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LOS ANGELES+ 6-10 AUG

THE PREMIER CONFERENCE & EXHIBITION ON
COMPUTER GRAPHICS & INTERACTIVE TECHNIQUES

FAST GPU-BASED TWO-WAY CONTINUOUS COLLISION HANDLING

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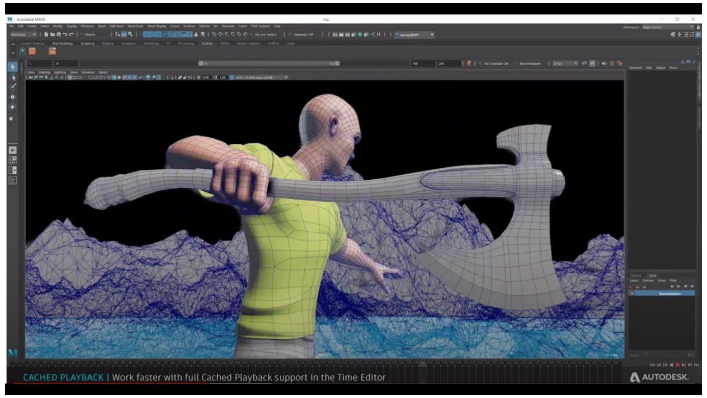
CONTACT SIMULATION IS EVERYWHERE

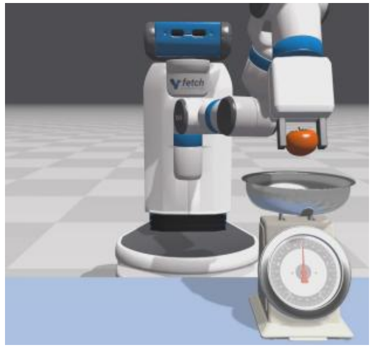


Movie production

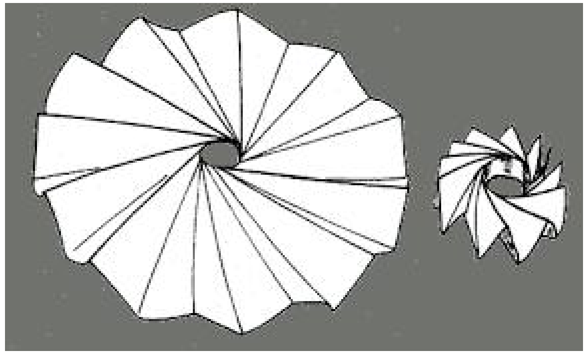
Video games

Virtual surgery

Fashion designing



Robotics



Smart material

CONTACT SIMULATION AND STEP-AND-PROJECT(SAP)

- For each time step *t*,

position at time *t*+1

position at time *t*

$$\mathbf{x}^{t+1} = \arg\min_{\mathbf{x}} E\big(\mathbf{x}, \mathbf{x}^t, \mathbf{v}^t\big),$$

$$\text{s.t.} \quad \mathcal{X}\big(\mathbf{x}^t, \mathbf{x}\big) \subset \Omega.$$

velocity at time *t*

trajectory from x^t to x

intersection-free region

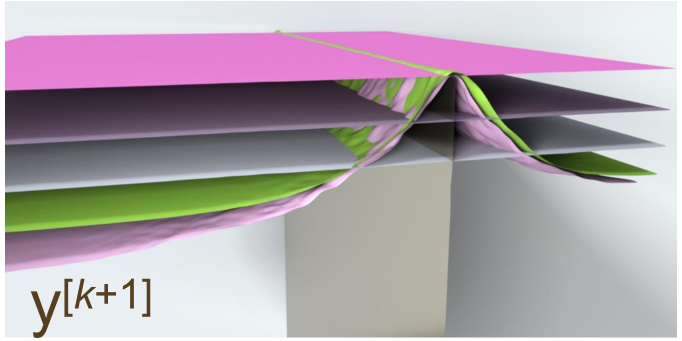
dynamics target

k-th quadratic approximation of *E* at the current position $x^{[k]}$

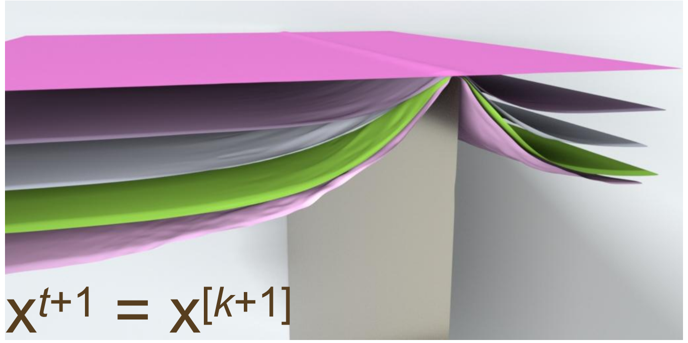
$$\begin{cases} \mathbf{y}^{[k+1]} = \arg\min_{\mathbf{y}} Q_k\big(\mathbf{y}, \mathbf{x}^{[k]}, \mathbf{x}^t, \mathbf{v}^t\big), \\ \mathbf{x}^{[k+1]} = \arg\min_{\mathbf{x}} D\big(\mathbf{x}, \mathbf{y}^{[k+1]}\big), \quad \text{s.t.} \ \mathcal{X}\big(\mathbf{x}^{[k]}, \mathbf{x}\big) \subset \Omega. \end{cases}$$

some distance metric between x and $y^{[k+1]}$

x^{t+1}

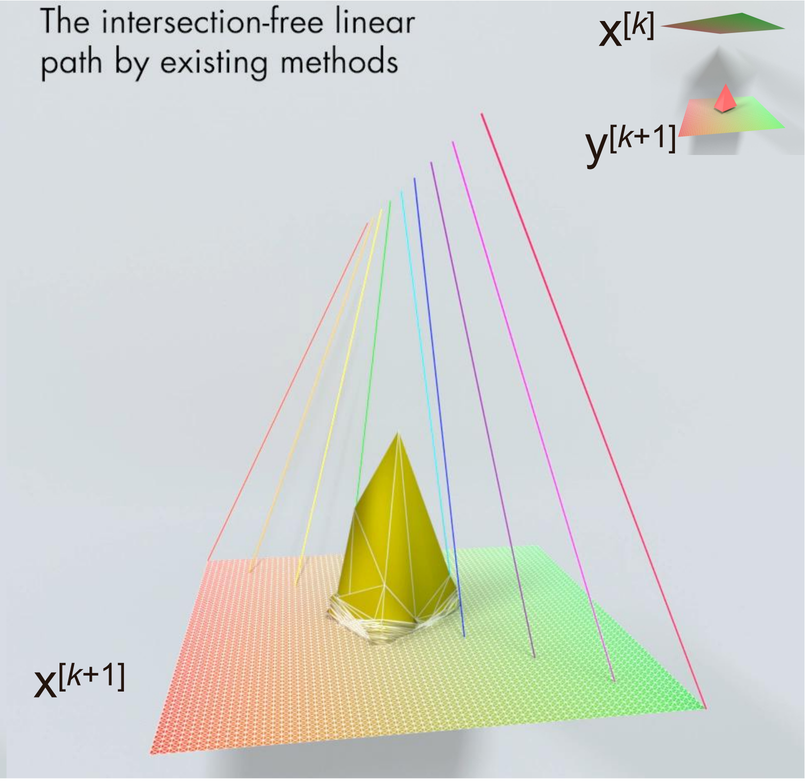


$y^{[k+1]}$



$x^{t+1} = x^{[k+1]}$

NON-RIGID IMPACT ZONE



General post collision handling module:

$$\mathbf{x}^{[k+1]} = \arg\min_{\mathbf{x}} D\left(\mathbf{x}, \mathbf{y}^{[k+1]}\right), \quad \text{s.t.} \;\; \mathcal{X}\left(\mathbf{x}^{[k]}, \mathbf{x}\right) \subset \Omega.$$

mass-weighted L_2 norm

Non-rigid impact zone solver:

$$\begin{cases} \mathbf{x}^{[k+1]} = \arg\min_{\mathbf{x}} D\left(\mathbf{x}, \mathbf{y}^{[k+1]}\right) = \frac{1}{2}\left\|\mathbf{x} - \mathbf{y}^{[k+1]}\right\|_{\mathbf{M}}^2, \\ \text{s.t.} \;\; \text{weak } \mathcal{X}_{\text{linear}}\left(\mathbf{x}^{[k]}, \mathbf{x}\right) \subset \Omega, \end{cases}$$

