



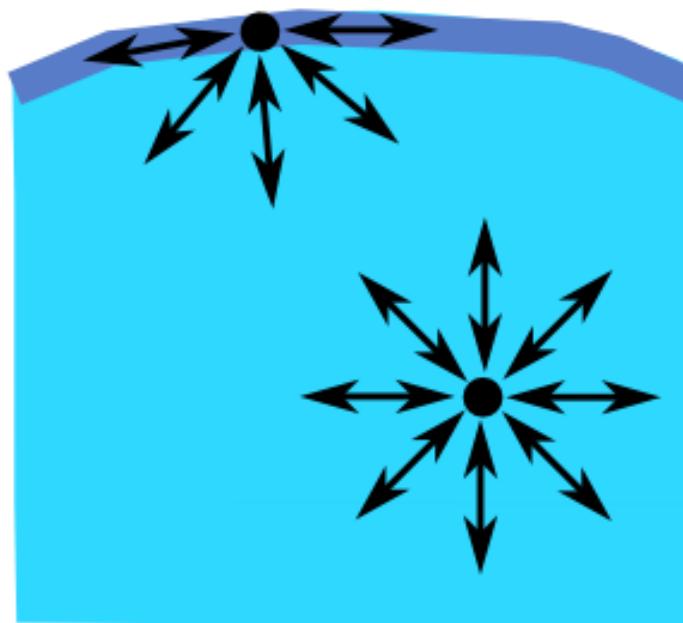
# Position-Based Surface Tension Flow

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(\* joint first authors)

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# Surface Tension



From [Wikipedia](#)

# Motivation



From [Wikipedia](#)

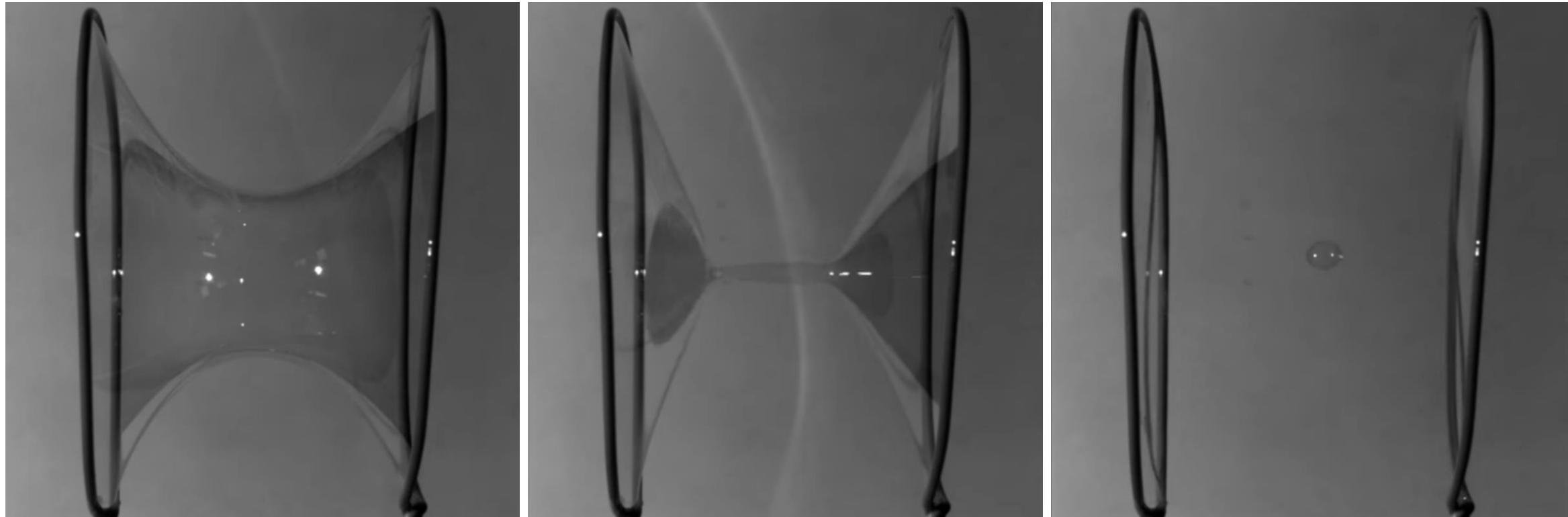


From [APS Physics](#)



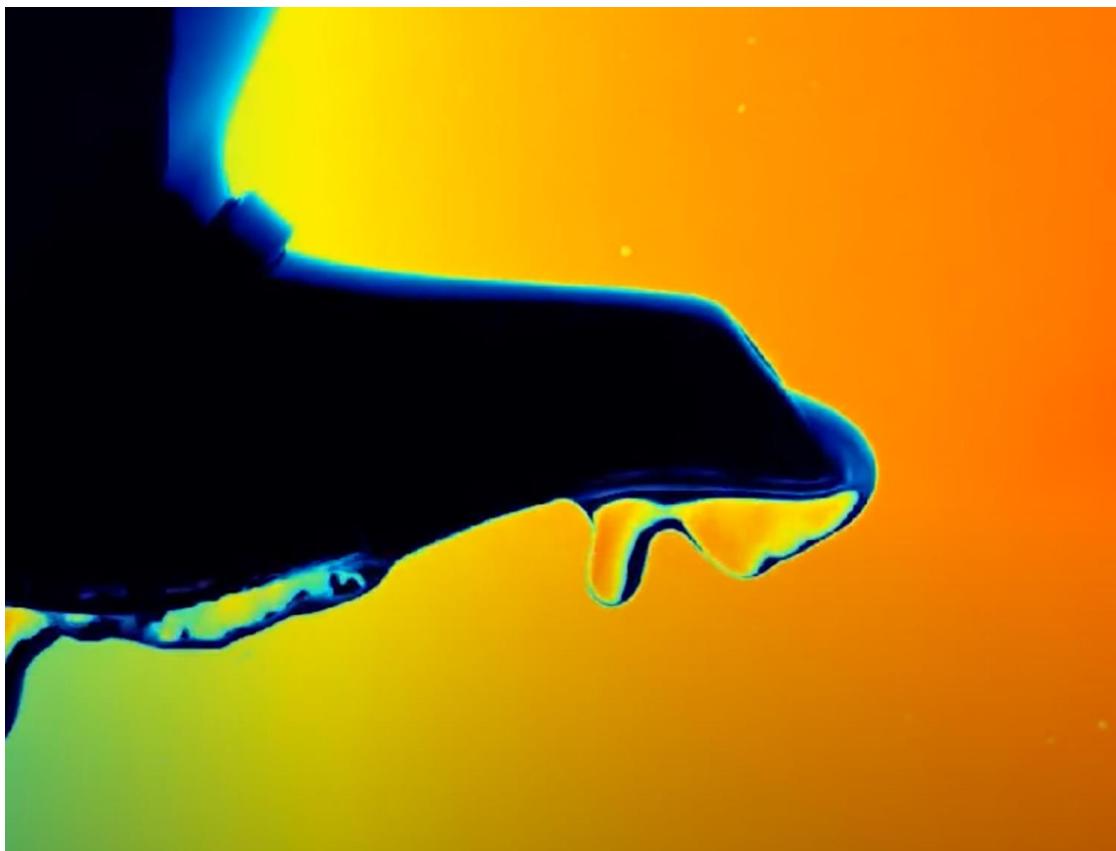
[Li et.al. 2019]

# Motivation



From [Soapbubble.dk](http://Soapbubble.dk)

# Motivation



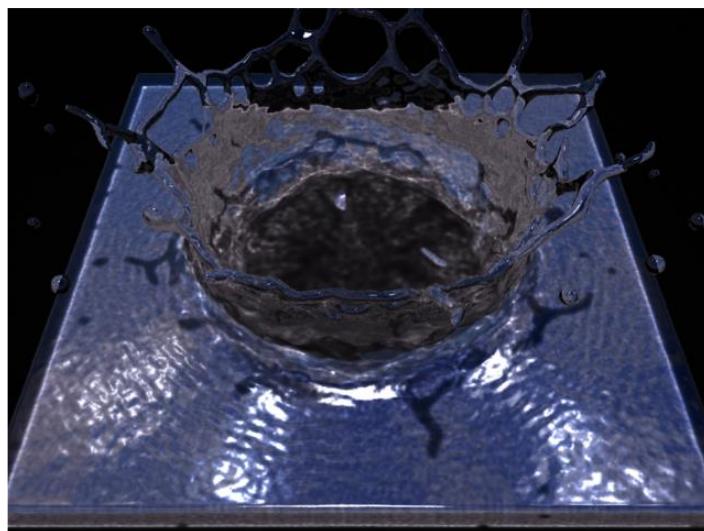
Teapot effect, from [TW Wien](#)



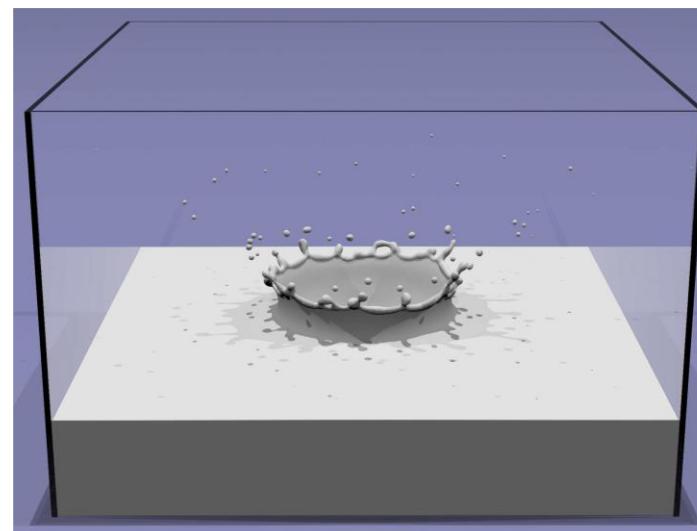
Capillary ratchet [\[Prakash et.al. 2008\]](#)

