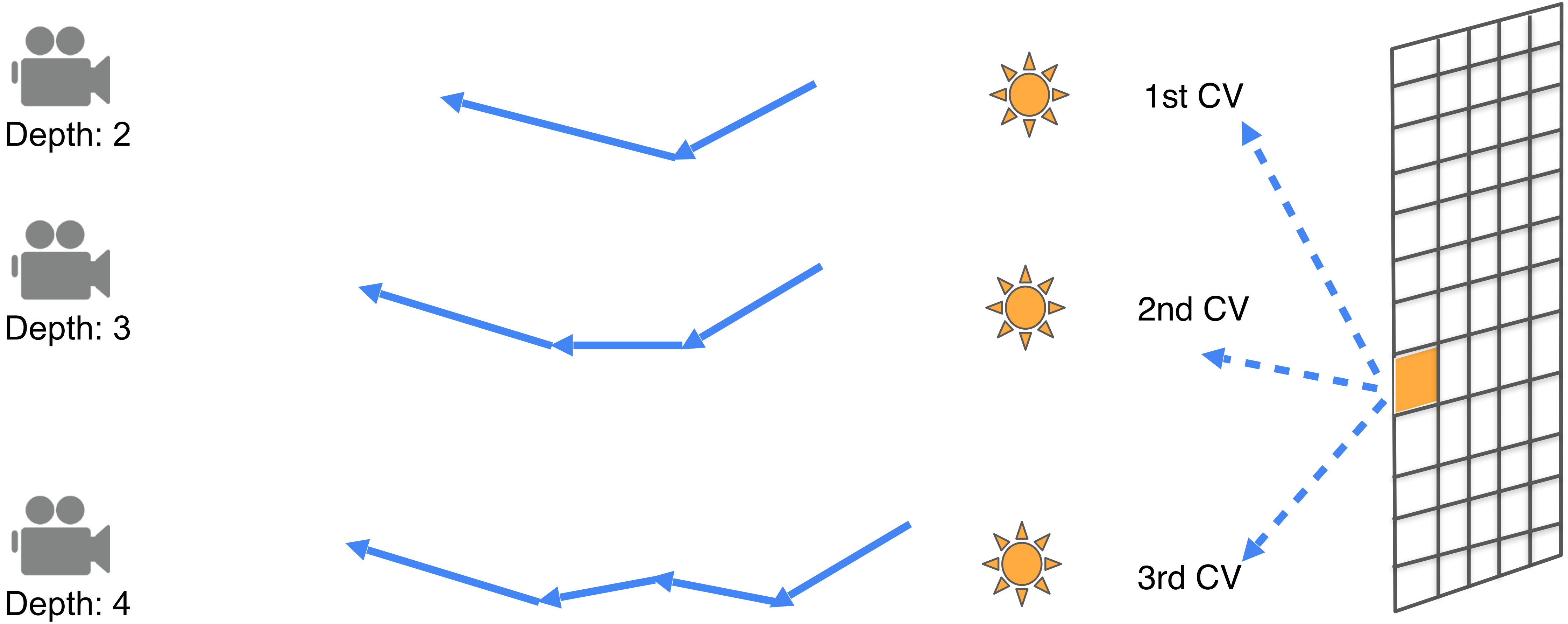


Fig. 1. Given an integrand (a), we first sample (b) $f(x)$ as in Monte Carlo (MC) integration. (c-d) Traditional MC estimator can be interpreted as fitting a *constant* model function to the sample values, with the integral of this constant function equals to F. (e) We, instead, propose to use a *non-constant* model function such as a polynomial, which is then fitted to the sampled values. (f) The resulting estimator is based on control variates; we add the analytical integral of the model function to MC integration of the difference between the original integrand and the model function. The bottom row shows renderings and the corresponding error images to demonstrate the impact of our regression approach against the traditional MC integration. The insets on the right compare our method with different orders (Ox) of polynomials. Our method has significant error reduction at equal time.

PROBLEM



PROBLEM

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$$\langle F \rangle_{\mathrm{mixCV}} = \sum_i \alpha_i + \frac{f(x) - \sum_i \alpha_i p_i(x)}{\sum_i c_i p_i(x)}$$

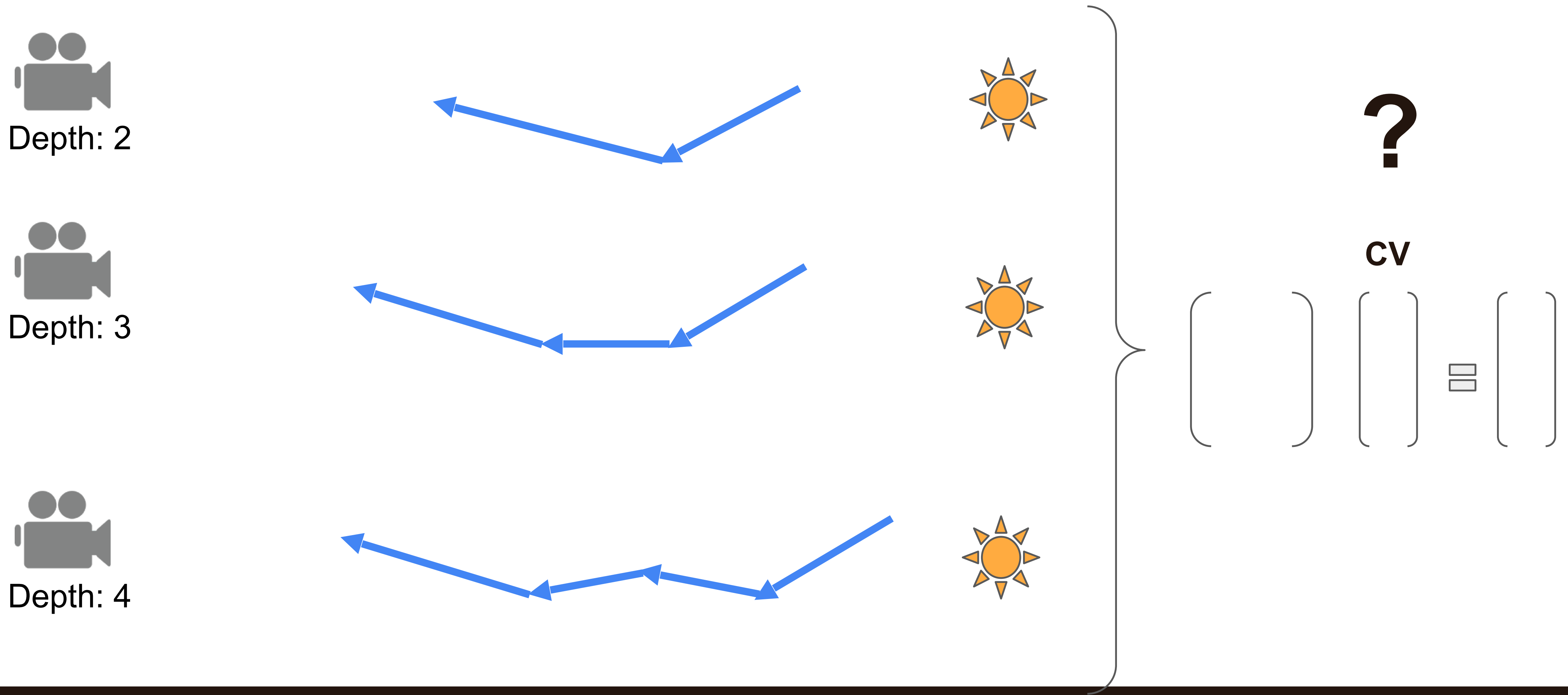
$$a_{ij} = \int \frac{p_i p_j}{\sum c_k p_k}$$