CCD-based constraint collection

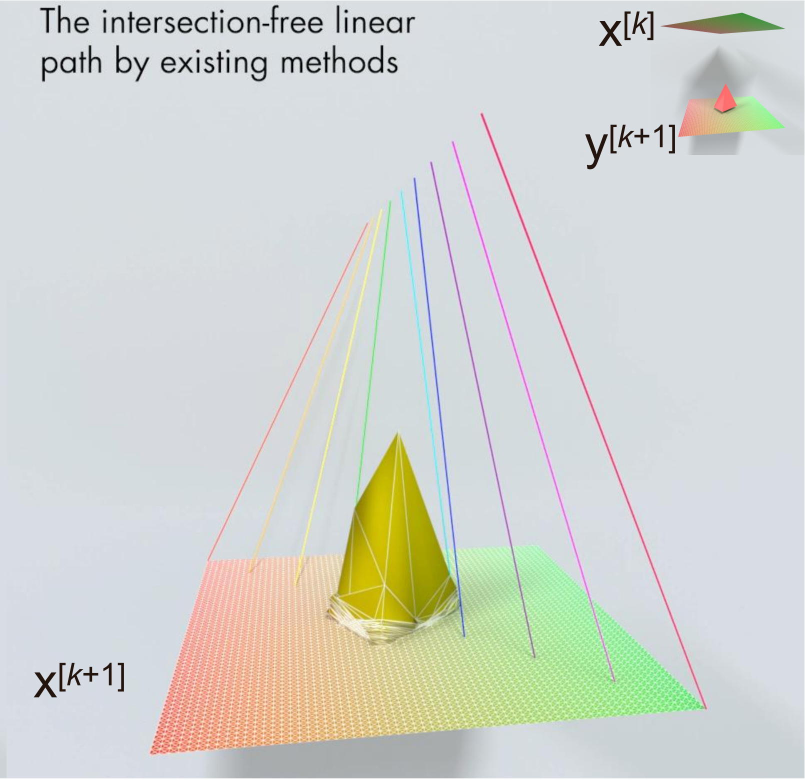
Constraint set Is not empty

Impact zone solver

Constraint set is empty

Obtain $\mathbf{x}^{[k+1]}$ and $\mathcal{X}_{\text{linear}}\left(\mathbf{x}^{[k]}, \mathbf{x}^{[k+1]}\right) \subset \Omega$

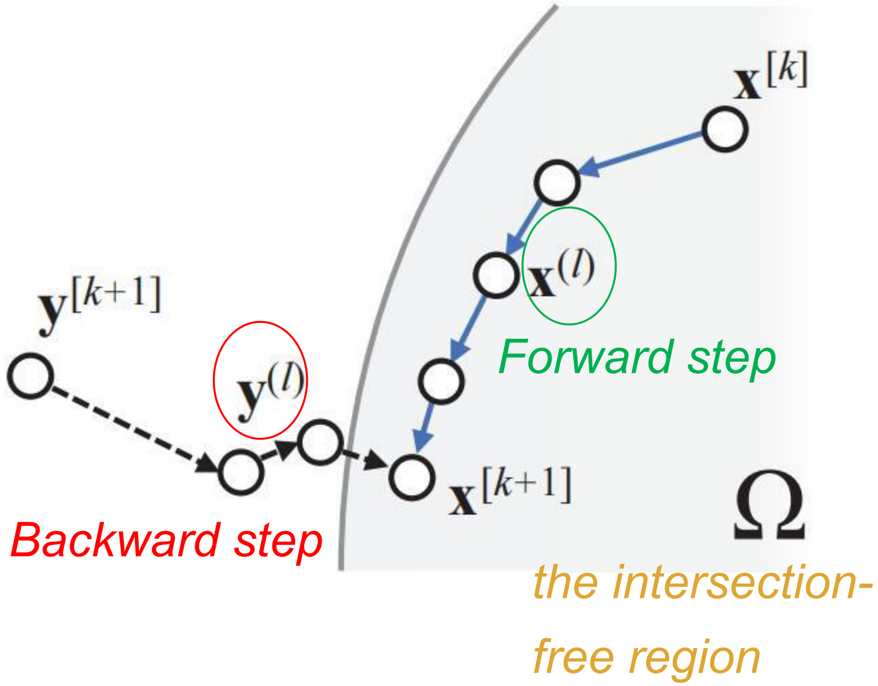
NON-RIGID IMPACT ZONE



Non-rigid impact zone solver: $\begin{cases} \mathbf{x}^{[k+1]} = \arg\min\limits_{\mathbf{x}} D\left(\mathbf{x}, \mathbf{y}^{[k+1]}\right) = \frac{1}{2}\left\|\mathbf{x} - \mathbf{y}^{[k+1]}\right\|_{\mathbf{M}}^2, \\ \text{s.t.} \quad \text{weak } \mathcal{X}_{\text{linear}}\left(\mathbf{x}^{[k]}, \mathbf{x}\right) \subset \Omega, \end{cases}$

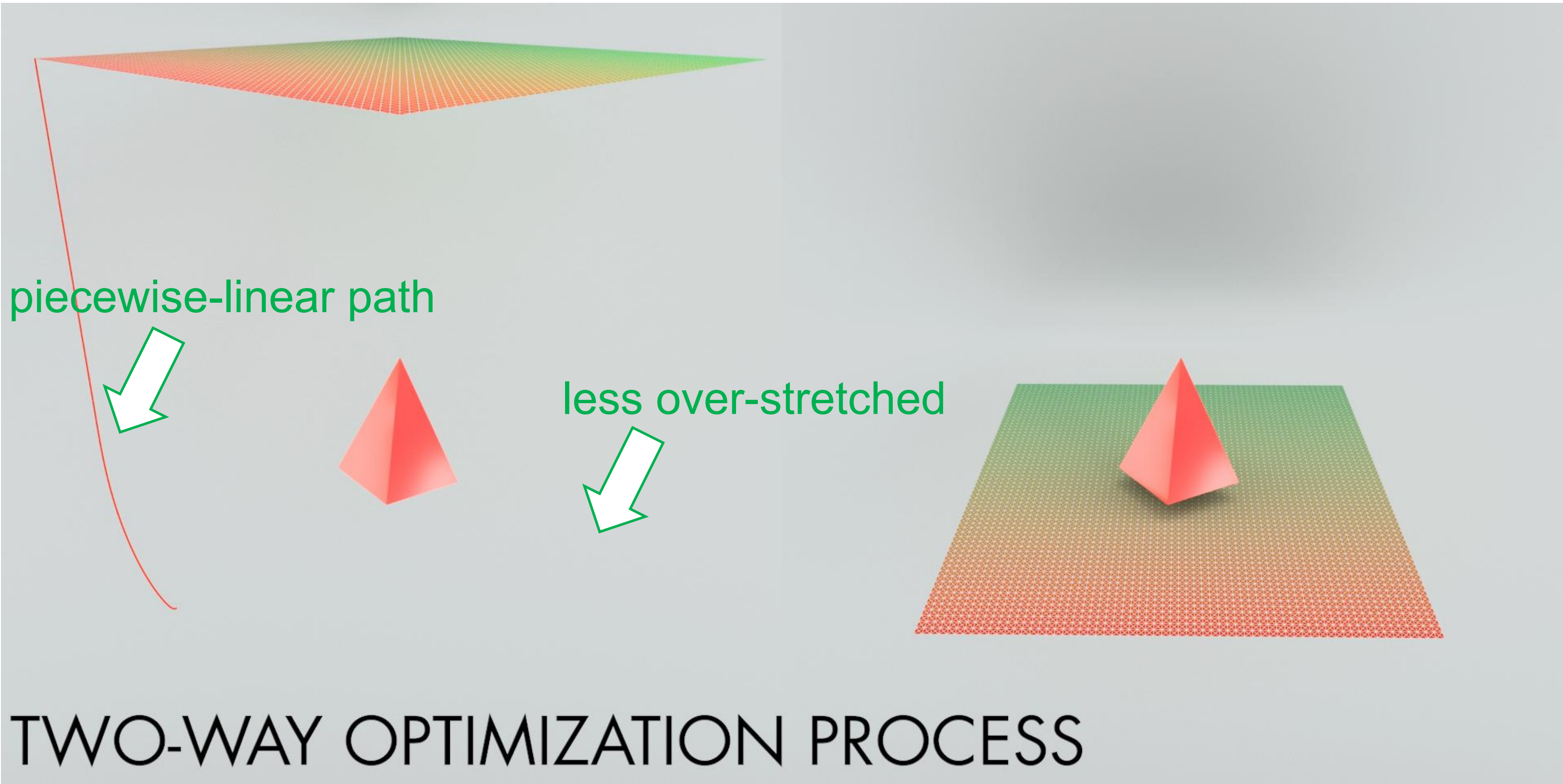
- The modified path is still linear (contacts happen at the same time)
- The mass-weighted L_2 norm: quadratic but is extrinsic
- Possible numerical issues and complexity on parallelization of CCD