# Previous Work

- Mesh-based methods
  - Accurately track fluid surface
  - Complex remeshing algorithm



[Yu et al. 2012]



[Zheng et al. 2015]



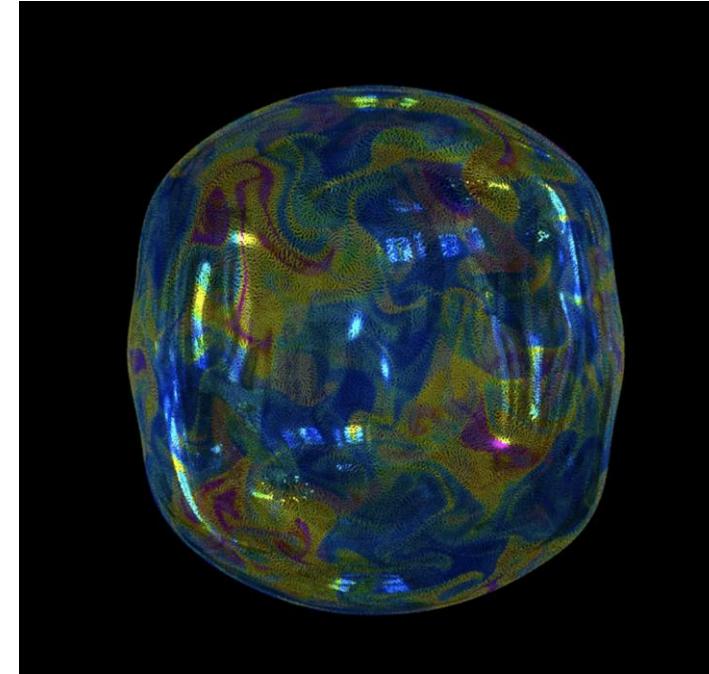
[Zhu et al. 2014]

# Previous Work

- Mesh-free particle methods (e.g. SPH)
  - Easily handle topological transitions
  - Unstable under large time step



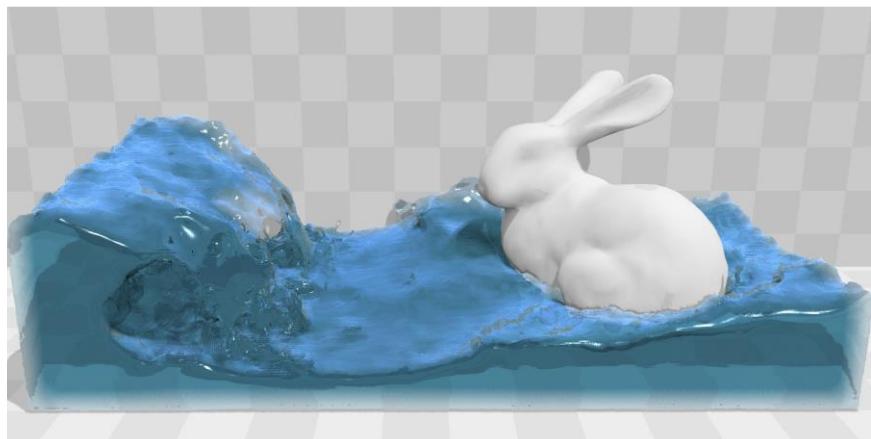
[Akinci et al. 2013]



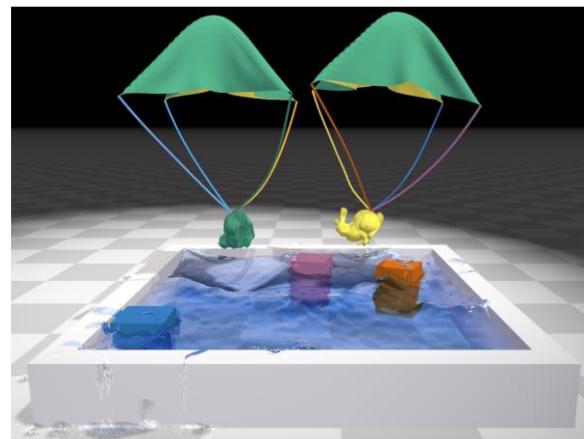
[Wang et al. 2021]

# Previous Work

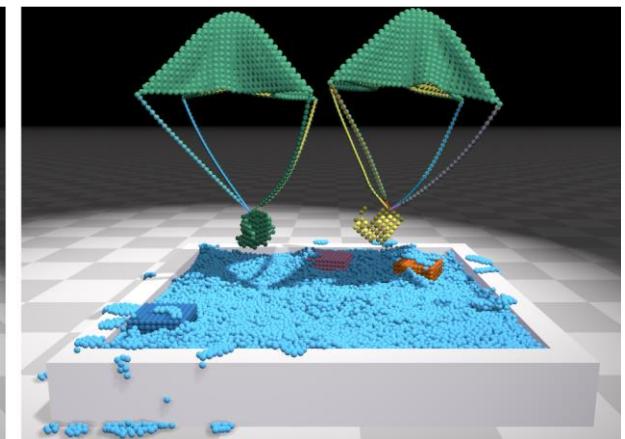
- Position based fluids (PBF)
  - Stable, fast, couple-friendly



[Macklin and Müller 2013]



[Macklin et al. 2014]



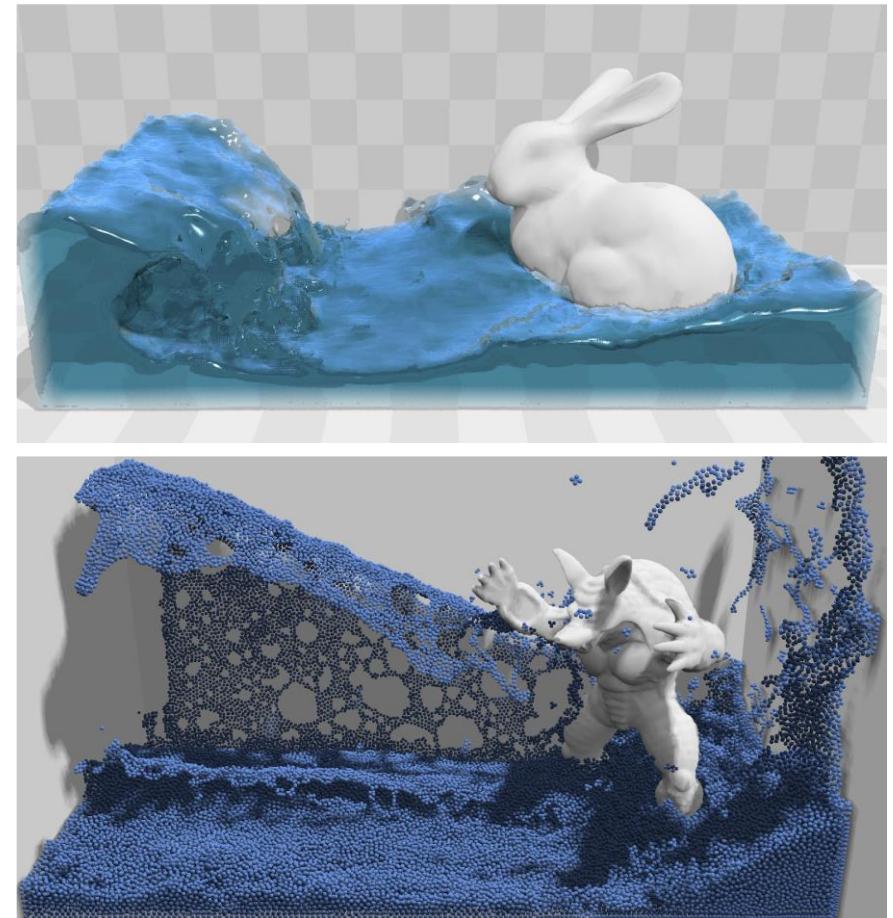
# Previous Work

- Position based fluids (PBF)
  - Density:  $\rho_i = \sum_j mW(\mathbf{p}_i - \mathbf{p}_j, h)$
  - Density constraint:  $C_i^\rho(\mathbf{p}) = \frac{\rho_i}{\rho_0} - 1$
  - Newton solve:  $C_i^\rho(\mathbf{p} + \Delta\mathbf{p}) = 0$
  - Artificial pressure:  
$$\Delta\mathbf{p} = \frac{m}{\rho_0} \sum_{j \in N(i)} \left( \frac{s_{\text{corr}}}{m} + \lambda_i + \lambda_j \right) \nabla W(\mathbf{p}_i - \mathbf{p}_j, h)$$

