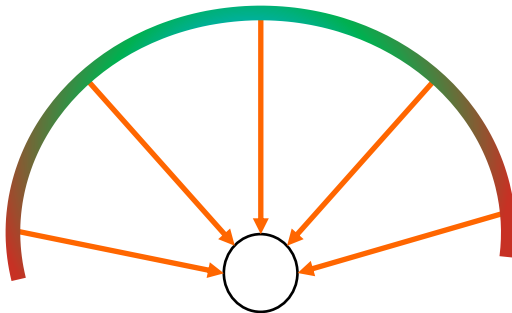


SPCBPT: SUBSPACE-BASED PROBABILISTIC CONNECTIONS FOR BIDIRECTIONAL PATH TRACING

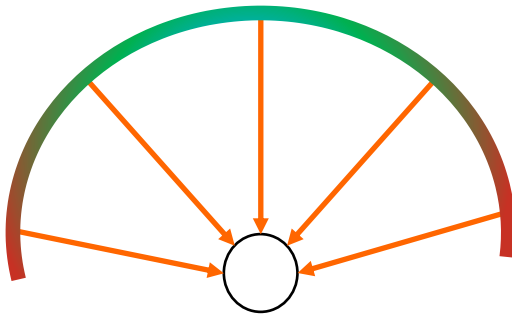
FUJIA SU, SHENG LI, GUOPING WANG

PATH INTEGRAL

- Outgoing radiance of a surface point x
 - Depends on its environment illumination



$$L(x) = \int_A f(x') dx'$$



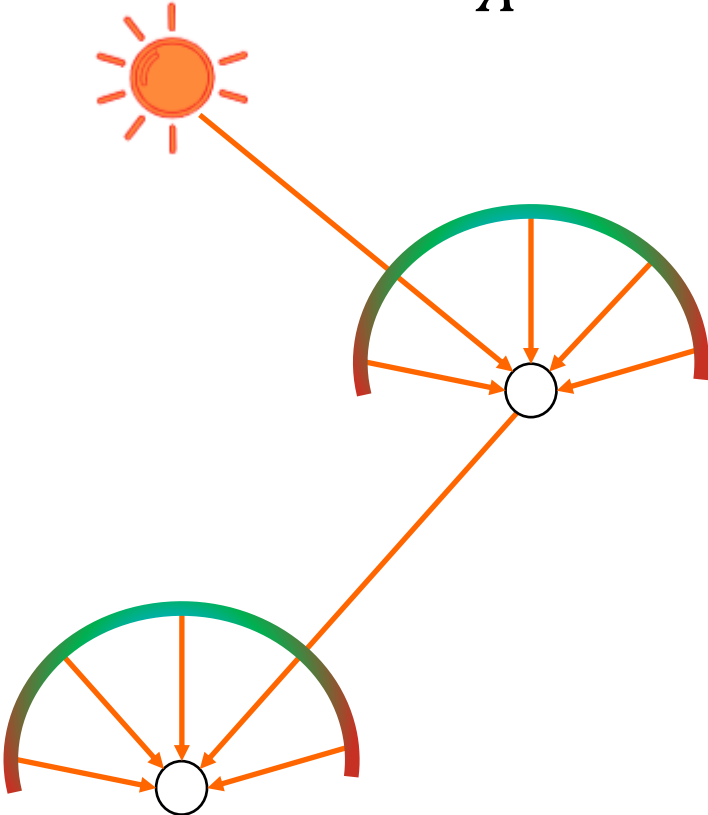
PATH INTEGRAL

- Outgoing radiance of a surface point x
 - Depends on its environment illumination
- Integrate the environment to estimate the radiance
 - Environment integral \Rightarrow Surface integral
 - Monte Carlo method

$$L(x) = \int_{A^2} f(x_1 x_2) dx_1 x_2$$

PATH INTEGRAL

- The radiance of environment surface also depends on its environment illumination \Rightarrow Recursion
- If the surface point is located on light source \Rightarrow stop the recursion and return the light source intensity.