TWO-WAY CONTINUOUS COLLISION HANDLING

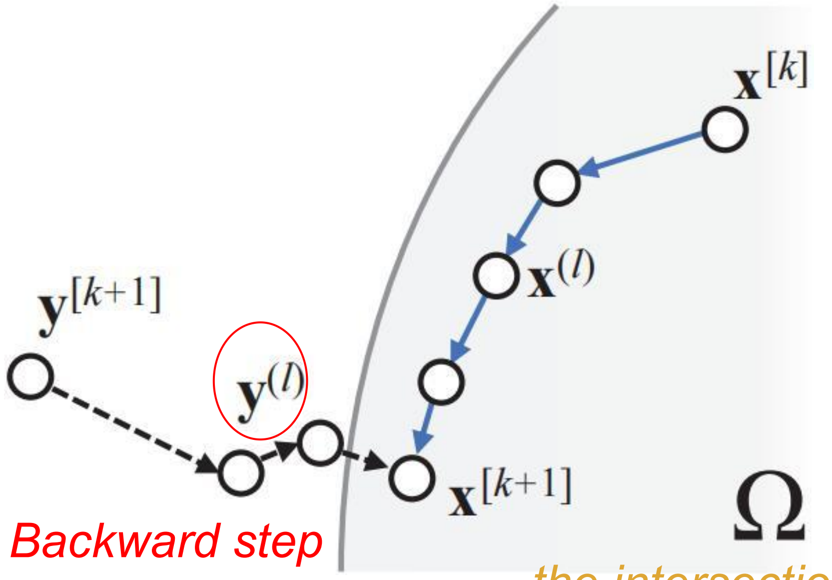


$x^{(l)}$ in the forward step

$y^{(l)}$ in the backward step



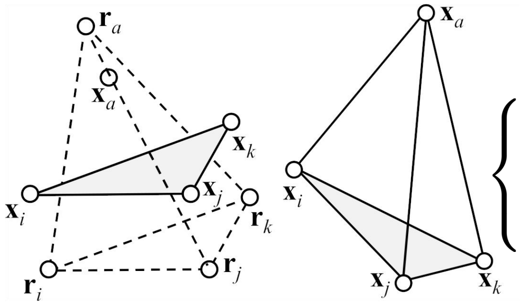
BACKWARD STEP



We roughly optimize:

$$\mathbf{y}^{(l+1)} = \arg\min_{\mathbf{y}} \frac{1}{2}\left\|\mathbf{y} - \mathbf{y}^{[k+1]}\right\|^2_{\mathbf{M}}, \quad \text{s.t. } \mathbf{c}(\mathbf{y}) \geq \mathbf{0}.$$

Class A of two classes of constraints: contact constraint



activation threshold

normal

$$\begin{cases} \mathbf{r}_a = \mathbf{x}_a + \frac{1}{2}\left(\delta - \|\mathbf{x}_a - b_i\mathbf{x}_i - b_j\mathbf{x}_j - b_k\mathbf{x}_k\|\right)\mathbf{n}, \\ \mathbf{r}_{i,j,k} = \mathbf{x}_{i,j,k} - \frac{1}{2}\left(\delta - \|\mathbf{x}_a - b_i\mathbf{x}_i - b_j\mathbf{x}_j - b_k\mathbf{x}_k\|\right)\mathbf{n}, \\ c(\mathbf{x}_a, \mathbf{x}_i, \mathbf{x}_j, \mathbf{x}_k) = \det(\partial\mathbf{x}/\partial\mathbf{r}) - 1 \geq 0. \end{cases}$$

consider the V-T pair collected at $x^{(l)}$

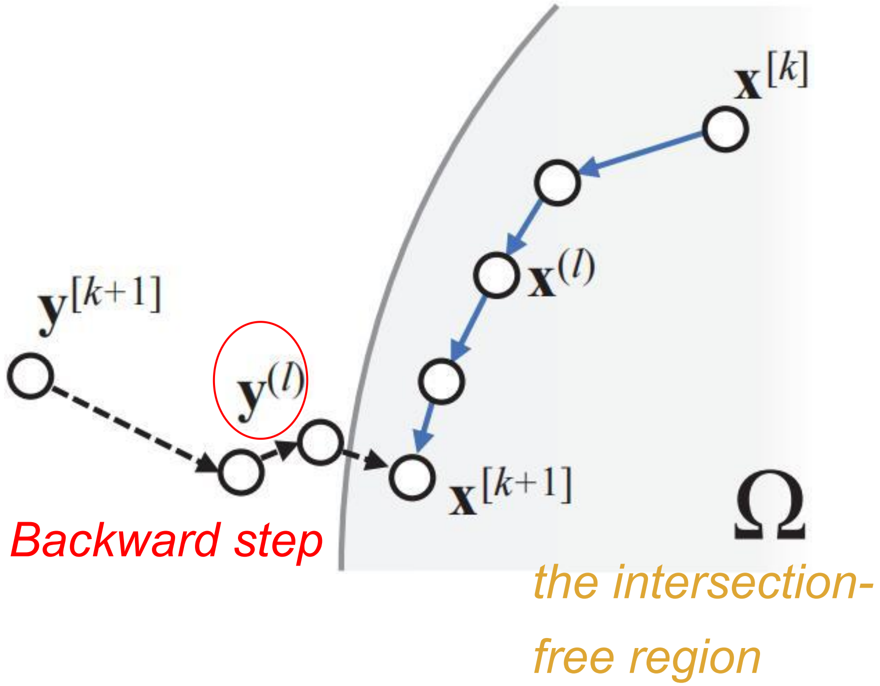
x_i,x_j on one edge

Class B of two classes of constraints: edge constraint

$$c(\mathbf{x}_i, \mathbf{x}_j) = \sigma - \left\|\mathbf{x}_i - \mathbf{x}_j\right\| \Big/ \left\|\mathbf{y}_i^{[k+1]} - \mathbf{y}_j^{[k+1]}\right\| \geq 0.$$

maximum violation ratio

BACKWARD STEP



We roughly optimize:

$$\mathbf{y}^{(l+1)} = \arg\min_{\mathbf{y}} \tfrac{1}{2}\left\|\mathbf{y} - \mathbf{y}^{[k+1]}\right\|_{\mathbf{M}}^2, \quad \text{s.t. } \mathbf{c}(\mathbf{y}) \geq \mathbf{0}.$$