Repulsive force term

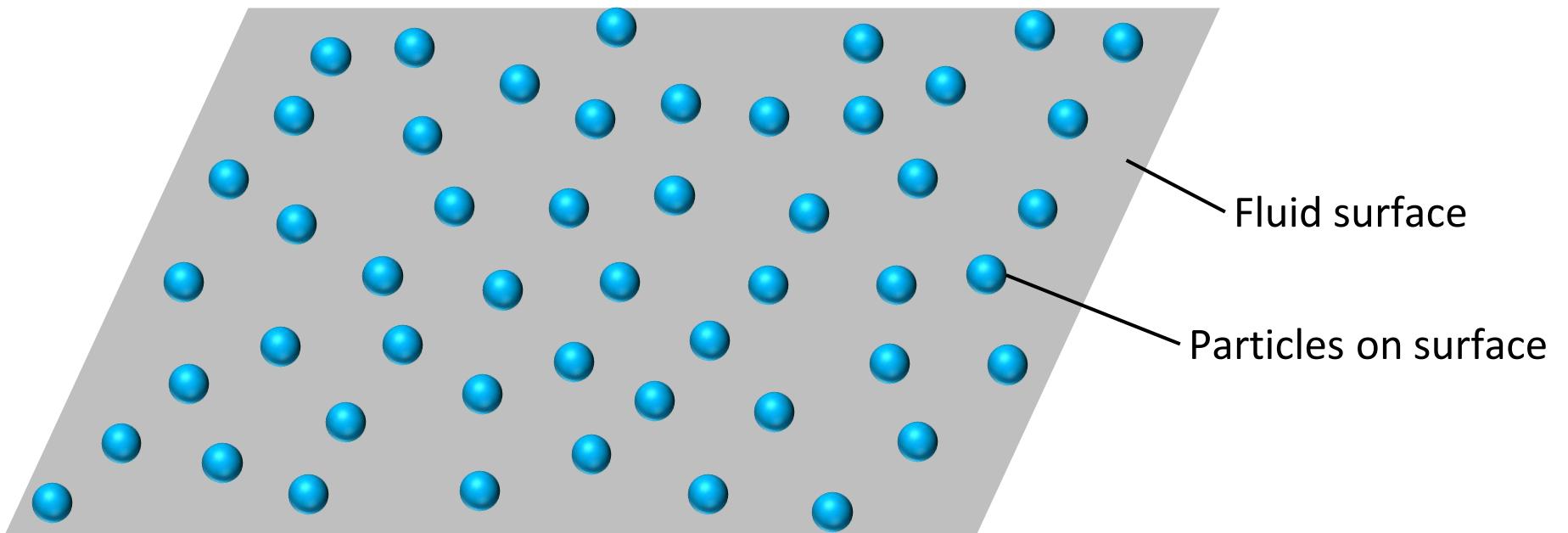
Surface tension as a side effect

Can only simulate surface tension  
in some extend



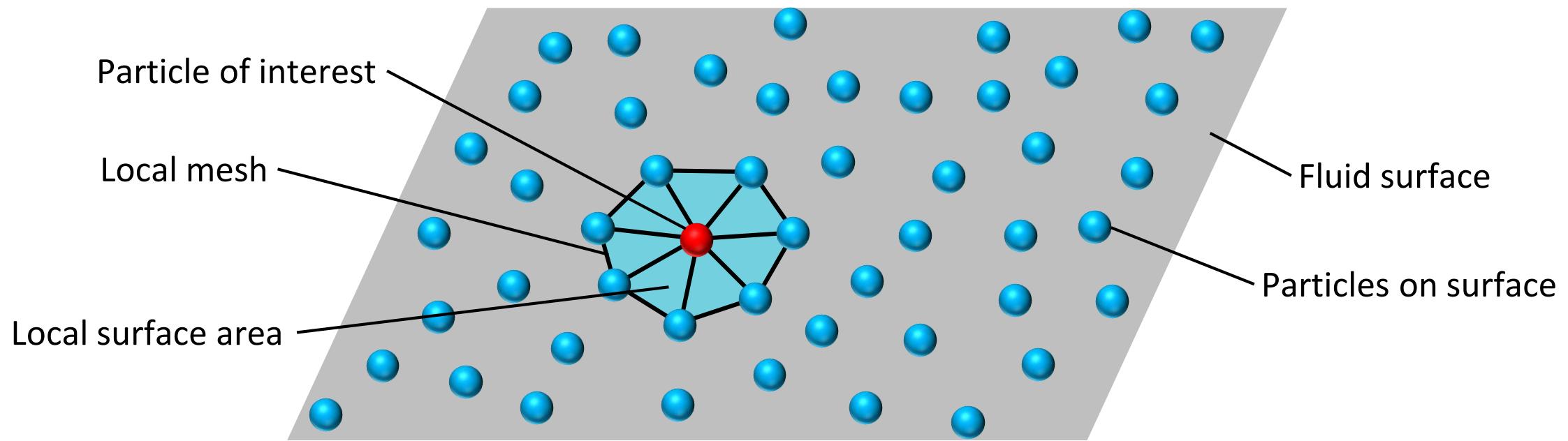
[Macklin and Müller 2013]

# Key Idea



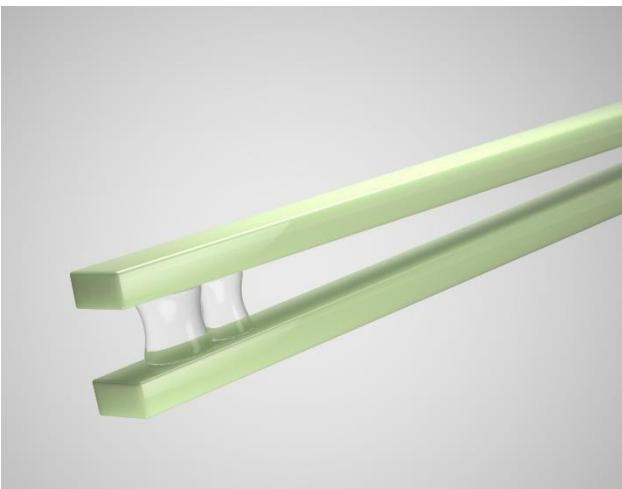
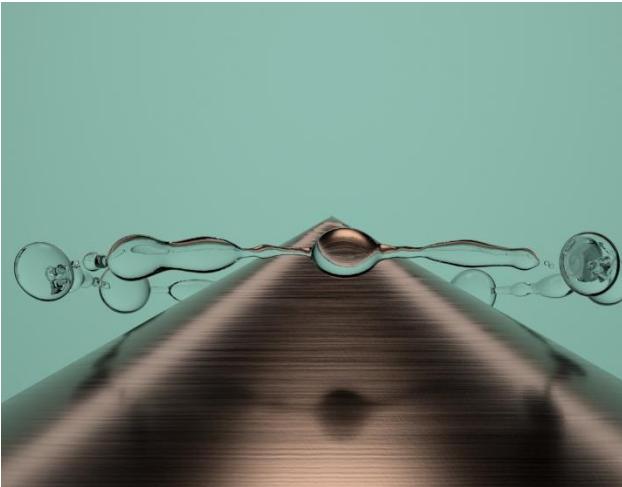
# Key Idea

- One-ring local meshes to represent local geometry of the surface
- Simulating surface tension effect by minimizing area



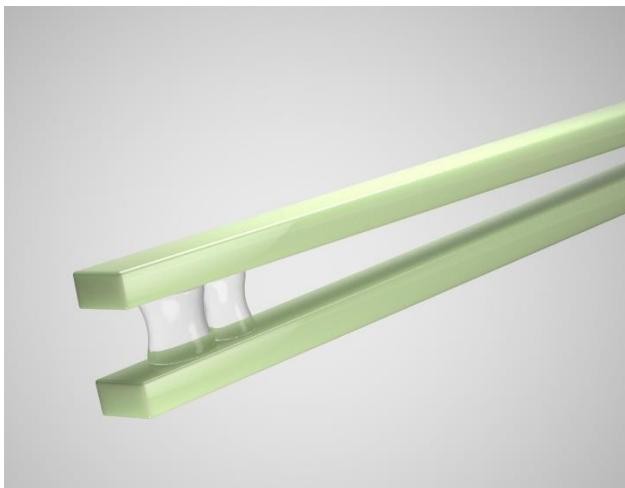
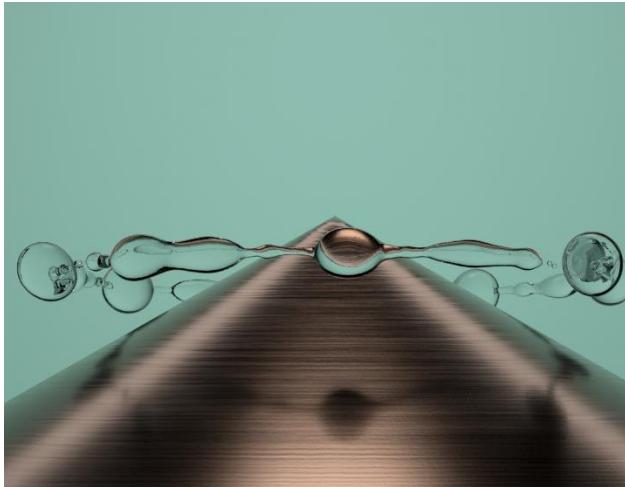
# Our Contribution

- Local mesh representation
- Surface tension constraint
- Topological transition



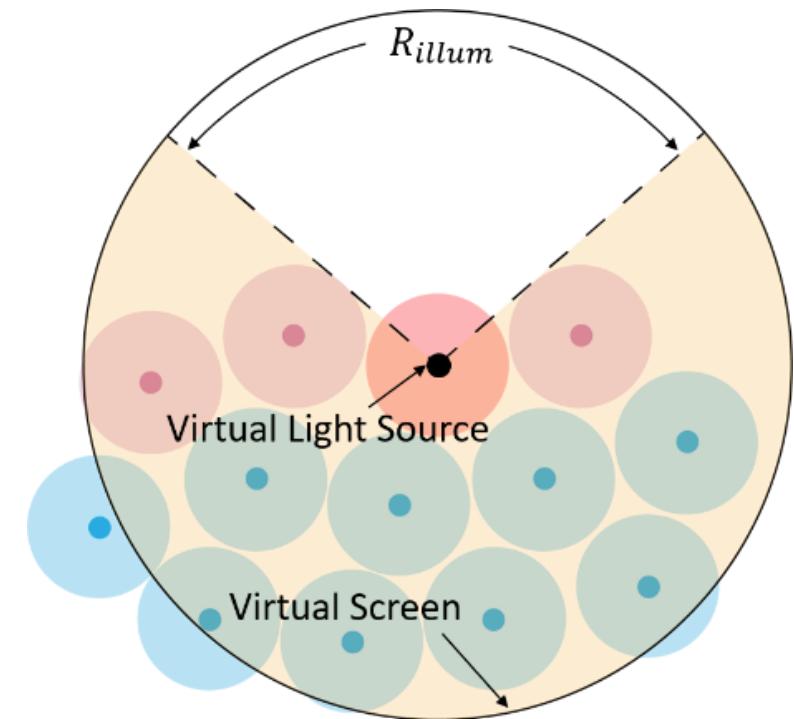
# Our Contribution

- Local mesh representation
- Surface tension constraint
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# Local Mesh Construction

- Detect surface particles [Shibata et al. 2015]
  - $R_{illum} > \beta$ , surface particle
  - $R_{illum} \leq \beta$ , interior particle