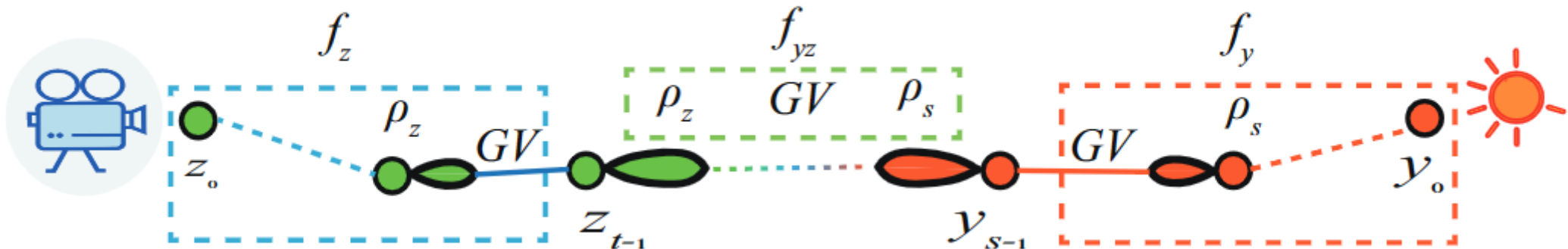
$$\begin{aligned} L(x) &= \int_A f(\bar{x}) d\bar{x} + \int_{A^2} f(\bar{x}) d\bar{x} \\ &+ \int_{A^3} f(\bar{x}) d\bar{x} \dots \\ &= \int_{\Omega} f(\bar{x}) d\bar{x} \end{aligned}$$

PATH INTEGRAL

- To compute the radiance, we need to integrate all the possible path from light source to the shading point.
- Integration domain increases as recursion
- Path space $\Omega = A + A^2 + A^3 \dots = \sum_{i=1} A^i$ is the union of the arbitrary dimensional of the scene surface A.

BIDIRECTIONAL PATH TRACING (BDPT)

- Sampling method for the path space $\Omega \Rightarrow$ Sampling method for rendering.
- Bidirectional path tracing:
 - Trace eye sub-path from camera
 - Trace light sub-path from light source
 - Connection to full path



Light Source

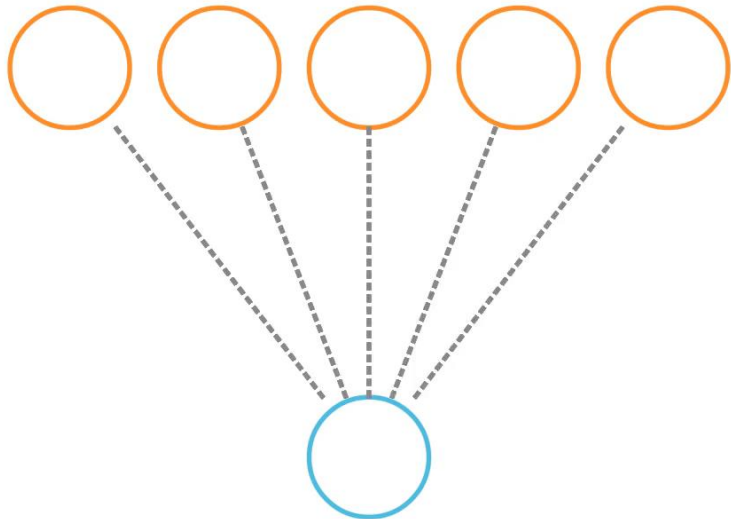
Camera

PROBABILISTIC CONNECTION

- Probabilistic connection is an extension of bidirectional path tracing, where:
 - Multiple light sub-paths are traced and cached for resampling.
 - Eye sub-path samples light sub-path from the cache.

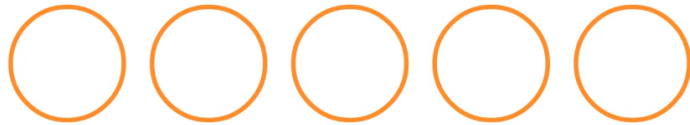
HOW TO IMPROVE?

- More light sub-paths to connect
- **Importance sample the candidate light sub-paths**



RELATED ALGORITHM

- Popov et al.[2015] selects the light sub-path by building PMF records in the scene.