For simplicity, we only include the linearized contact constraints in the formulation:

$$\mathcal{L}(\mathbf{y}, \boldsymbol{\lambda}) = \frac{1}{2}\left\|\mathbf{y} - \mathbf{y}^{[k+1]}\right\|_{\mathbf{M}}^2 - \left(\mathbf{c}\left(\mathbf{x}^{(l)}\right) + \mathbf{J}^{(l)}\left(\mathbf{y} - \mathbf{x}^{(l)}\right)\right)^{\mathsf{T}}\boldsymbol{\lambda},$$

$$\begin{cases} \nabla_{\mathbf{y}}\mathcal{L} = \mathbf{M}(\mathbf{y} - \mathbf{y}^{[k+1]}) - \left(\mathbf{J}^{(l)}\right)^{\mathsf{T}}\boldsymbol{\lambda} = \mathbf{0}, \\ \boldsymbol{\lambda} \geq 0 \perp \mathbf{c}\left(\mathbf{x}^{(l)}\right) + \mathbf{J}^{(l)}\left(\mathbf{y} - \mathbf{x}^{(l)}\right) \geq \mathbf{0}. \end{cases}$$

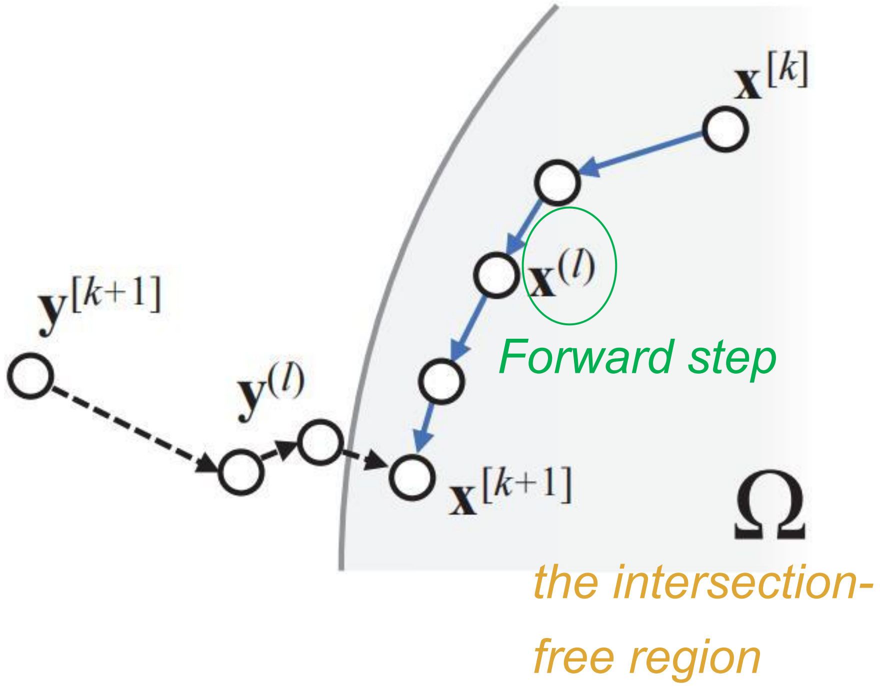
$$\boldsymbol{\lambda} \geq \mathbf{0} \perp \mathbf{c}\left(\mathbf{x}^{(l)}\right) + \mathbf{J}^{(l)}\mathbf{M}^{-1}\left(\mathbf{J}^{(l)}\right)^{\mathsf{T}}\boldsymbol{\lambda} + \mathbf{J}^{(l)}\left(\mathbf{y}^{[k+1]} - \mathbf{x}^{(l)}\right) \geq \mathbf{0}.$$

initialize $\boldsymbol{\lambda}^{(l)} \leftarrow \mathbf{0}$,

inexactly solve: $\boldsymbol{\lambda}^{(l)} \geq \mathbf{0} \perp \mathbf{c}\left(\mathbf{x}^{(l)}\right) + \mathbf{J}^{(l)}\mathbf{M}^{-1}\left(\mathbf{J}^{(l)}\right)^{\mathsf{T}}\boldsymbol{\lambda}^{(l)} + \mathbf{J}^{(l)}\left(\mathbf{y}^{(l)} - \mathbf{x}^{(l)}\right) \geq \mathbf{0}$,

after l iterations, $\mathbf{y}^{(l+1)} \approx \mathbf{y}^{[k+1]} + \sum_{i=0}^{l}\left(\mathbf{J}^{(i)}\right)^{\mathsf{T}}\boldsymbol{\lambda}^{(i)}$.

FORWARD STEP



Here we want: $\mathbf{x}_i^{(l+1)} = \mathbf{x}_i^{(l)} + \alpha_i^{(l+1)}\left(\mathbf{y}_i^{(l+1)} - \mathbf{x}_i^{(l)}\right),$
s.t. $(1-t)\mathbf{x}^{(l)} + t\mathbf{x}^{(l+1)} \in \Omega, \forall\, t \in [0,1]$ and l.

Instead of CCD tests, how can we do that?

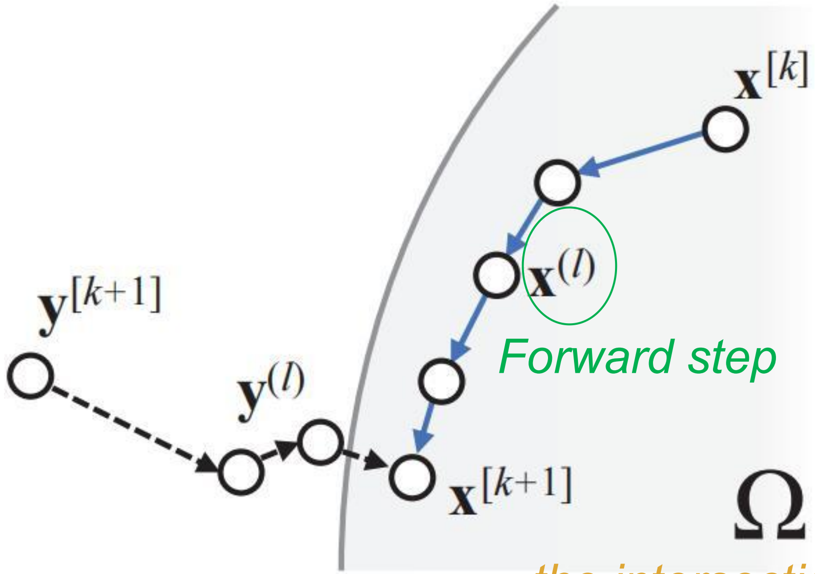
$$D_i = \min_{\{a,b\}\in\mathcal{P}} \texttt{dist}\left(\mathbf{x}_a^{(l)}, \mathbf{x}_b^{(l)}\right) \le D^{(l)}, \forall a, b : a \neq b \text{ and } i \in a \cup b,$$

$$\forall i: \left\|\mathbf{x}_i^{(l)} - \mathbf{x}_i^{(l+1)}\right\| < D_i/2 \Rightarrow (1-t)\mathbf{x}^{(l)} + t\mathbf{x}^{(l+1)} \in \Omega, \forall\, t \in [0,1],$$