$$\underbrace{\begin{pmatrix} \boxed{a_{11}} & \cdots & a_{1N} \\ \vdots & \ddots & \vdots \\ a_{N1} & \cdots & \boxed{a_{NN}} \end{pmatrix}}_{A} \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_N \end{pmatrix} = \underbrace{\begin{pmatrix} \boxed{b_1} \\ \vdots \\ b_N \end{pmatrix}}_{b}$$

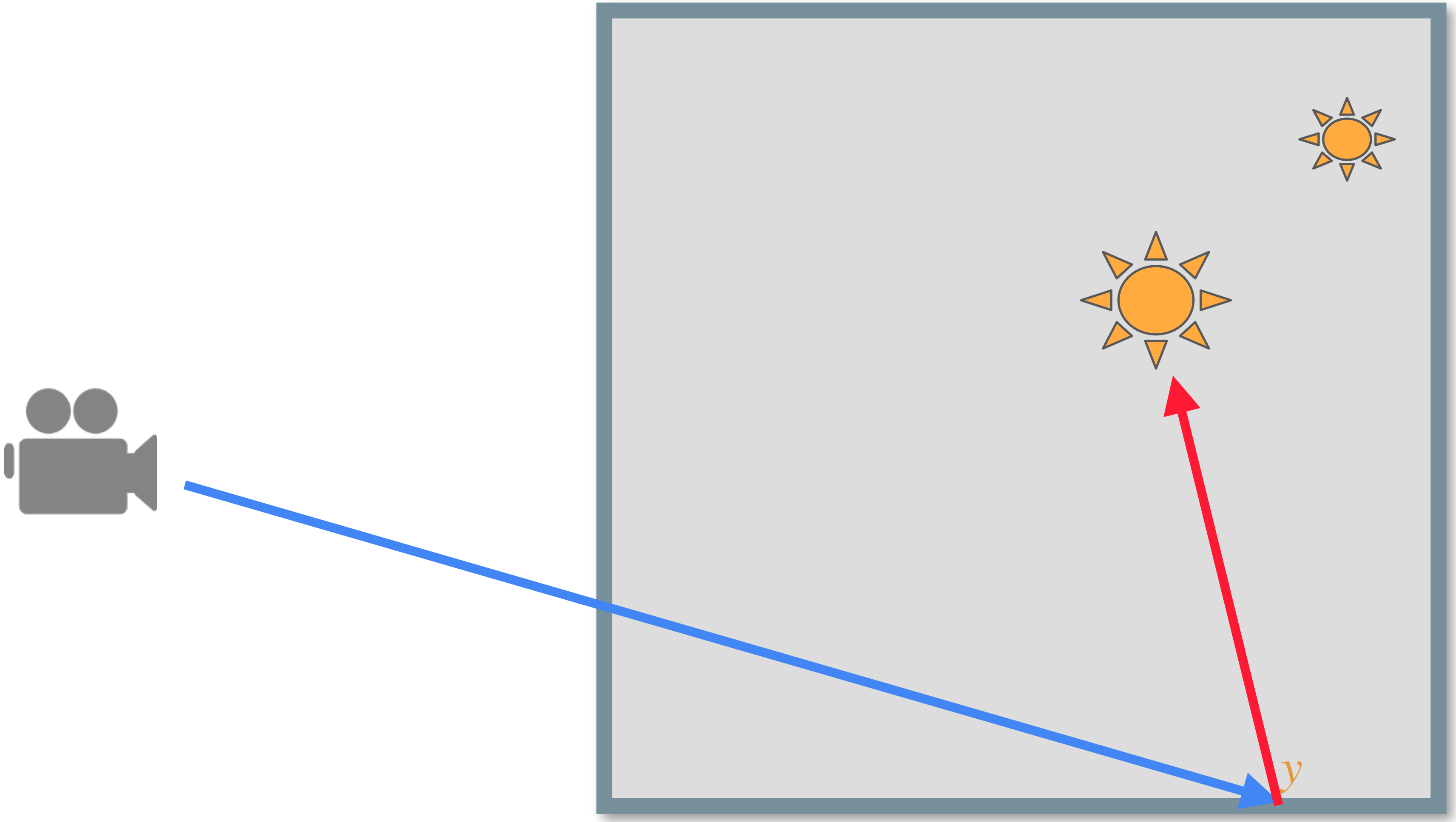
$$b_i = \int \frac{p_i f}{\sum c_k p_k}$$