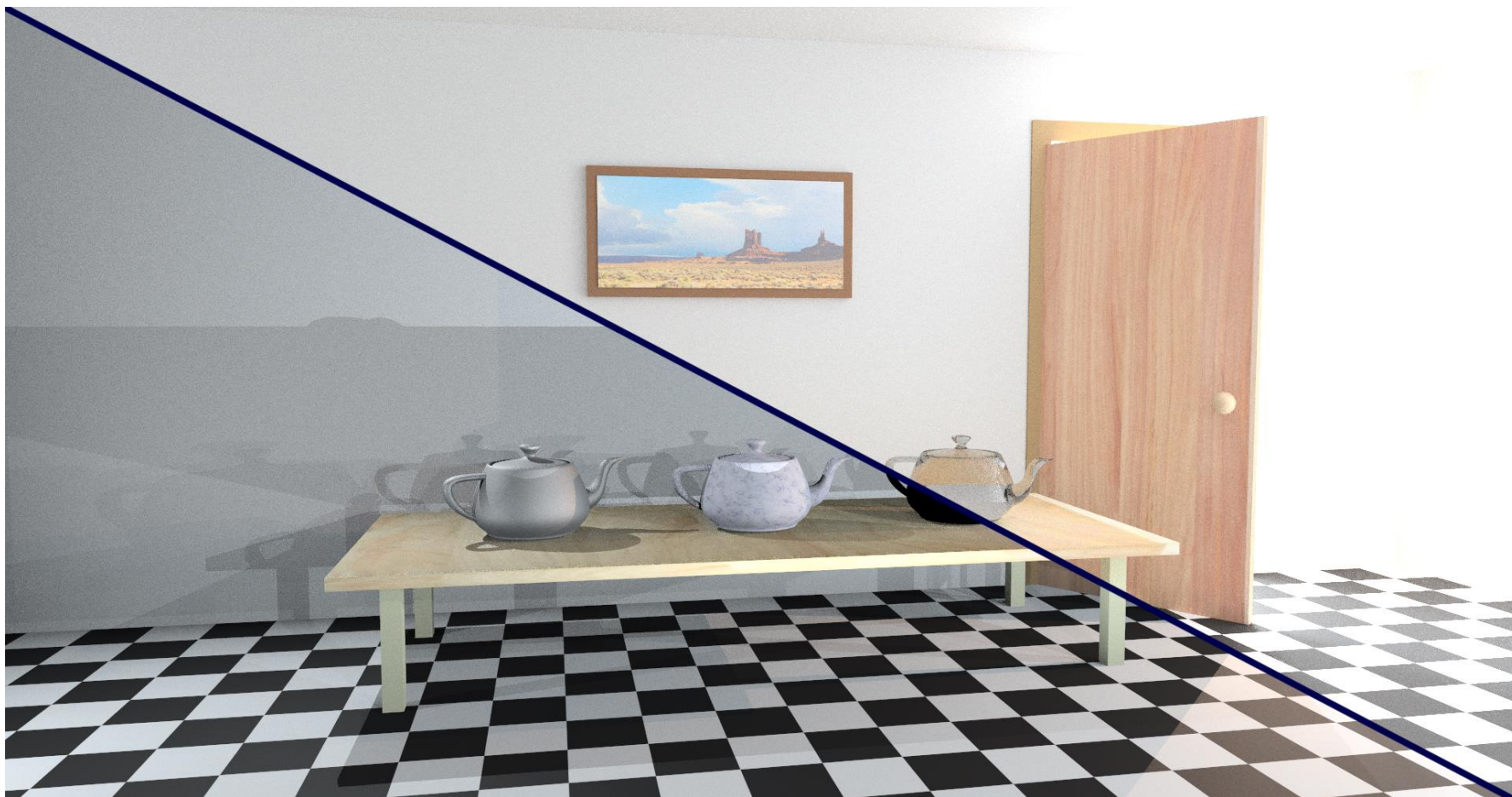


Camera

RECORD-BASED METHOD: CHALLENGE

- Limited candidate sub-paths
- Expensive reconstruction overhead

Reference



M=200