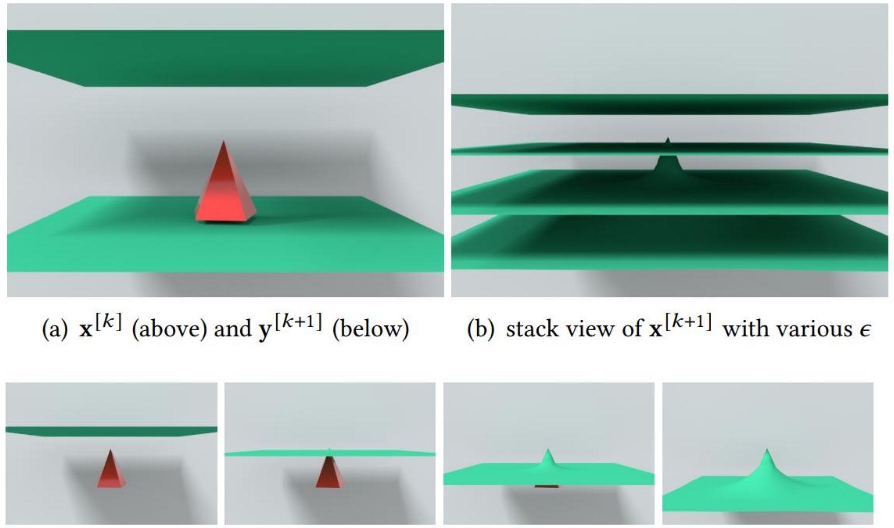
damping factor

$$\begin{cases} \alpha_i^{(l+1)} = \min\left(0.5\gamma D_i \Big/ \left\|\mathbf{y}_i^{(l+1)} - \mathbf{x}_i^{(l)}\right\|, 1\right), \\ \mathbf{x}_i^{(l+1)} = \mathbf{x}_i^{(l)} + \alpha_i^{(l+1)}\left(\mathbf{y}_i^{(l+1)} - \mathbf{x}_i^{(l)}\right), \\ r_i^{(l+1)} = r_i^{(l)}\left(1 - \alpha_i^{(l+1)}\right). \end{cases}$$

FORWARD STEP



Termination condition : $\|\mathbf{r}^{(l+1)}\|_\infty < \epsilon$



(a) $\mathbf{x}^{[k]}$ (above) and $\mathbf{y}^{[k+1]}$ (below)

(b) stack view of $\mathbf{x}^{[k+1]}$ with various ϵ

(c) $\mathbf{x}^{[k+1]}$ with ϵ = 0.75

(d) $\mathbf{x}^{[k+1]}$ with ϵ = 0.50

(e) $\mathbf{x}^{[k+1]}$ with ϵ = 0.25

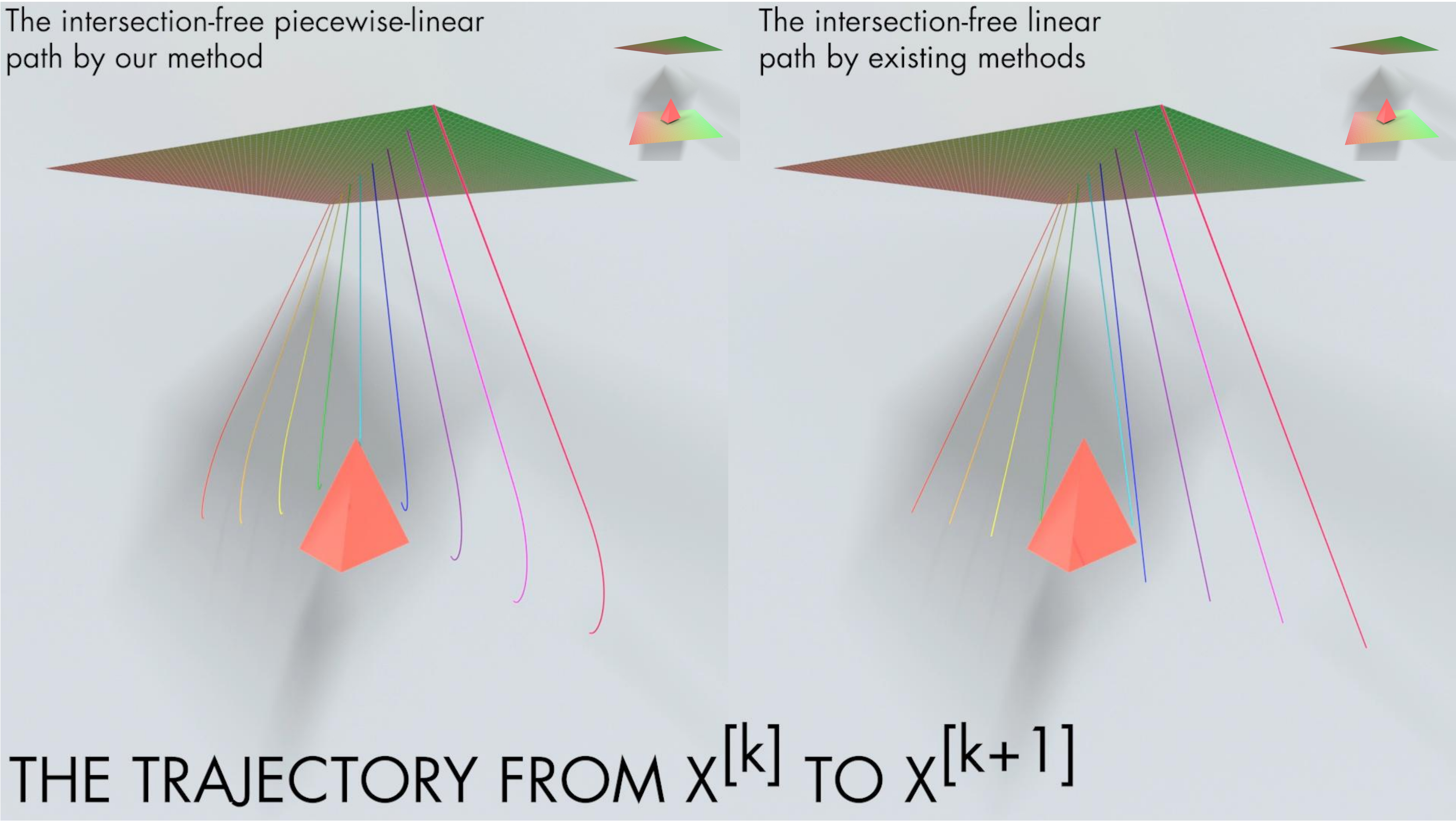
(f) $\mathbf{x}^{[k+1]}$ with ϵ = 0.0001

damping factor

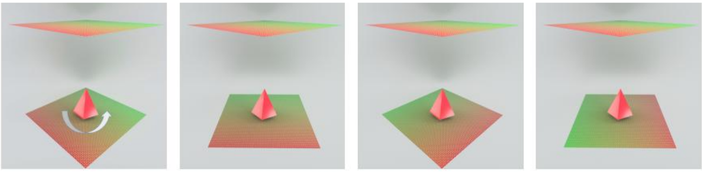
termination metric

$$\begin{cases} \alpha_i^{(l+1)} = \min\left(0.5\gamma D_i \Big/ \left\|\mathbf{y}_i^{(l+1)} - \mathbf{x}_i^{(l)}\right\|, 1\right), \\ \mathbf{x}_i^{(l+1)} = \mathbf{x}_i^{(l)} + \alpha_i^{(l+1)}\left(\mathbf{y}_i^{(l+1)} - \mathbf{x}_i^{(l)}\right), \\ \boxed{r_i^{(l+1)}} = r_i^{(l)}\left(1 - \alpha_i^{(l+1)}\right). \end{cases}$$

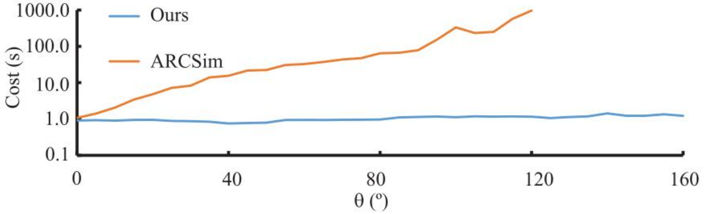
RESULTS


The intersection-free piecewise-linear path by our method
The intersection-free linear path by existing methods
THE TRAJECTORY FROM $X^{[k]}$ TO $X^{[k+1]}$

RESULTS



(a) $\mathbf{x}^{[k]}$ and $\mathbf{y}^{[k+1]}$ when $\theta = 0°$ (b) $\mathbf{x}^{[k]}$ and $\mathbf{y}^{[k+1]}$ when $\theta = 45°$ (c) $\mathbf{x}^{[k]}$ and $\mathbf{y}^{[k+1]}$ when $\theta = 90°$ (d) $\mathbf{x}^{[k]}$ and $\mathbf{y}^{[k+1]}$ when $\theta = 135°$



(e) Time cost comparison with ARCSim [Narain et al. 2012]