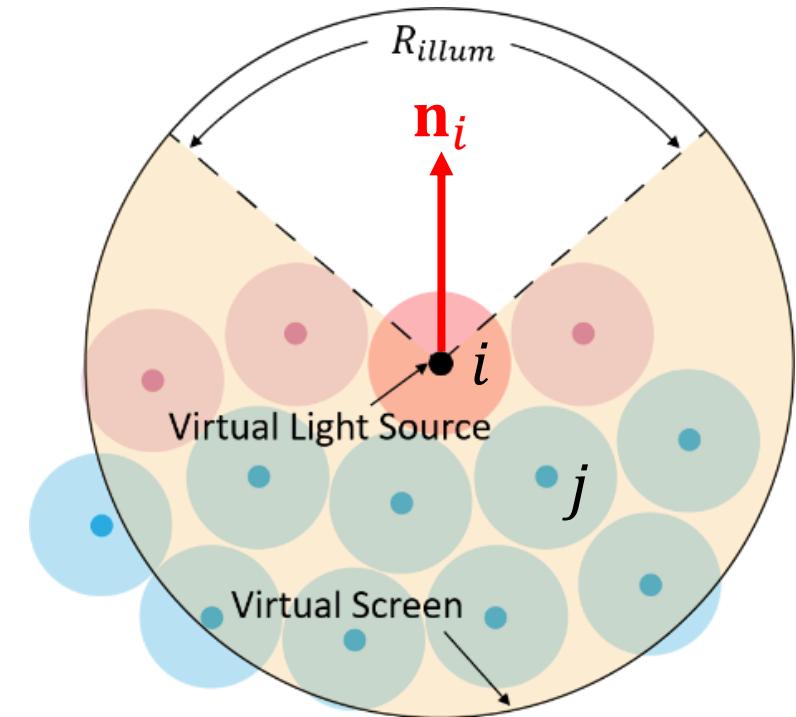
# Local Mesh Construction

- Calculate normal vectors

- Color field:  $c(\mathbf{p}) = \sum_{j \in N(\mathbf{p})} \frac{m}{\rho_j} W(\mathbf{p} - \mathbf{p}_j, h)$

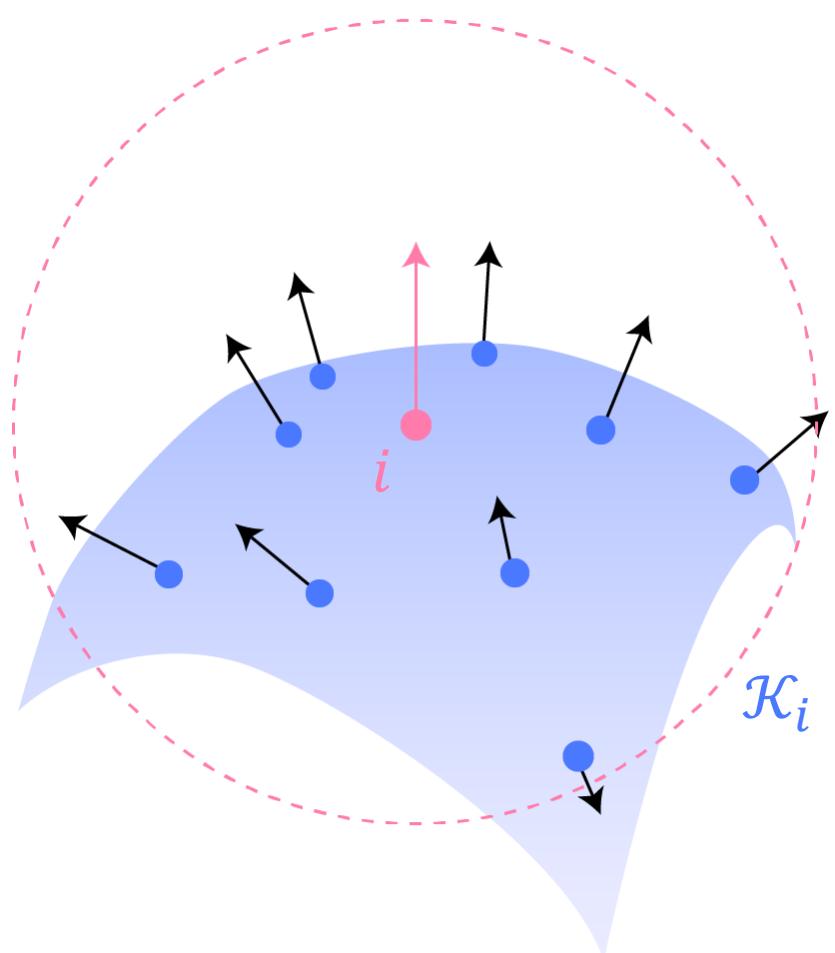
- Color gradient:

$$\mathbf{n}_i = -\nabla c(\mathbf{p}_i) = -\sum_{j \in N(\mathbf{p})} \frac{m}{\rho_j} \nabla W(\mathbf{p}_i - \mathbf{p}_j, h)$$



# Local Mesh Construction

- Construct local meshes

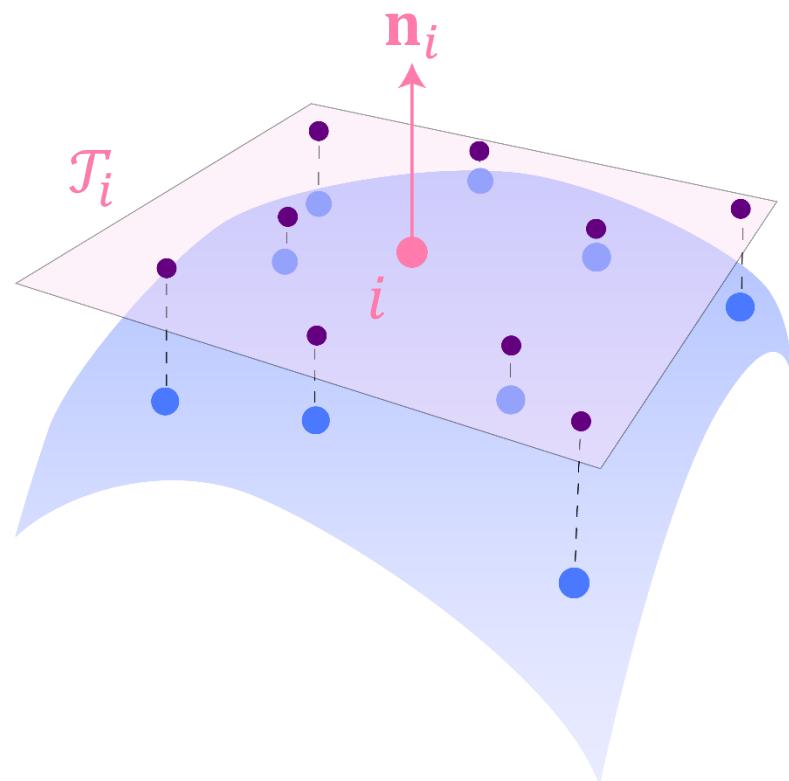


## Neighbor search.

- $\mathcal{K}_i$ : surface particles within kernel radius

# Local Mesh Construction

- Construct local meshes

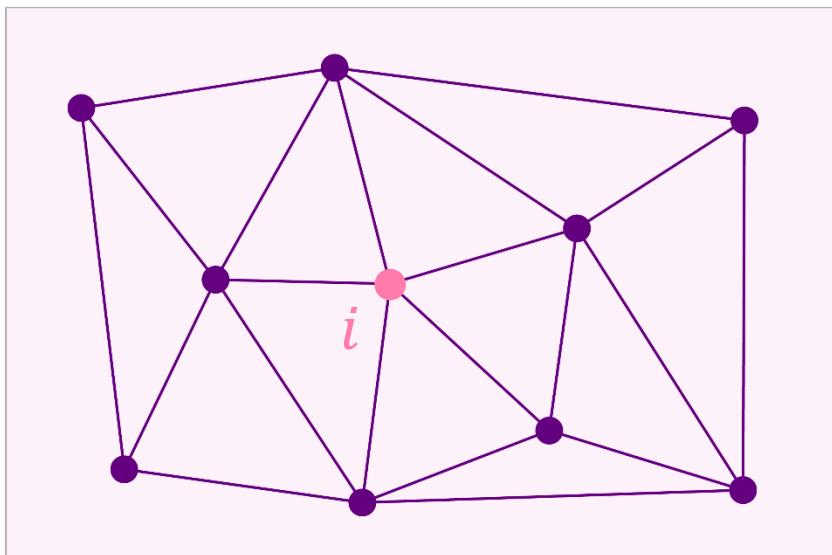


## Particle projection.

- $\mathcal{T}_i$ : tangent plane
- Project  $\mathcal{K}_i$  and  $i$  onto  $\mathcal{T}_i$