#N NUMBER OF SAMPLING TECHS

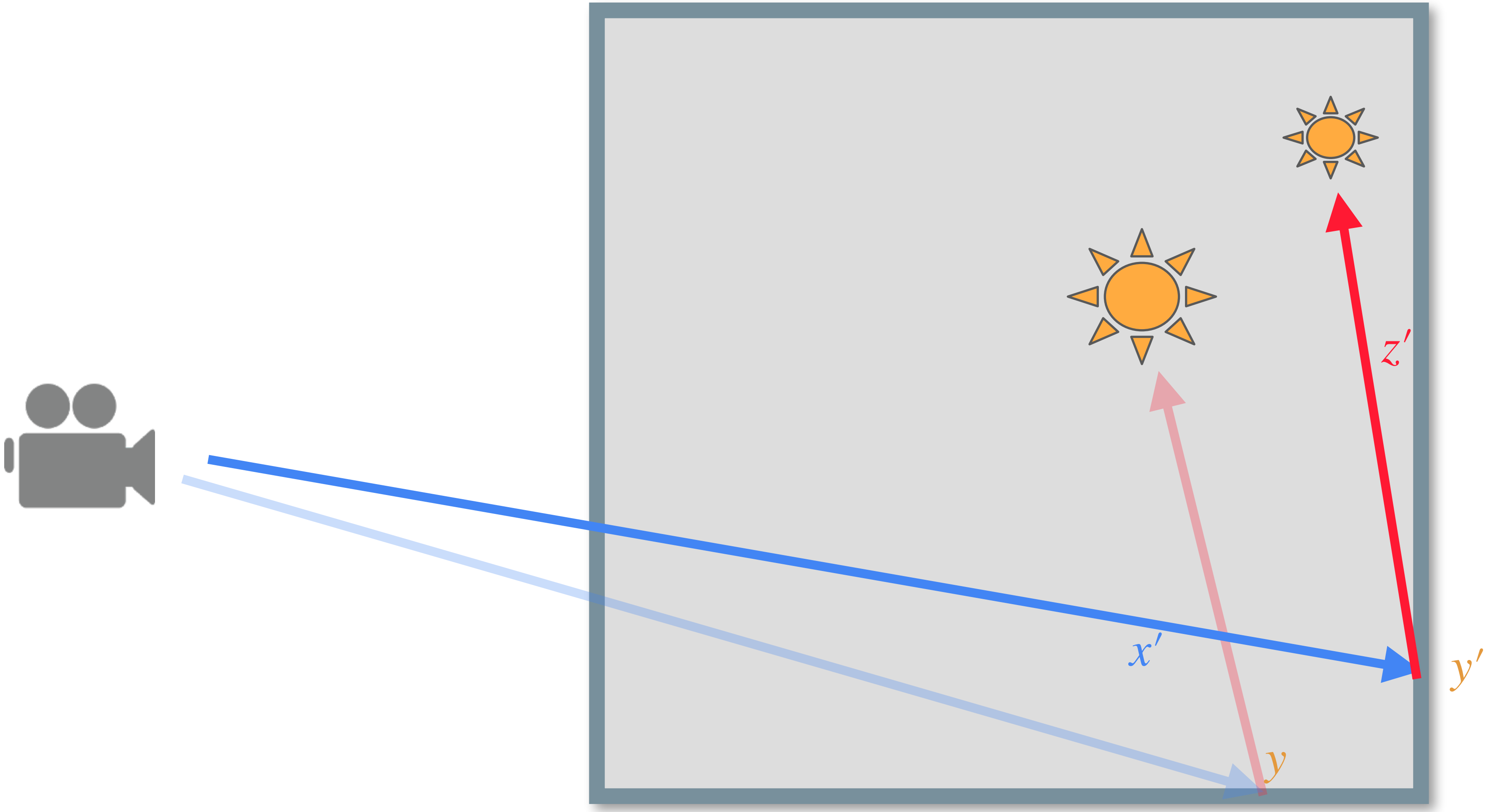
PROBLEM



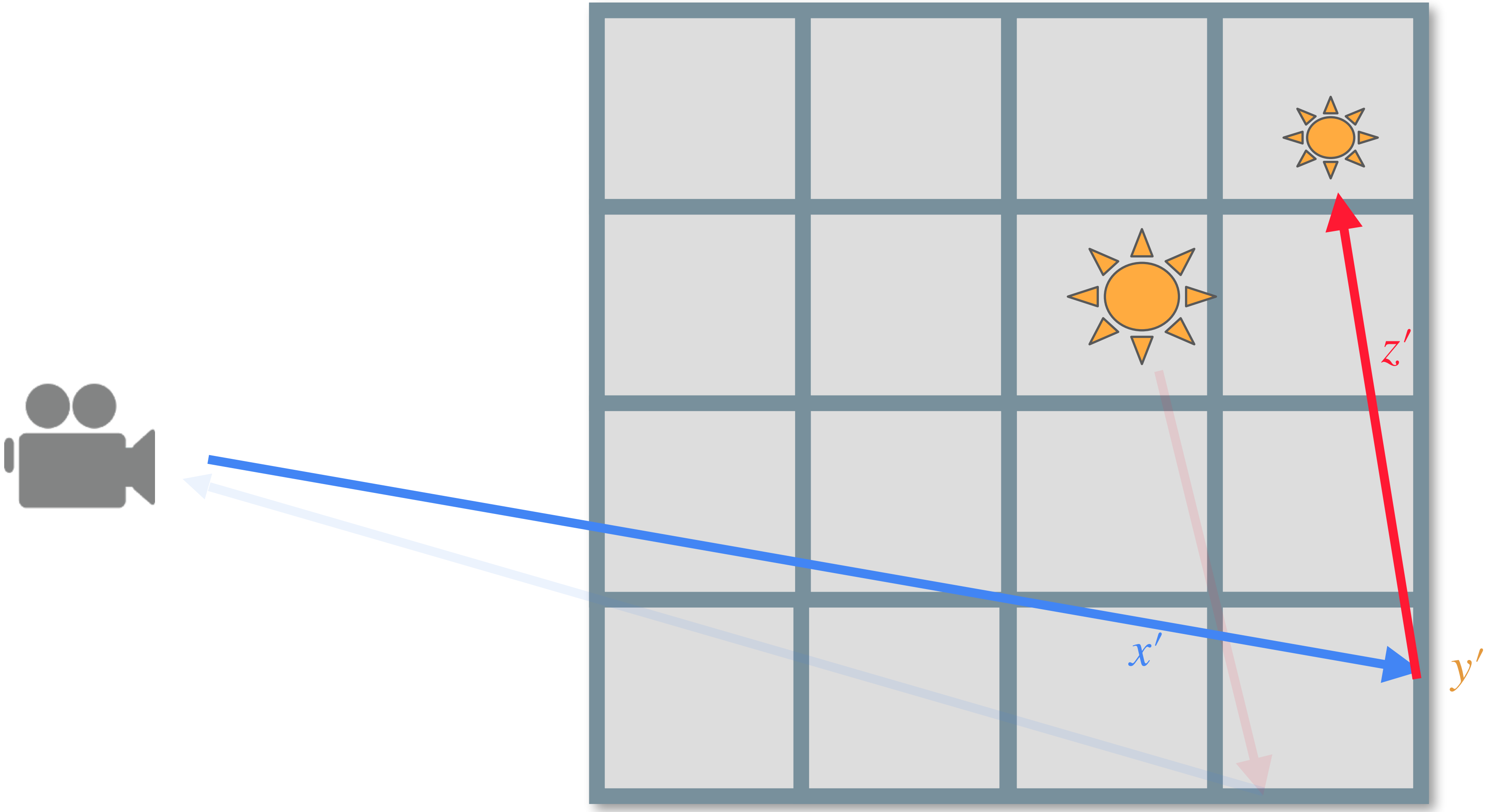
OURS



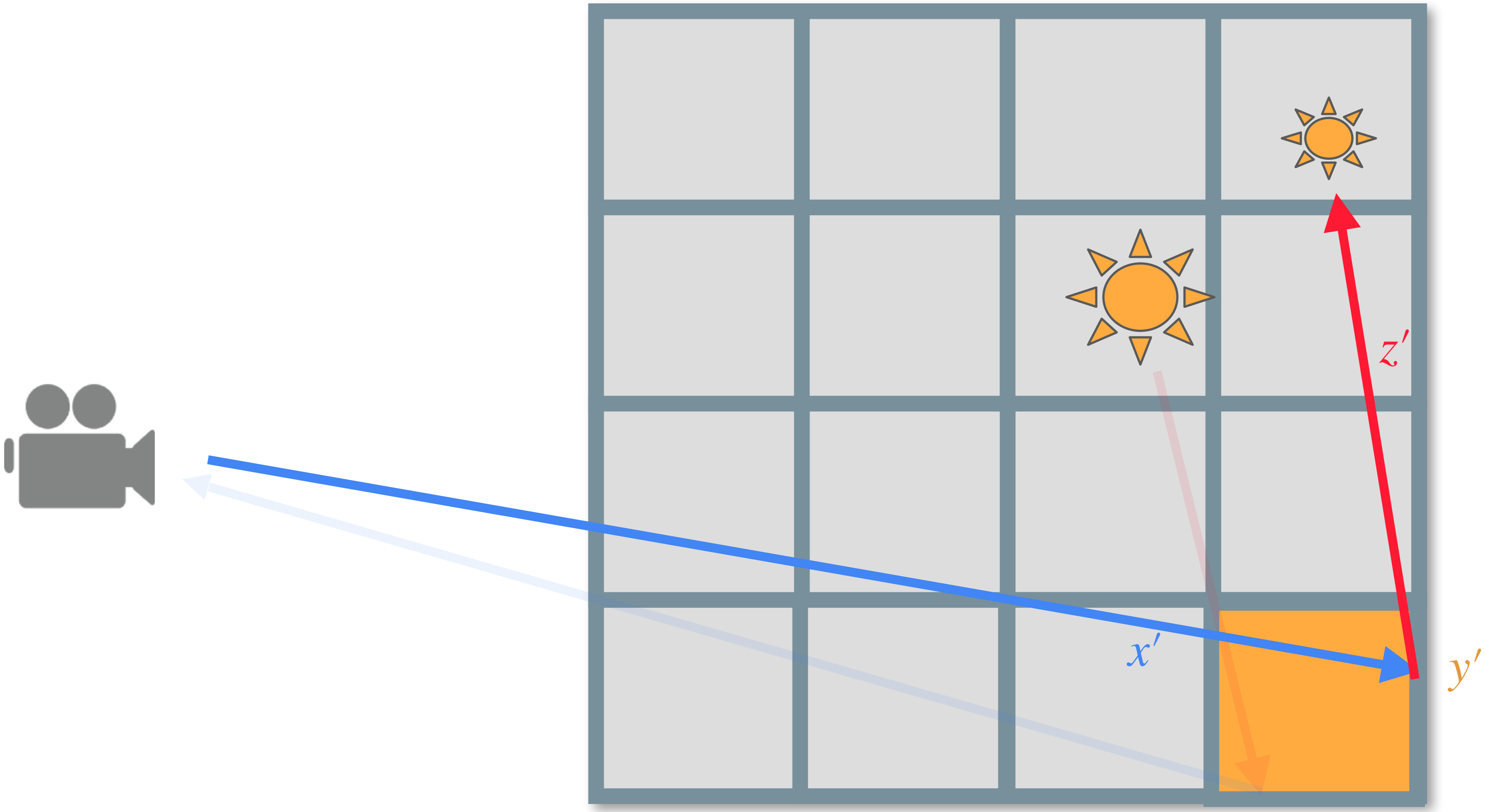
OURS



OURS

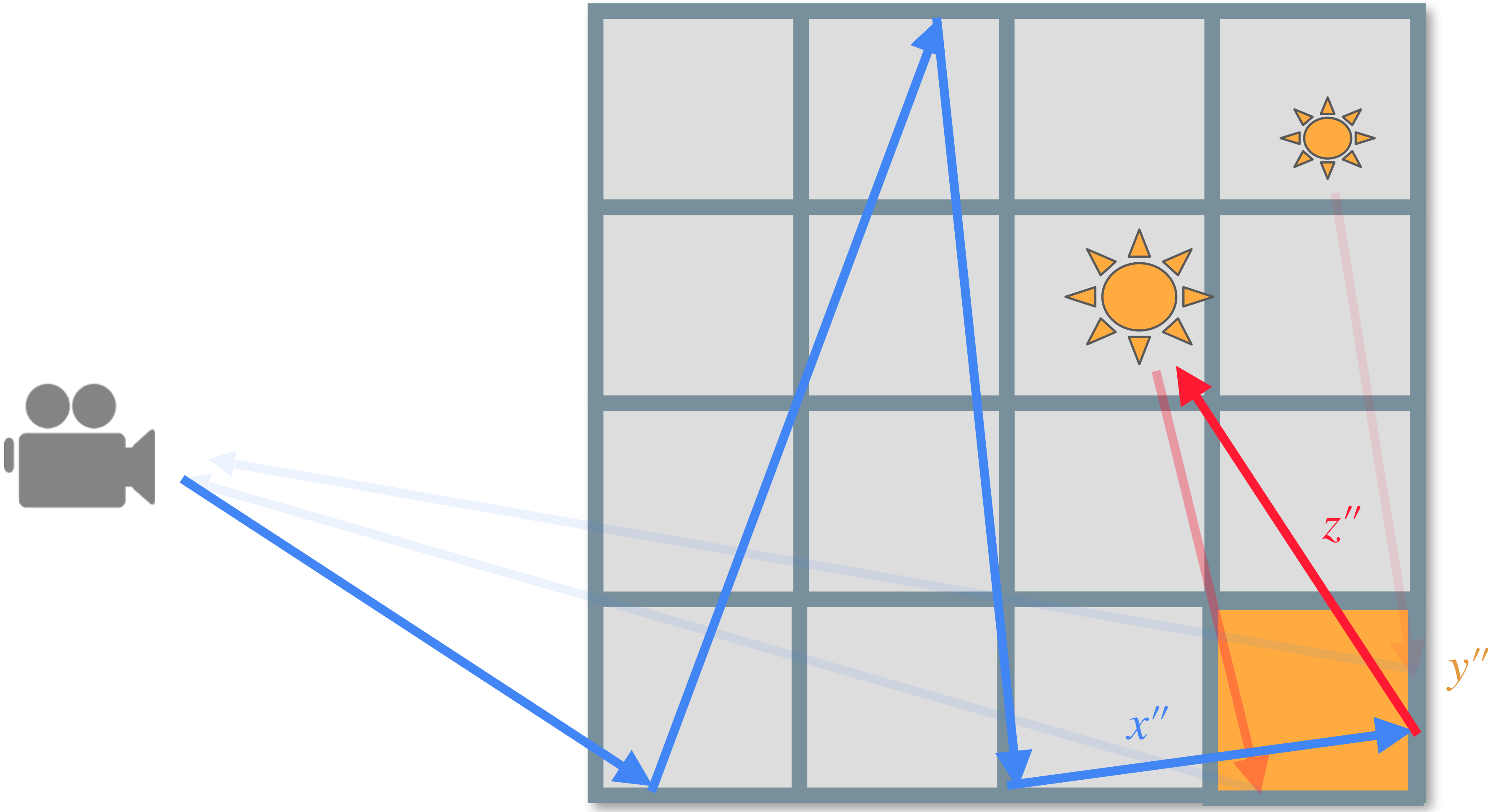


OURS



$$\sum_{i}^{N} \alpha_i p_i(z \mid y, x)$$

OURS



$$\sum_{i}^{N} \alpha_i p_i(z \mid y, x)$$

RESULTS

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PREVIOUS WORK

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Bayesian online regression for adaptive direct illumination sampling

PETR VÉVODA*, Charles University, Prague and Render Legion, a. s.
IVO KONDAPANENI*, Charles University, Prague
JAROSLAV KŘIVÁNEK, Charles University, Prague and Render Legion, a. s.

No learning
RMSE 0.0031 (0.0014 for DI only)

Our method
RMSE 0.0012, *6.7x speedup* (0.000062, *510x* for DI only)

No learning

Our method

Fig. 1. Equal-time comparison (60 s) of path-traced global illumination solutions computed using our learning-based direct illumination sampling method (right) and a baseline sampling method without learning (left). While both methods start off by sampling lights proportionally to rough estimates of their unoccluded contribution, our method progressively incorporates information about their actual contributions, including visibility, dramatically reducing image variance.