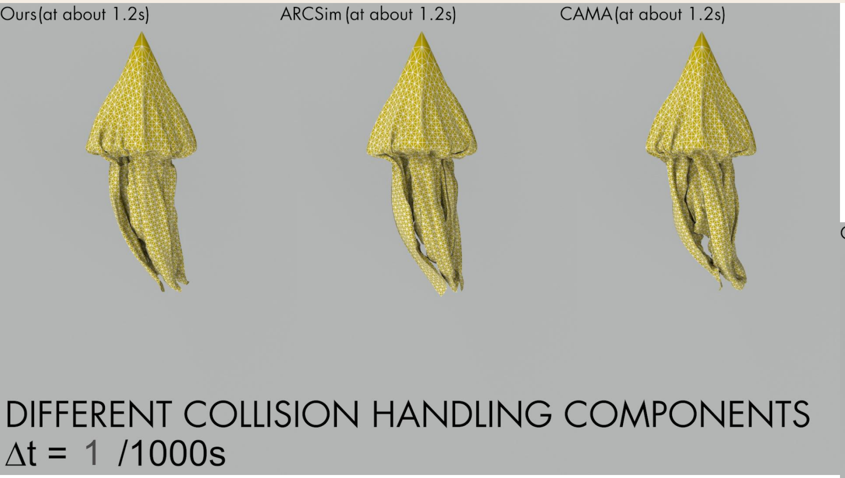
RESULTS OF VARIOUS PAIRED INPUTS

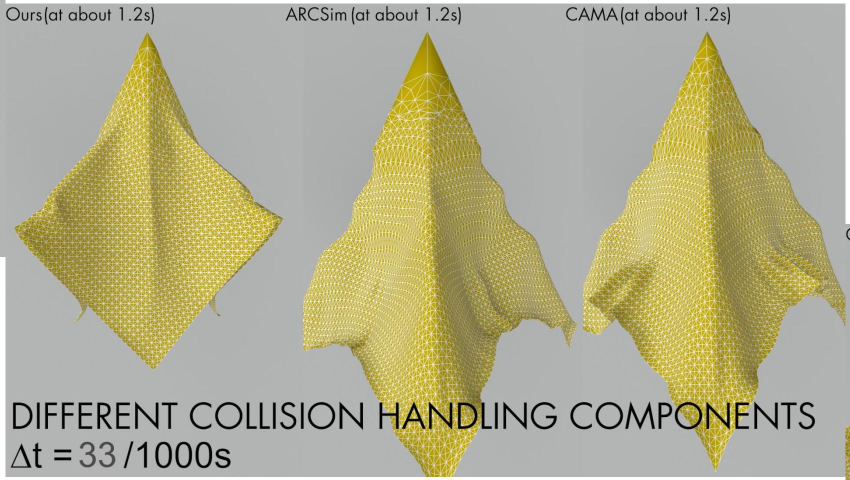
RESULTS OF VARIOUS PAIRED INPUTS

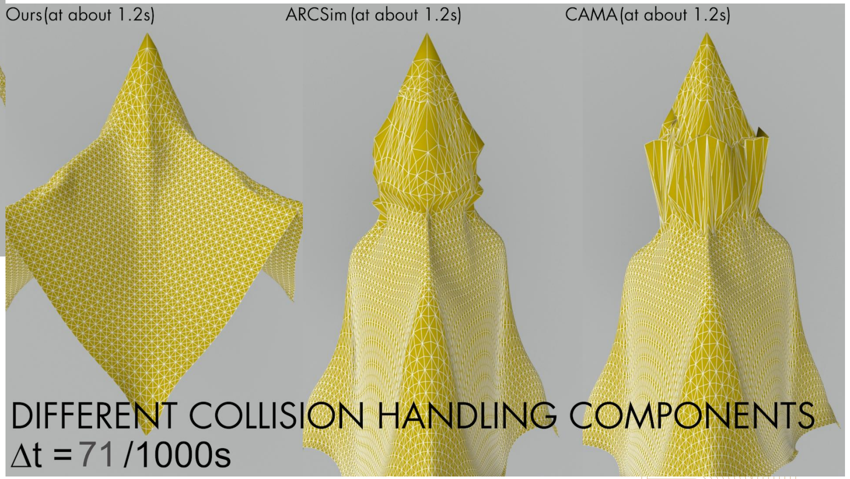
RESULTS OF VARIOUS PAIRED INPUTS

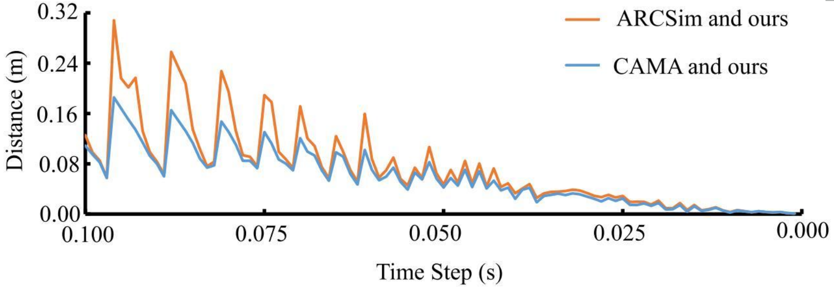
RESULTS OF VARIOUS PAIRED INPUTS

RESULTS


Ours(at about 1.2s)
ARCSim (at about 1.2s)
CAMA(at about 1.2s)
DIFFERENT COLLISION HANDLING COMPONENTS
Δt = 1 /1000s


Ours(at about 1.2s)
ARCSim (at about 1.2s)
CAMA(at about 1.2s)
DIFFERENT COLLISION HANDLING COMPONENTS
Δt = 33 /1000s


Ours(at about 1.2s)
ARCSim (at about 1.2s)
CAMA(at about 1.2s)
DIFFERENT COLLISION HANDLING COMPONENTS
Δt = 71 /1000s