# Local Mesh Construction

- Construct local meshes

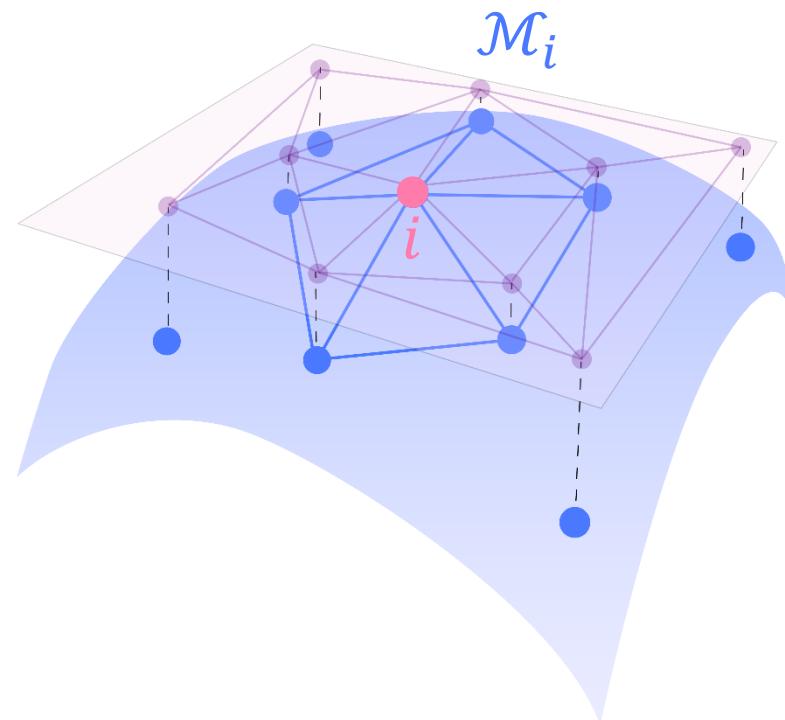


**Delaunay triangulation.**

- Carry out 2D triangulation on  $\mathcal{T}_i$

# Local Mesh Construction

- Construct local meshes

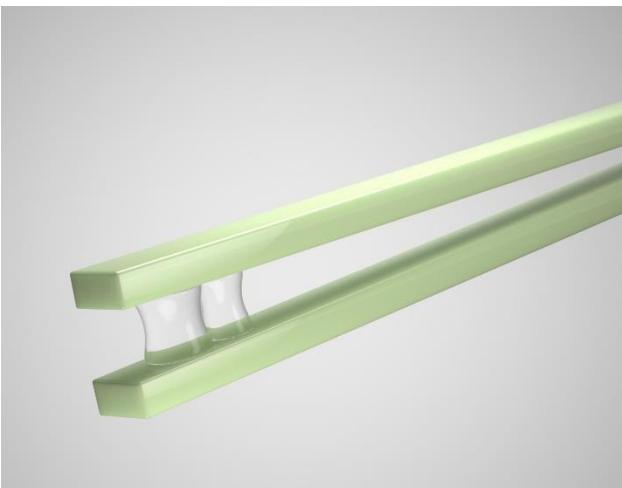
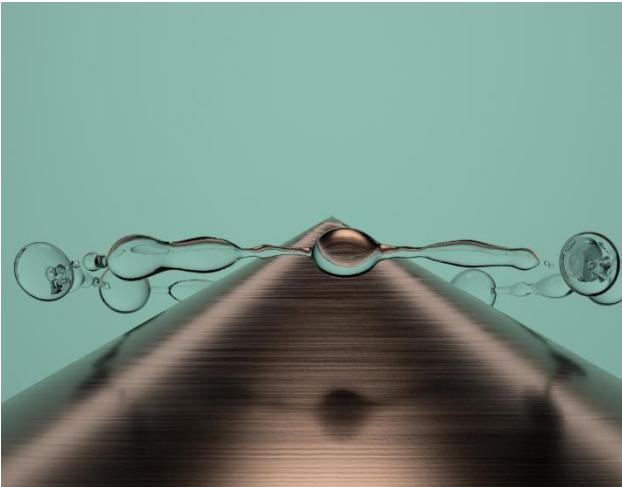


## Local mech construction.

- Take the 1-ring mesh
- Restore particles' 3D positions

# Our Contribution

- Local mesh representation
- Surface tension constraint
- Topological transition



# Surface Tension Constraint

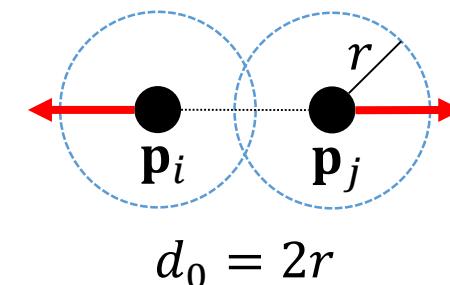
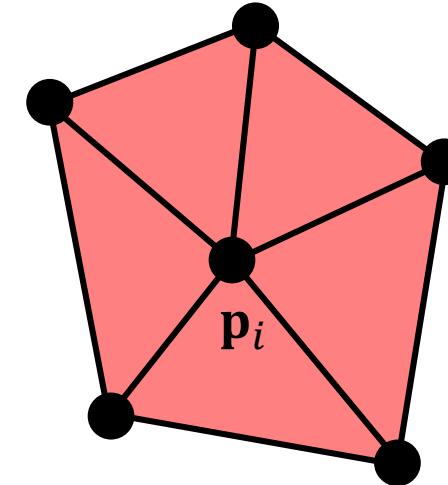
- **Area constraint**

$$C_i^A(\mathbf{p}) = \sum_{t \in T(i)} \frac{1}{2} \|(\mathbf{p}_{t^2} - \mathbf{p}_{t^1}) \times (\mathbf{p}_{t^3} - \mathbf{p}_{t^1})\|$$

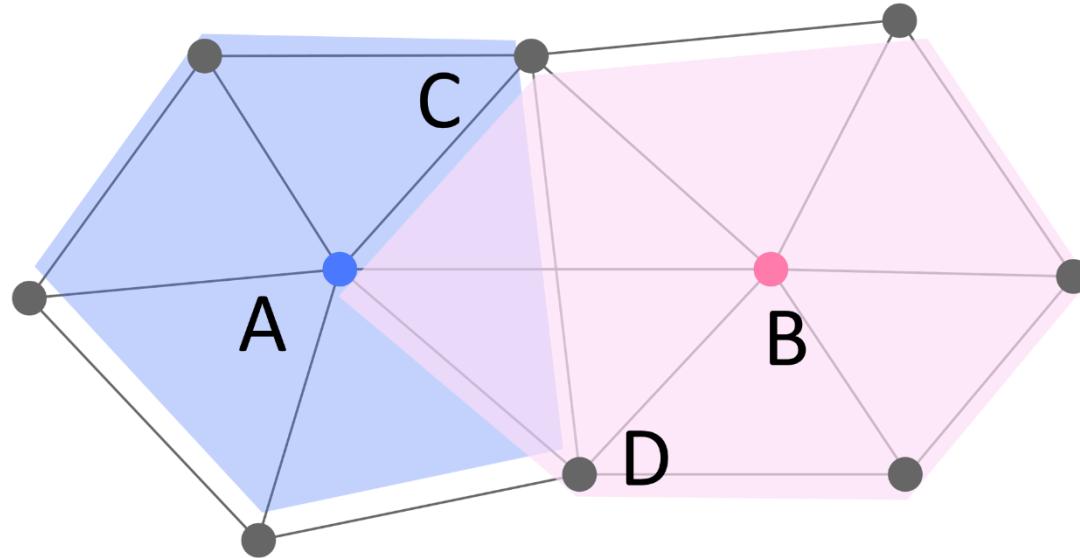
- Minimizes the local area
- **Distance constraint**

$$C_{ij}^D(\mathbf{p}) = \min\{0, \|\mathbf{p}_i - \mathbf{p}_j\| - d_0\}$$

- Keeps a uniform distribution



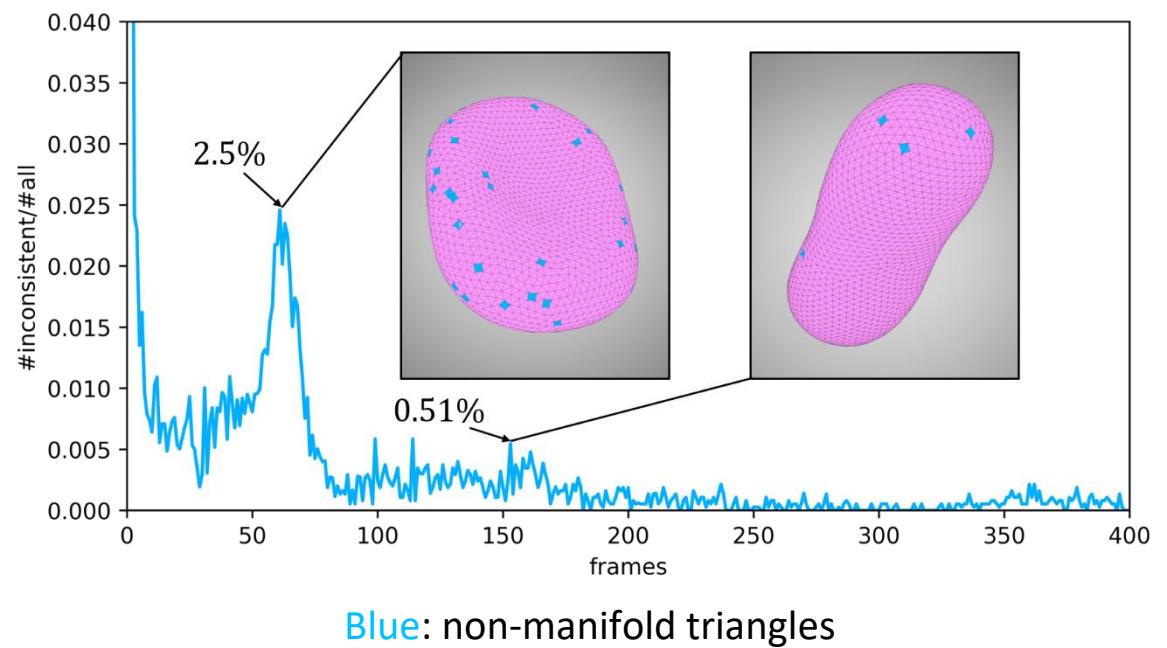
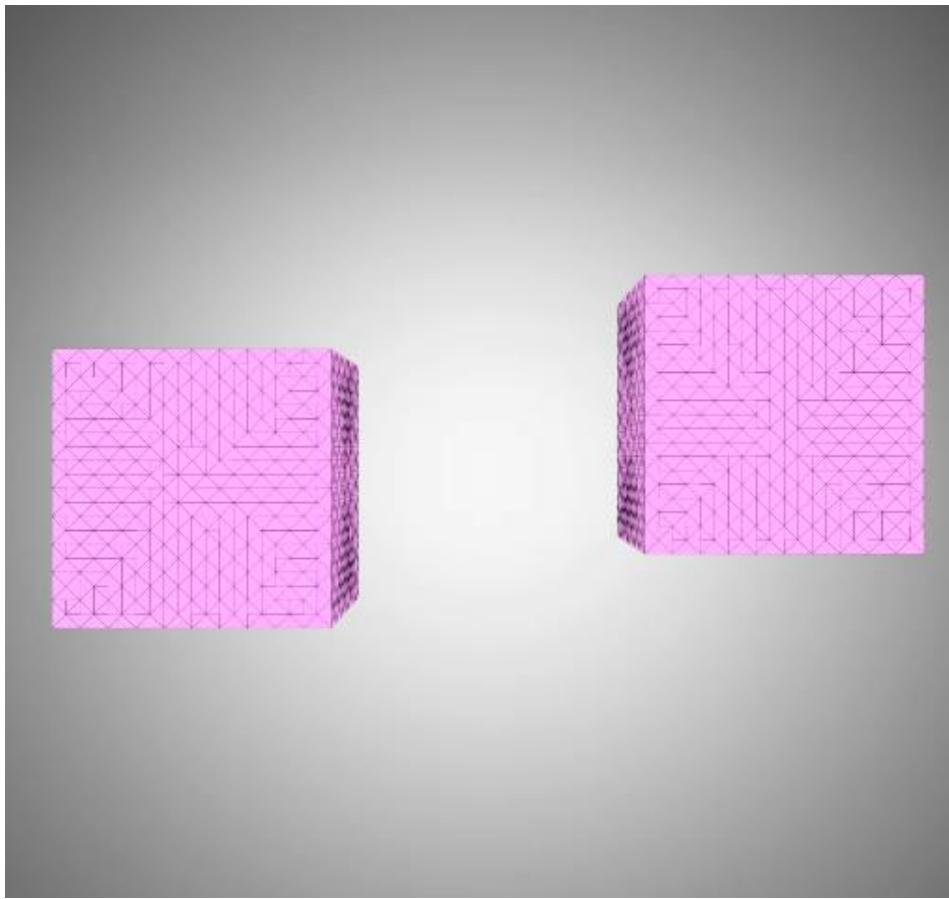
# Local Mesh Inconsistency