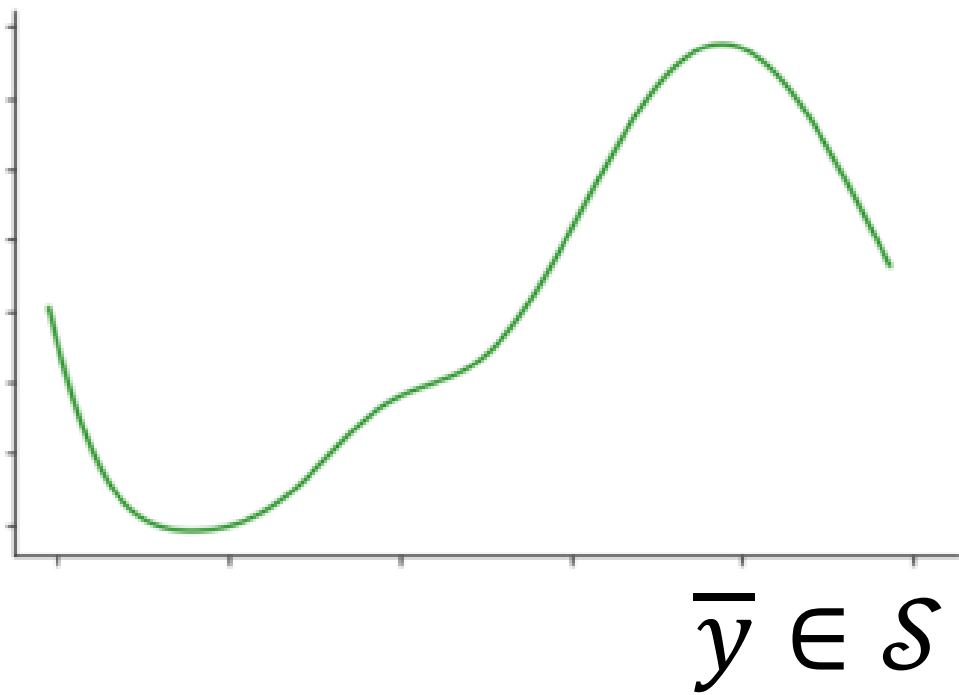


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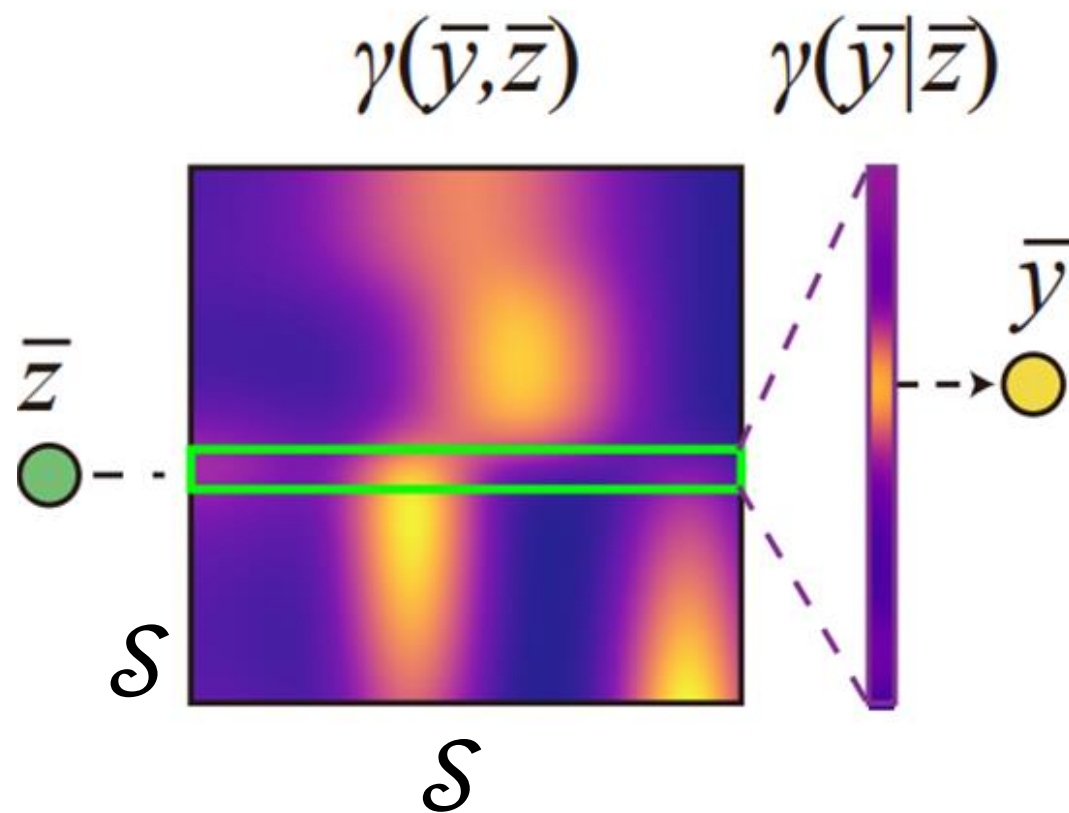
SUBSPACE-BASED PROBABILISTIC CONNECTIONS OUR METHOD