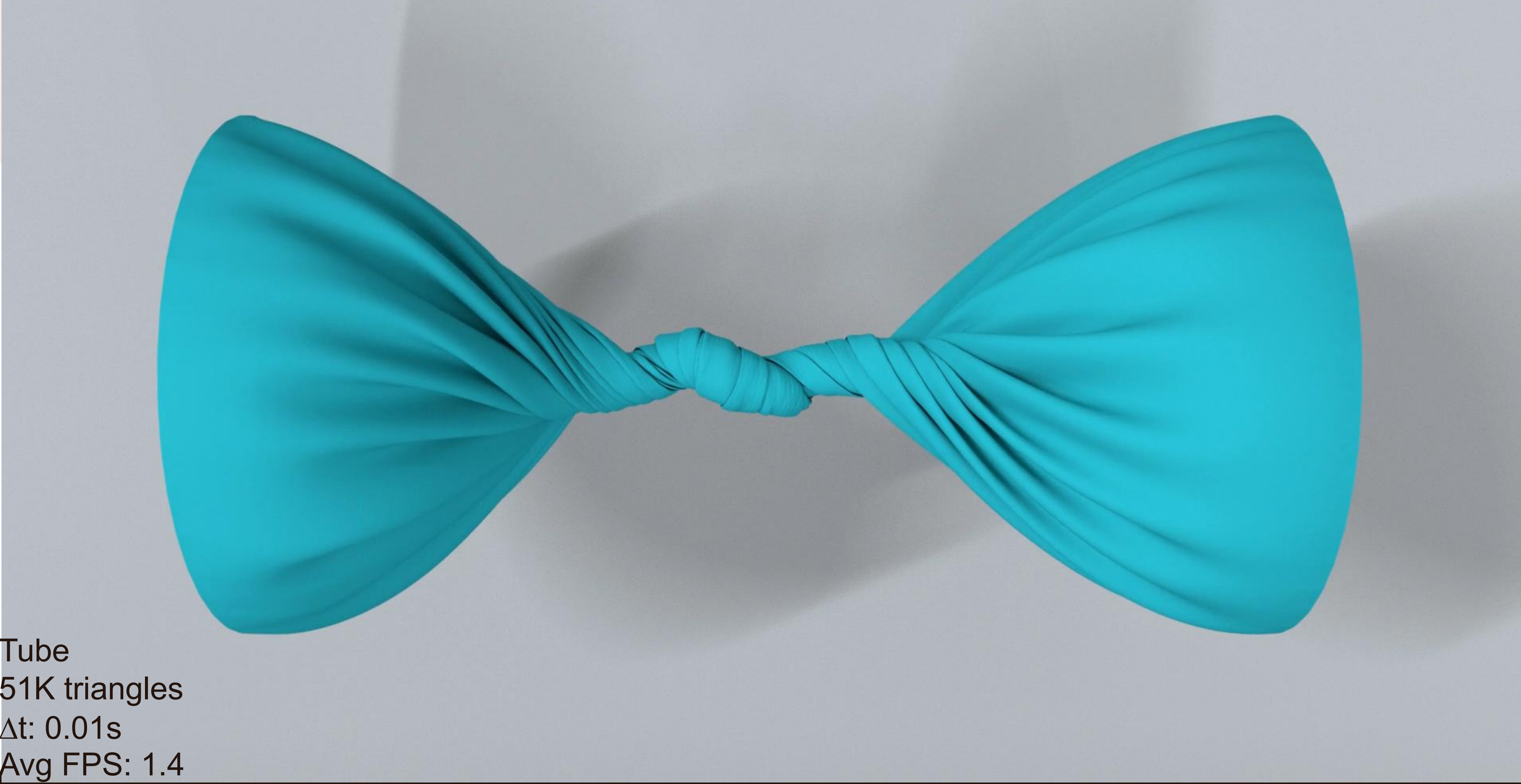

0.32
0.24
0.16
0.08
0.00
Distance (m)
0.100
0.075
0.050
0.025
0.000
Time Step (s)
ARCSim and ours
CAMA and ours

More results:

1. codimensional simulation

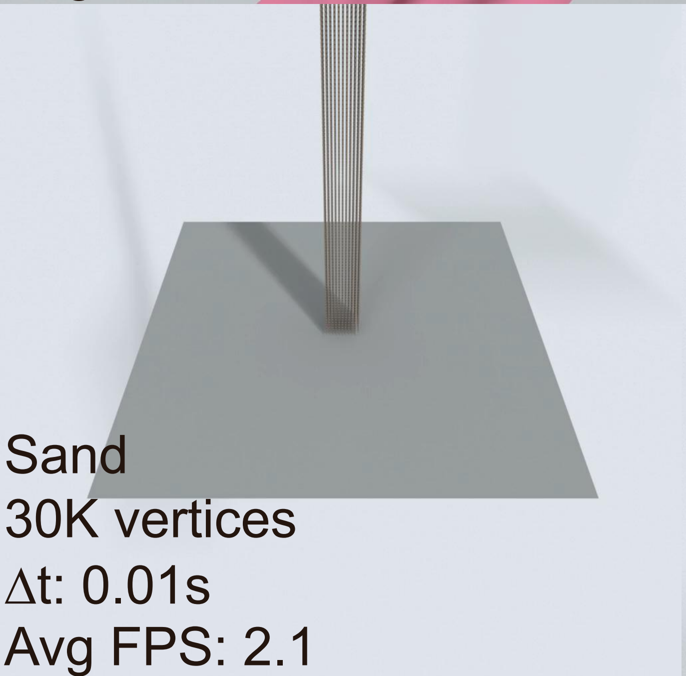
- E.g., the current dynamics part based on Newton's method:

$$\begin{cases} \mathbf{y}^{[k+1]} = \arg\min_{\mathbf{y}} Q_k\left(\mathbf{y}, \mathbf{x}^{[k]}, \mathbf{x}^t, \mathbf{v}^t\right), \\ \text{where } Q_k = E\left(\mathbf{x}^{[k]}, \mathbf{x}^t, \mathbf{v}^t\right) + \mathbf{b}^{[k]\top}(\mathbf{y} - \mathbf{x}^{[k]}) + \frac{1}{2}(\mathbf{y} - \mathbf{x}^{[k]})^\top \mathbf{G}^{[k]}(\mathbf{y} - \mathbf{x}^{[k]}). \end{cases}$$