## Non-manifold triangles:

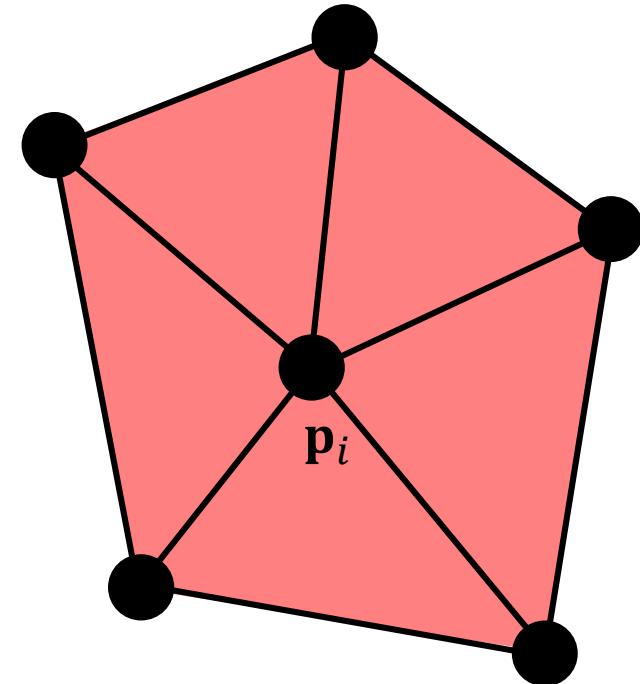
- Not in exact 3 local meshes
- $\Delta ACD$ ,  $\Delta ACB$  and  $\Delta ADB$

# Local Mesh Inconsistency



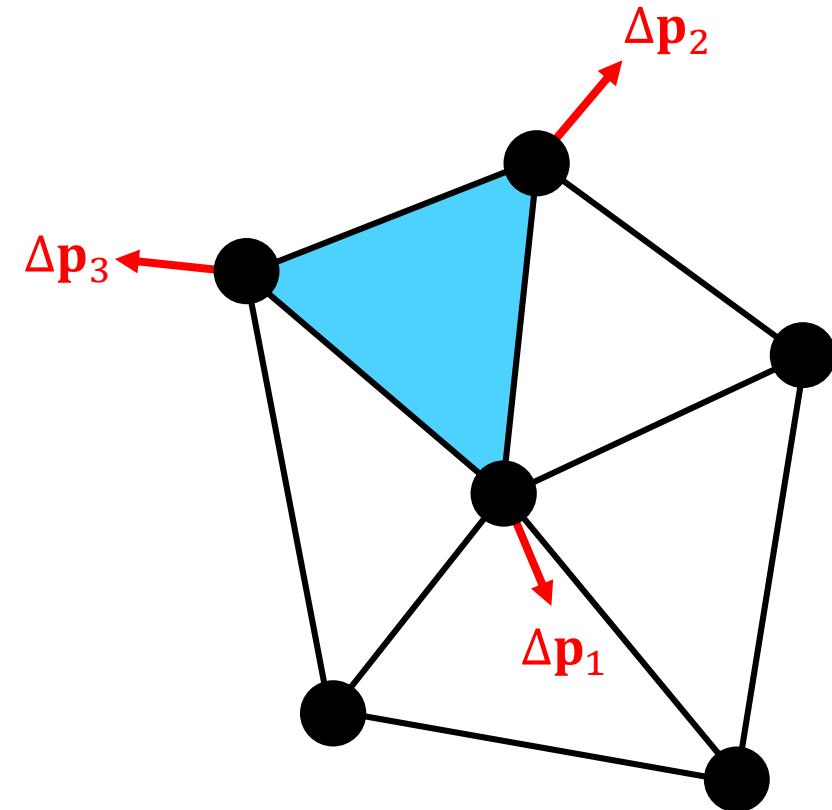
# Local Mesh Inconsistency

- Though inconsistency exists:
  - Minimizes local area



# Local Mesh Inconsistency

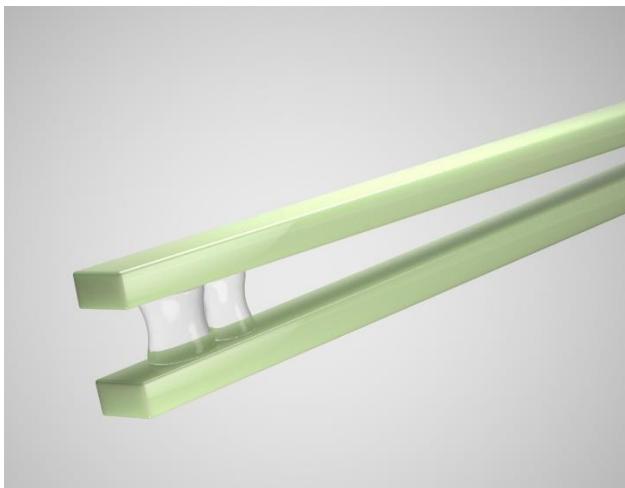
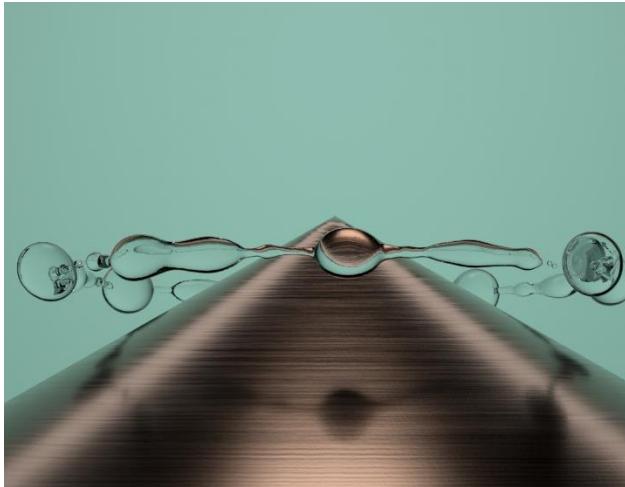
- Though inconsistency exists:
  - Minimizes local area
  - Conserves momentum