$$p(\bar{y}|\bar{z})$$



LIGHT SELECTION FUNCTION

- Sub-path space $\mathcal{S} := \cup_t A^t$
 - A : Scene Surface
- Given eye sub-path \bar{z} , sample light sub-path \bar{y}
 - Pdf $p(\bar{y}|\bar{z})$: distribution in \mathcal{S}
 - Sub-path $\bar{y}, \bar{z} \in \mathcal{S}$

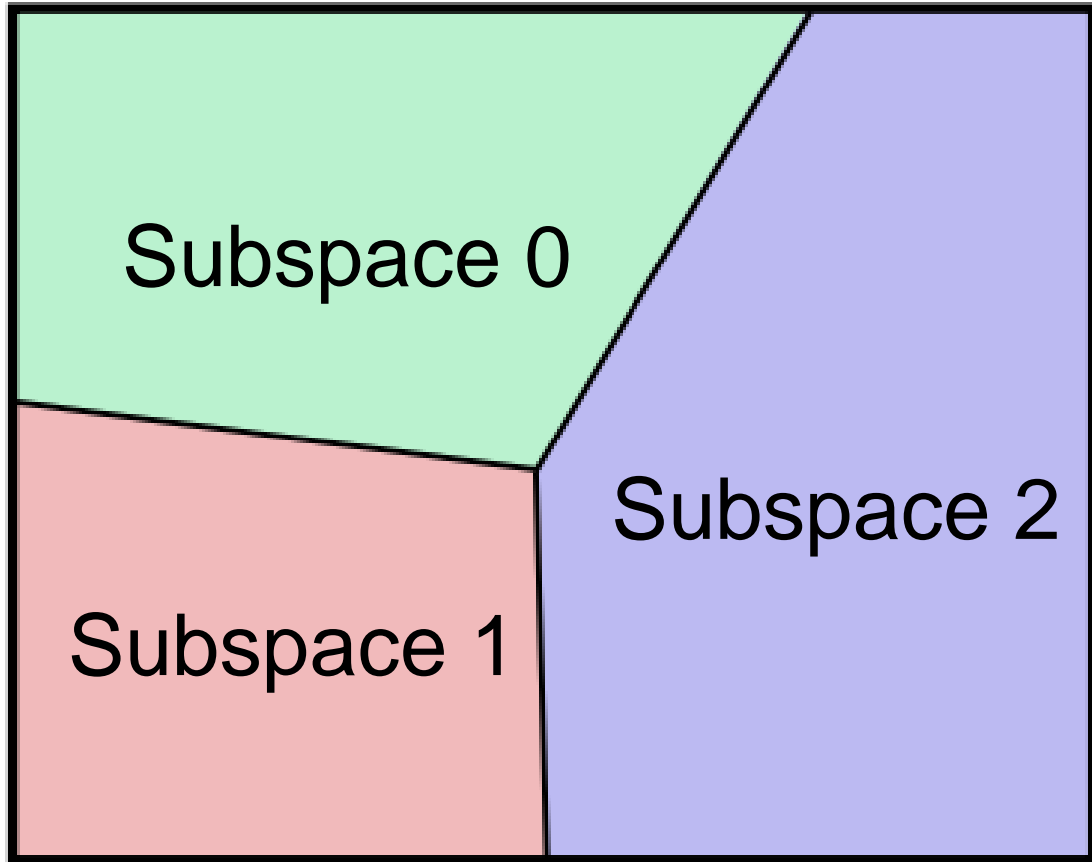


LIGHT SELECTION FUNCTION $\gamma(\bar{y}, \bar{z})$

- Define in $\mathcal{S} \times \mathcal{S}$
- $\gamma(\bar{y}, \bar{z})$: sampling probability for (\bar{y}, \bar{z})
- Marginalization of $\gamma(\bar{y}, \bar{z})$
- \Leftrightarrow Distribution of light sub-path sampling $p(\bar{y} | \bar{z})$