OUR CV

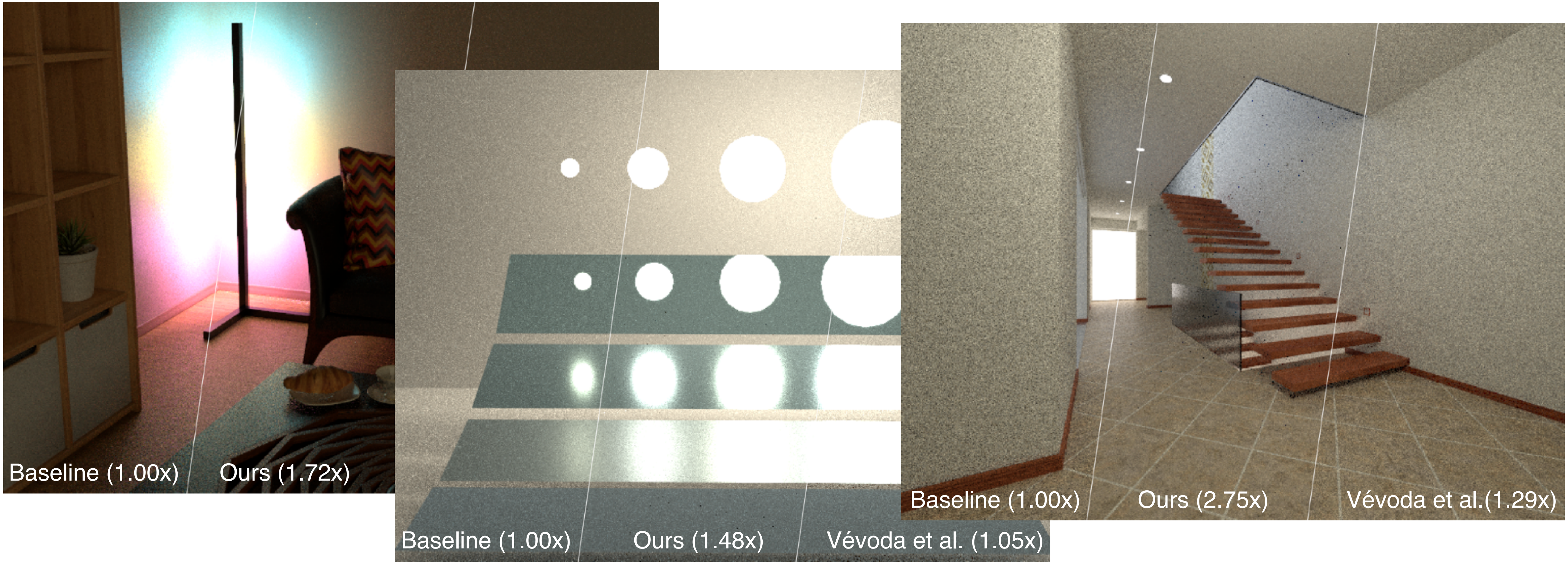
Bsdf Mixture

+

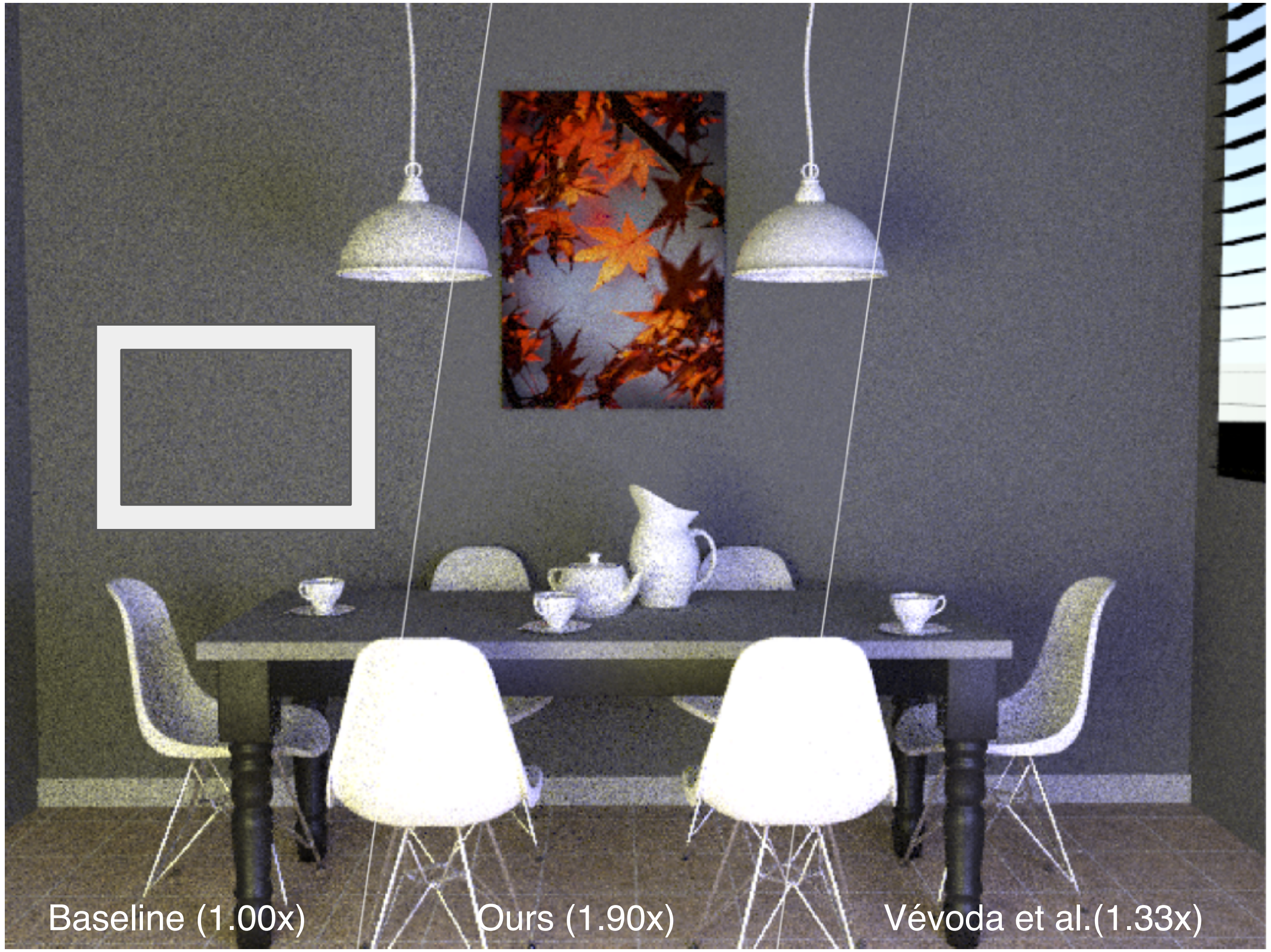
Vévoda et al.