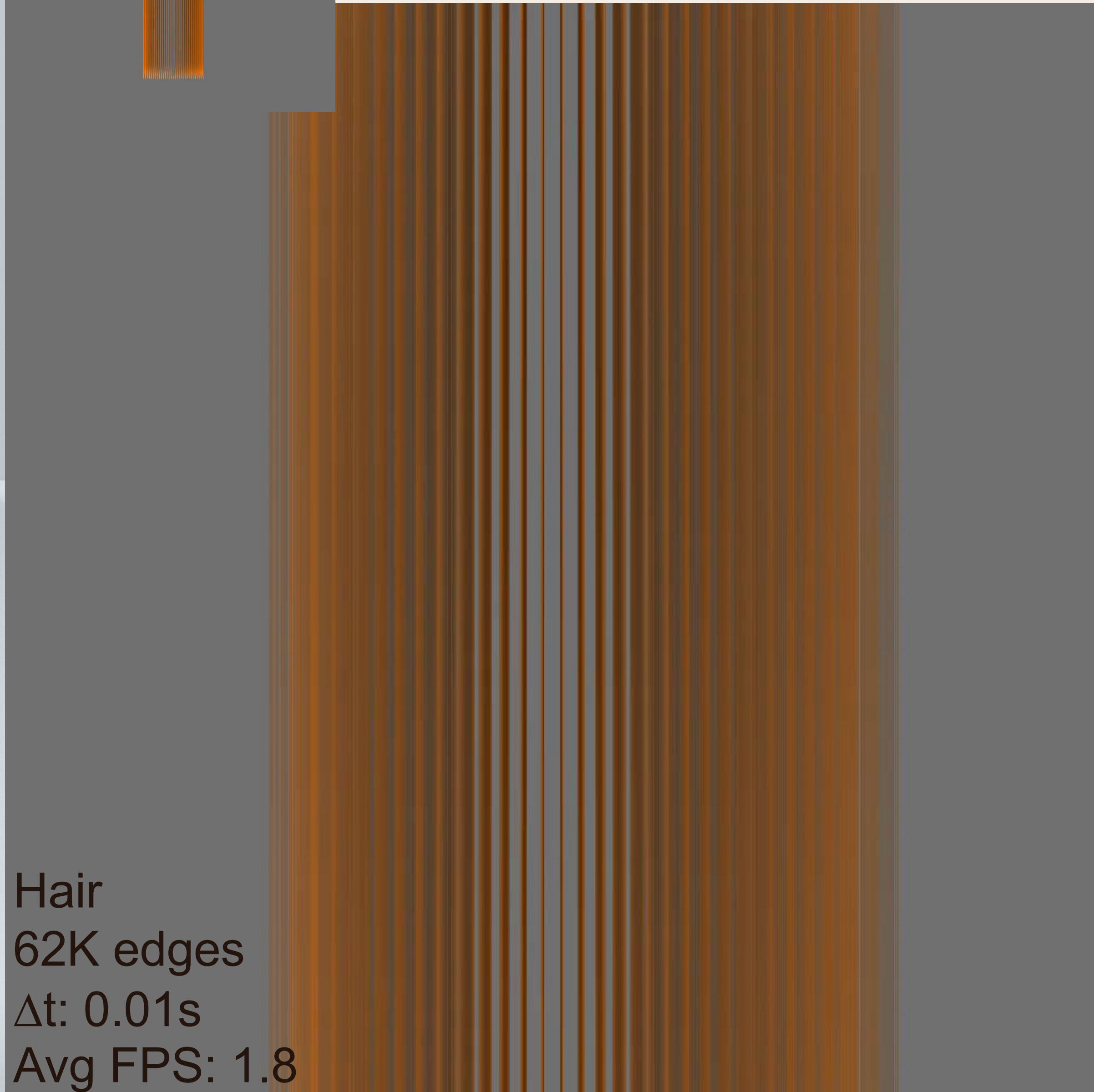


Tube
51K triangles
Δt: 0.01s
Avg FPS: 1.4


Bow knot
142K triangles
Δt: 0.01s
Avg FPS: 11.7


Sand
30K vertices
Δt: 0.01s
Avg FPS: 2.1


Mat
88K tetrahedra
Δt: 0.01s
Avg FPS: 8.6

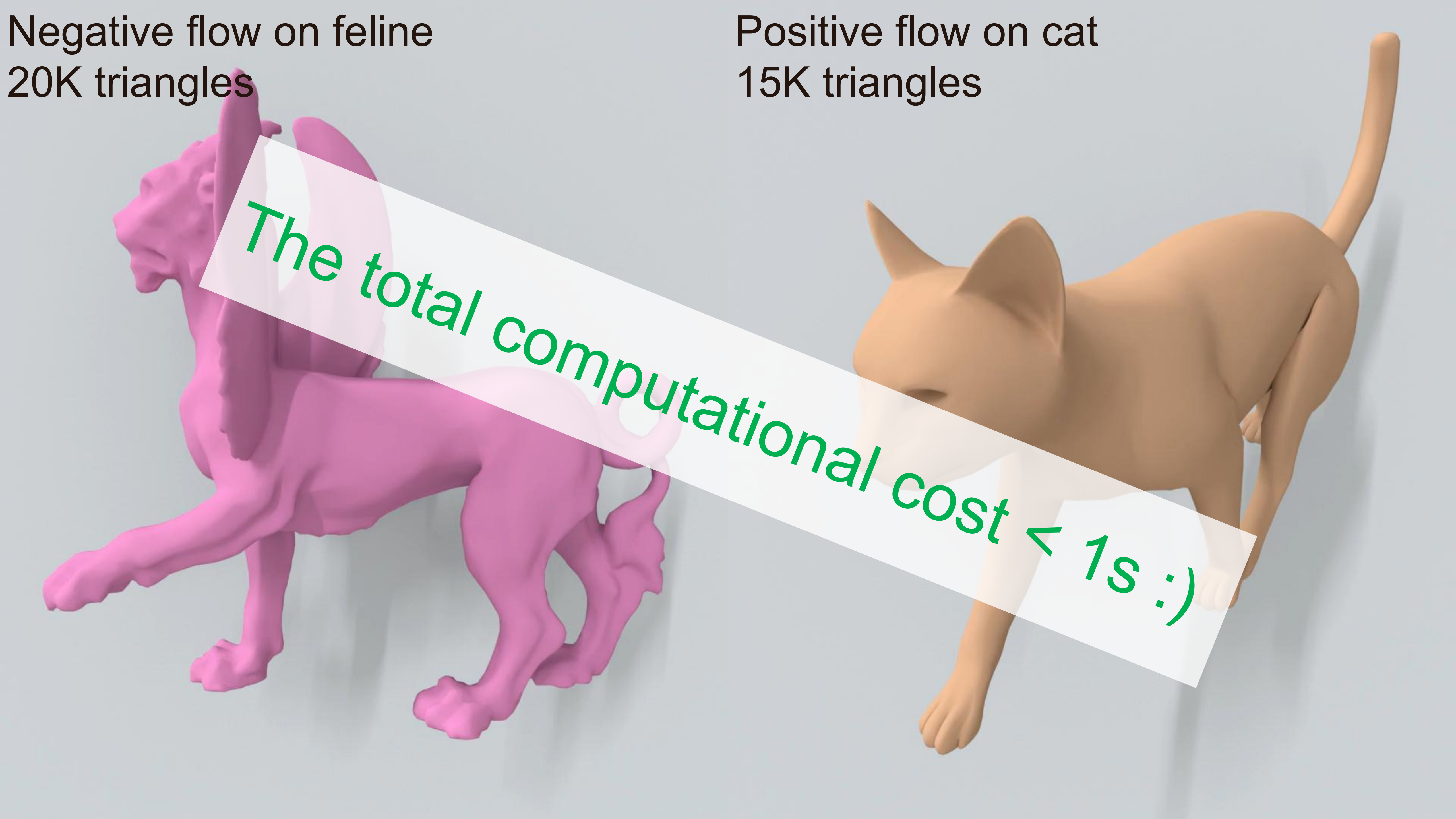

Hair
62K edges
Δt: 0.01s
Avg FPS: 1.8

More results:

1. codimensional simulation
2. geometry processing

- E.g., the "dynamics" part of the globally injective normal flow computation:

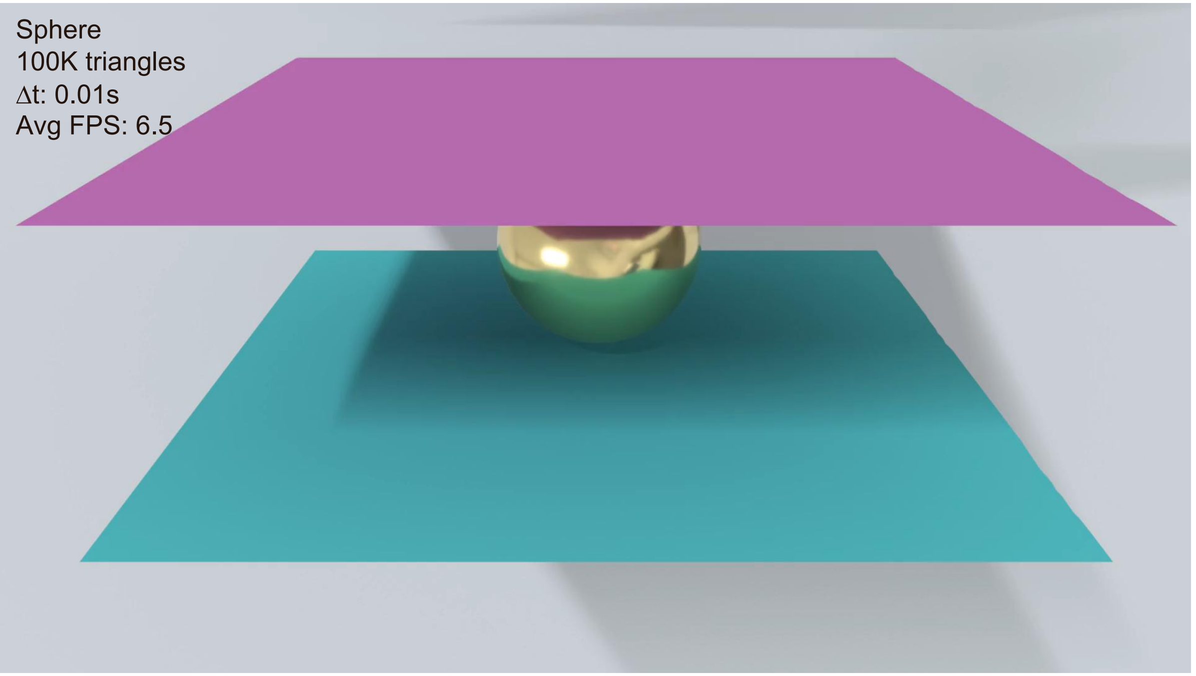
$$\begin{cases} \mathbf{y}_0^{[k+1]} = \mathbf{x}^{[k]} \pm \beta \mathbf{n}^{[k]}, \\ \mathbf{y}_{i+1}^{[k+1]}(v) = \mathbf{y}_i^{[k+1]}(v) + \alpha \triangle \mathbf{y}_i^{[k+1]}(v), \\ \mathbf{y}^{[k+1]} = \mathbf{y}_3^{[k+1]}. \end{cases}$$



Negative flow on feline
20K triangles
Positive flow on cat
15K triangles
The total computational cost < 1s :)

More results:

1. codimensional simulation
2. geometry processing
3. frictional contact



FUTURE WORK

- Can SAP be improved to be globally convergent?
  [Conn et al. 1988; Lin and Moré 1999]
- Better friction
- Better performance
- Coupling with rigid body or fluid

Thank You!