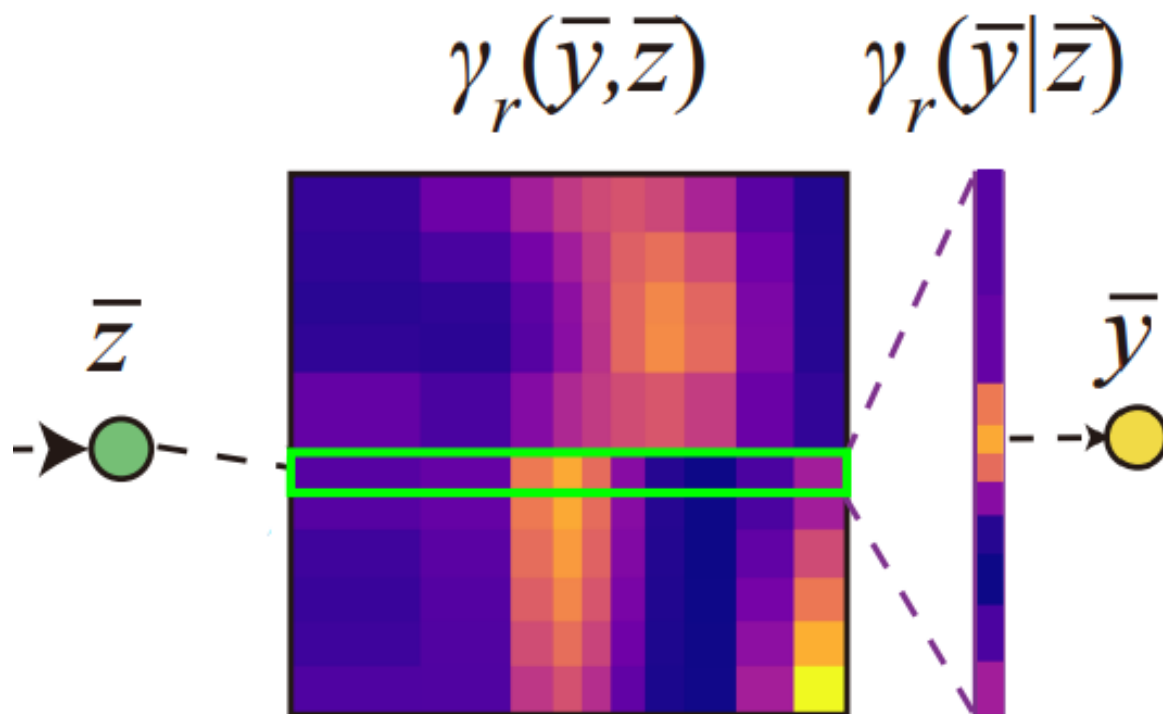
TARGET

- Find a light selection function
 - Avoid distribution reconstruction
 - Easy to sample
 - Easy to evaluate
 - Massive candidate sub-paths



SUBSPACE

- Subset of sub-path space
- Group of similar sub-paths
- Share the sampling weight
 - Light subspace for light sub-path
 - Eye subspace for eye sub-path



SUBSPACE-BASED LIGHT SELECTION

- Sample the light sub-path based on
 - Eye subspace of \bar{z} : $\kappa_{\mathcal{E}}(\bar{z})$
 - Light subspace of \bar{y} : $\kappa_{\mathcal{L}}(\bar{y})$
- But, is a simple matrix enough?