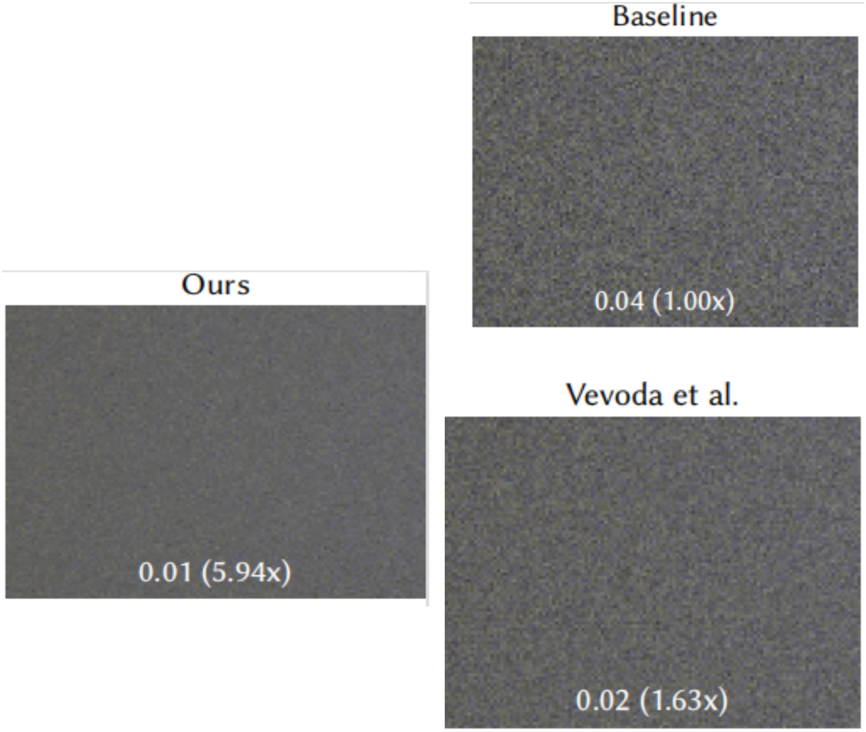
Ours

PATH TRACING (EQUAL TIME)



PATH TRACING (EQUAL TIME)





PATH TRACING (EQUAL TIME)