$$\Delta\mathbf{p}_1 + \Delta\mathbf{p}_2 + \Delta\mathbf{p}_3 = \mathbf{0}$$

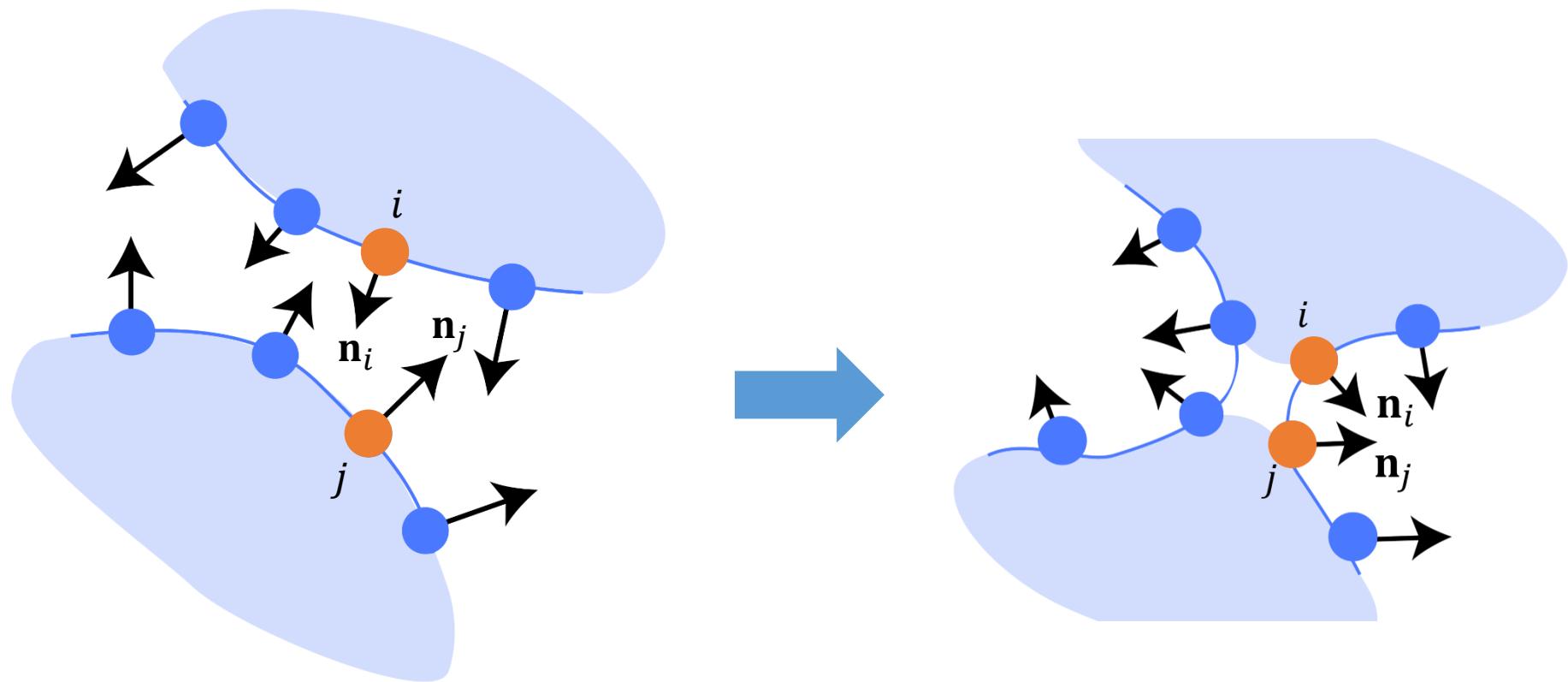
# Our Contribution

- Local mesh representation
- Surface tension constraint
- Topological transition



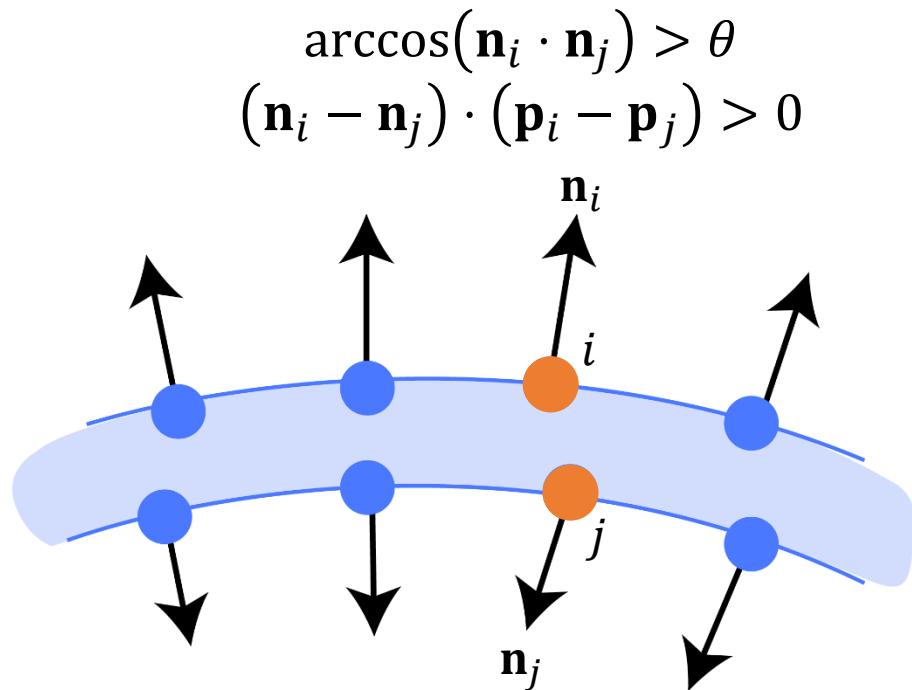
# Topological Transition

- Merging



# Topological Transition

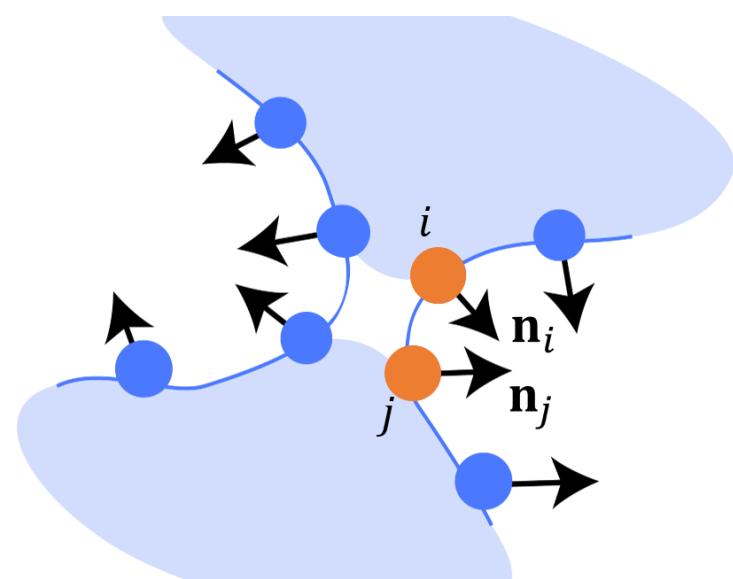
- Merging



Avoid merging in thin film

$$\arccos(\mathbf{n}_i \cdot \mathbf{n}_j) > \theta$$
$$(\mathbf{n}_i - \mathbf{n}_j) \cdot (\mathbf{p}_i - \mathbf{p}_j) \leq 0$$

v.s.