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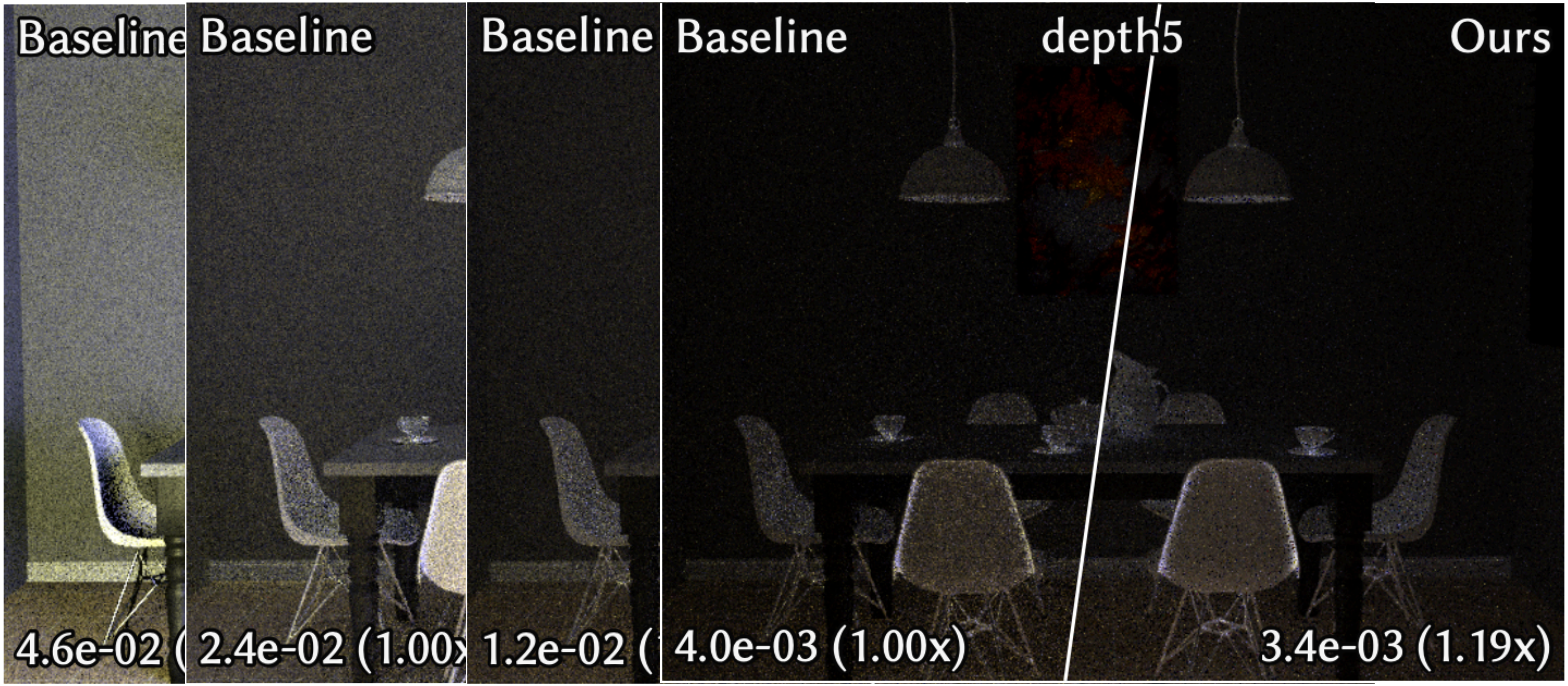
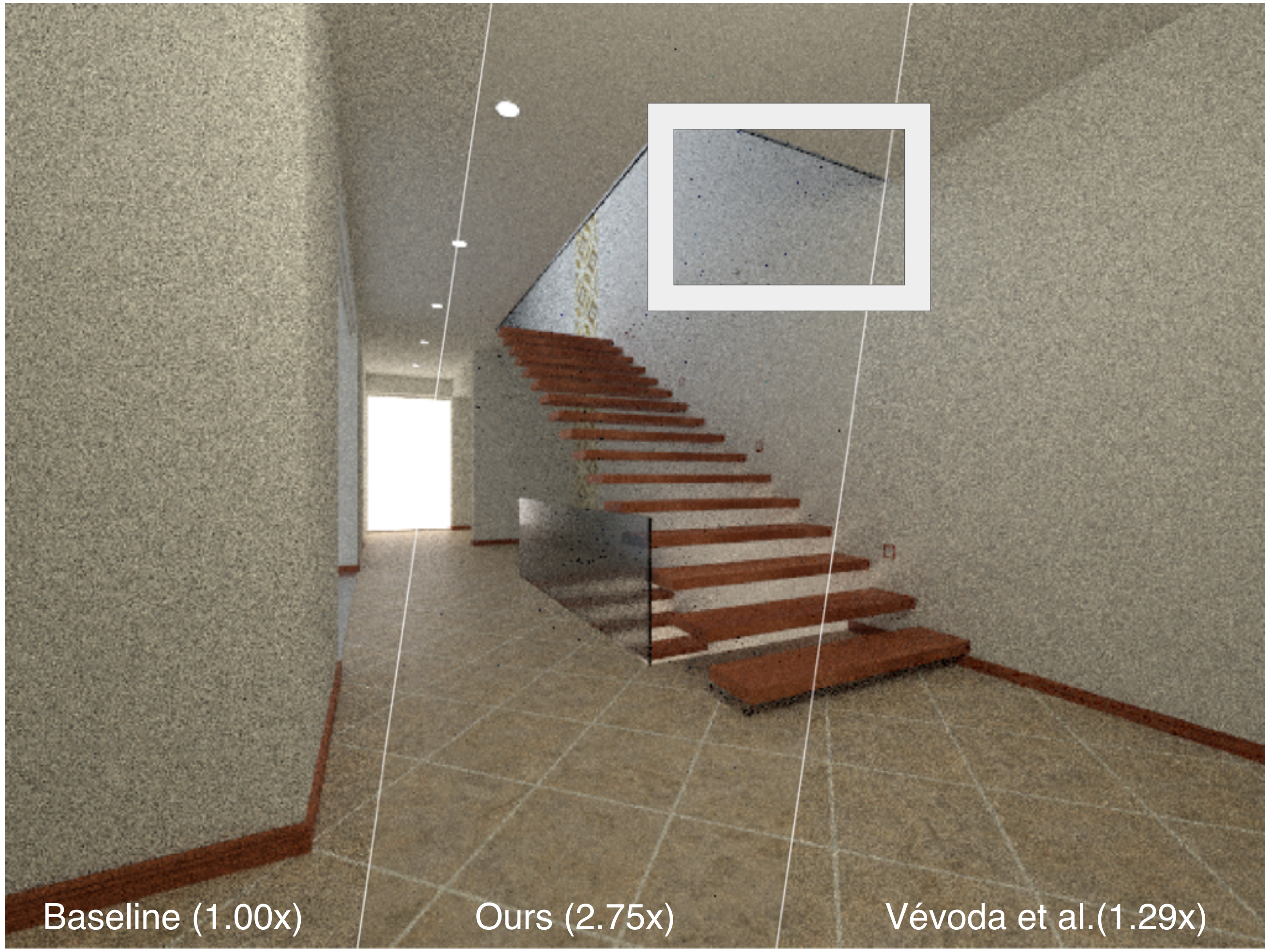
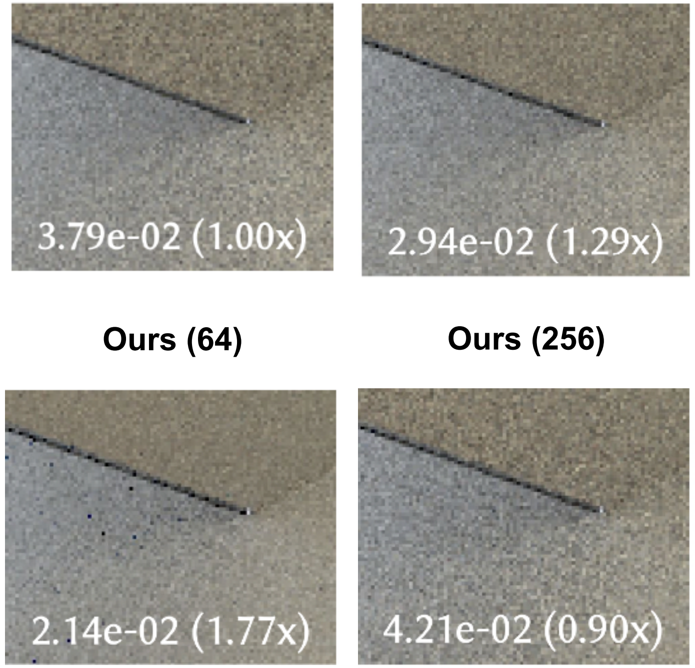


PATH TRACING (EQUAL TIME)



Baseline **Vévoda et al.**

Ours (64) **Ours (256)**



OVERHEAD

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Ours vs baseline			
	Equal-sample	*Equal-time*	*Time overhead*
Dining Room	2.20×	1.90×	15.46%
RGB Sofa	2.17×	2.03×	11.72%
Bathroom	1.03×	1.01×	13.65%
Veach MIS	1.78×	1.60×	16.82%
Modern Hall	3.52×	3.28×	15.27%
Ours vs Vévoda et al.			
	Equal-sample	*Equal-time*	*Time overhead*
Dining Room	1.63×	1.41×	15.31%
RGB Sofa	1.11×	0.98×	15.42%
Bathroom	1.00×	0.92×	17.12%
Veach MIS	1.64×	1.41×	27.30%
Modern Hall	2.62×	2.32×	18.26%

FUTURE WORK

- **Better spatial subdivision criteria**
- **More applications…**
  - Path Guiding
  - Spectral Rendering
  - Differentiable Rendering

SUMMARY

- **Generic theory:**
  - Optimise one CV for multiple integrals
  - Optimal MIS + optimised mixture sampling
  - Applicable to any sampling technique
- **Practical use case**
  - Multi-light & Bsdf sampling

THANK YOU!

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Paper and source code

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