# Topological Transition



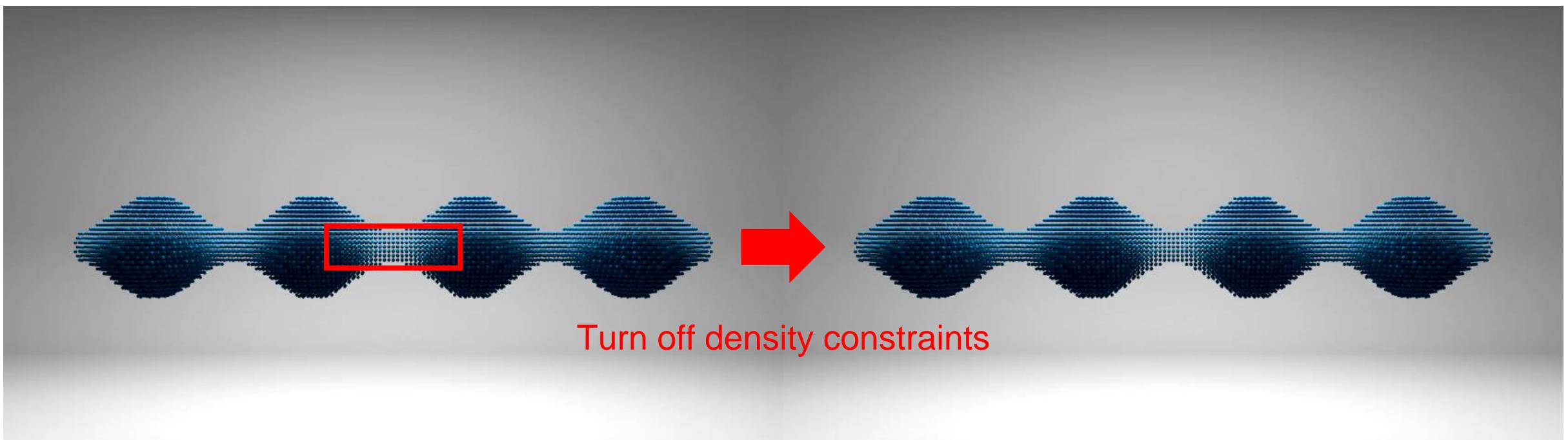
# Topological Transition

- Splitting

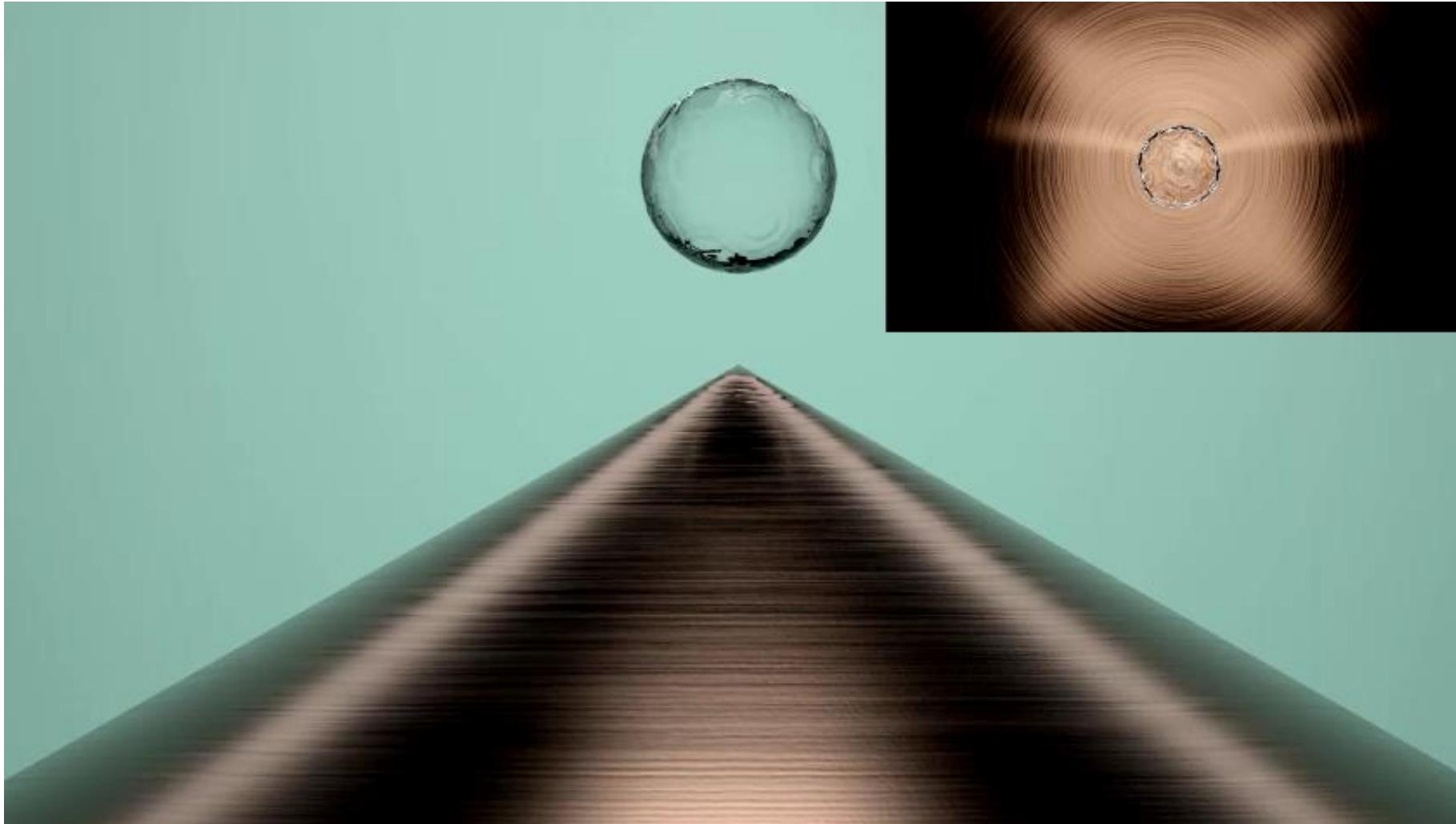


# Topological Transition

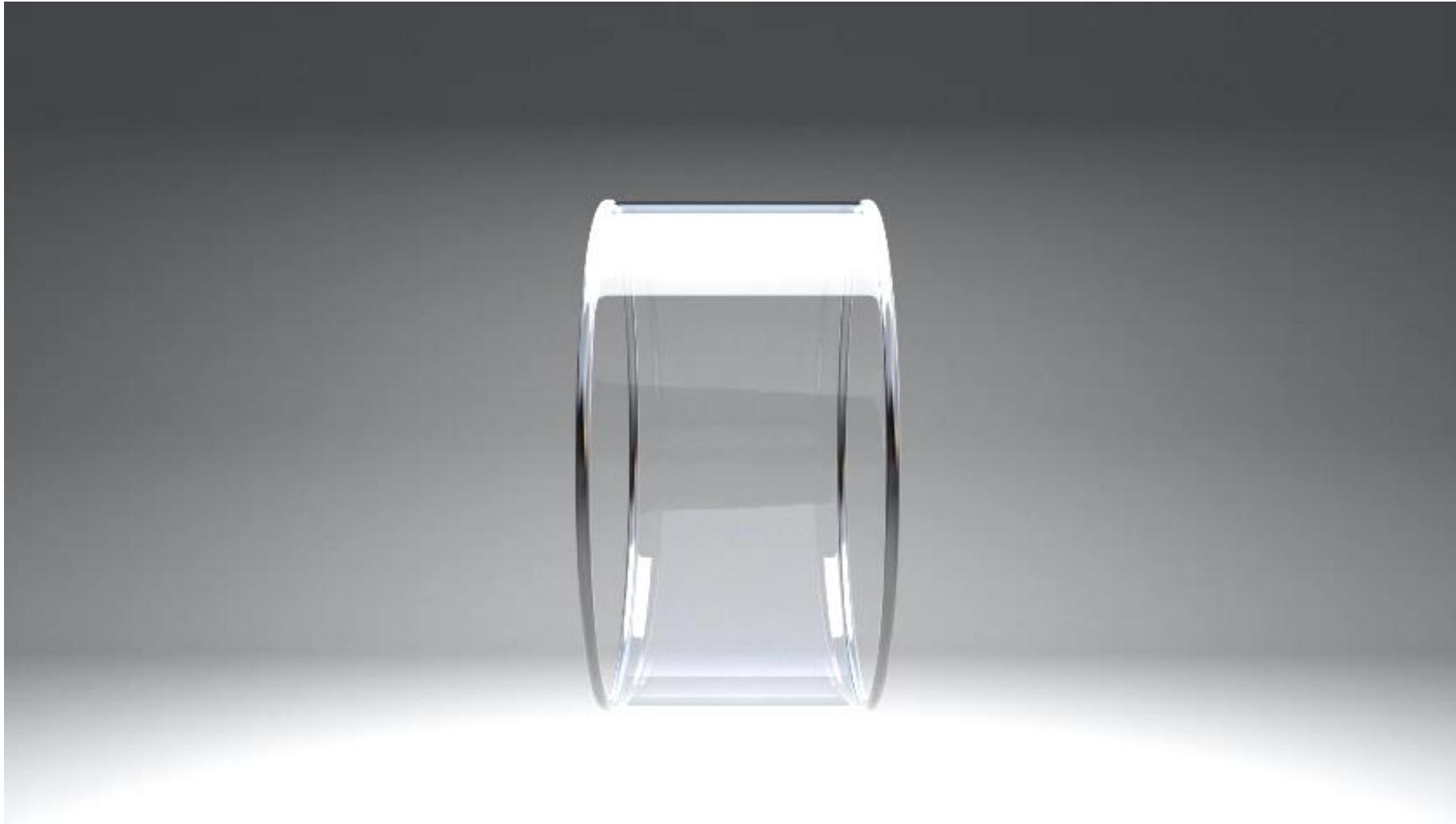
- Splitting
  - Eliminate filaments



# Examples



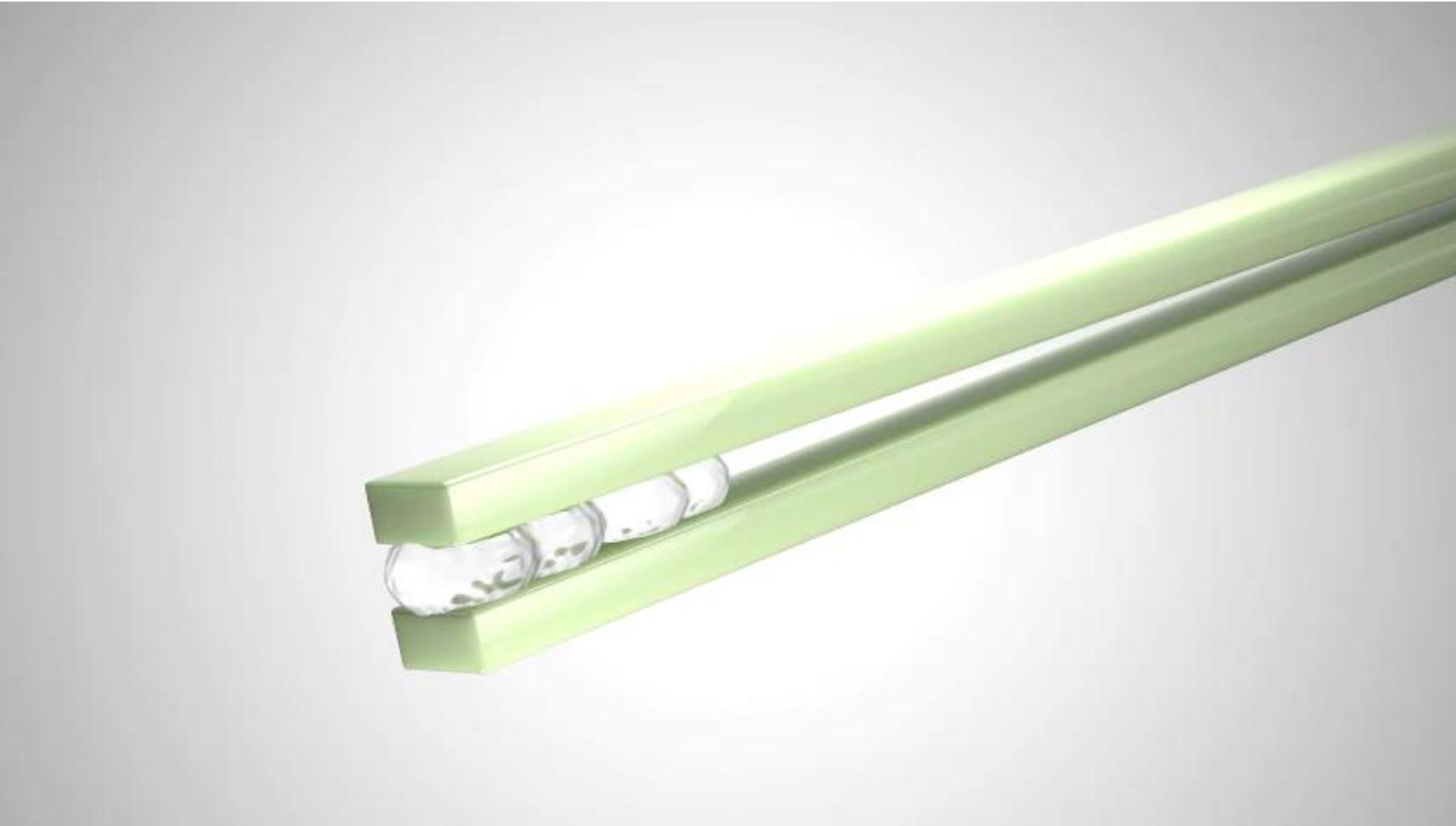
# Examples



# Examples



# Examples



# Summary

- Simulating surface tension effect in PBF framework
  - Local mesh construction
  - Surface tension constraint
  - Topological transition

# Limitations

- Relies on accurate surface particle detection
- Lack of acceleration strategies
- Not physically accurate (inherited from PBD)



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# Thank you!

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