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SUBSPACE-BASED PROBABILISTIC CONNECTIONS

OPTIMAL LIGHT SELECTION

SUBSPACE-BASED PROBABILISTIC CONNECTIONS

EQUATION FOR PROBABILISTIC CONNECTIONS

EQUATION FOR PROBABILISTIC CONNECTIONS

$$I = \int_{\Omega} f(\bar{x}) d\mu(\bar{x})$$

EQUATION FOR PROBABILISTIC CONNECTIONS

$$I = \int_{\Omega} \sum_t w_t(\bar{x}) f(\bar{x}) d\mu(\bar{x})$$

With MIS weighting function

$$\sum_t w_t(\bar{x}) = 1$$

EQUATION FOR PROBABILISTIC CONNECTIONS

$$I = \sum_t \int_{\Omega} w_t(\bar{y}\bar{z}) f(\bar{y}\bar{z}) d\mu(\bar{y}\bar{z})$$

In probabilistic connections, full path is constructed by connecting sub-path pair

$$\bar{x} = \bar{y}\bar{z}$$

EQUATION FOR PROBABILISTIC CONNECTIONS

$$I = \sum_t \int_{A^t} \int_{\mathcal{A}} w_t(\bar{y}\bar{z}) f(\bar{y}\bar{z}) d\mu(\bar{y}) d\mu(\bar{z})$$

Strategy t uses eye sub-path of t vertices and light sub-path of arbitrary length to generate the full path

$$\bar{z} \in A^t, \bar{y} \in \cup_s A^s = \mathcal{A}$$

EQUATION FOR PROBABILISTIC CONNECTIONS

$$I = \sum_{\underline{t}} \int_{A^{\underline{t}}} I(\bar{\underline{z}}) d\mu(\bar{\underline{z}})$$

$$I(\bar{\underline{z}}) = \int_{A^{\underline{t}}} w_{\underline{t}}(\bar{\underline{y}}\bar{\underline{z}}) f(\bar{\underline{y}}\bar{\underline{z}}) d\mu(\bar{\underline{y}})$$

OPTIMAL LIGHT SELECTION FUNCTION

$$w_t(\bar{y}\bar{z})f(\bar{y}\bar{z})$$

OPTIMAL LIGHT SELECTION FUNCTION

$w_t(\bar{y}\bar{z})$: MIS weighting function

$f(\bar{y}\bar{z})$: Path contribution

OPTIMAL LIGHT SELECTION FUNCTION

$w_t(\bar{y}\bar{z})$: MIS weighting function

$f_z(\bar{z})$
depends on \bar{z} only

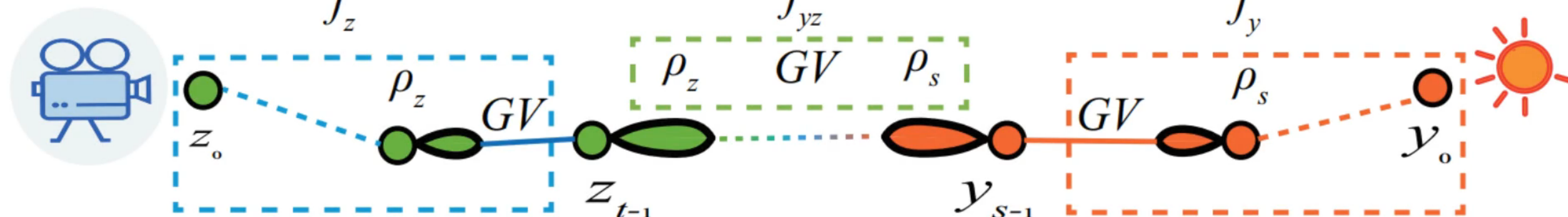
$f_{yz}(\bar{y}, \bar{z})$
depends on \bar{z} & \bar{y}

$f_y(\bar{y})$
depends on \bar{y} only

canceled out in $p(\bar{y}|\bar{z})$

Expensive to evaluate

Easy to get



REQUIREMENT

- The subspace-based light selection function $\gamma_r(\bar{y}, z)$ should
 - Depend on the eye subspace $\kappa_{\mathcal{E}}(\bar{z})$ and the light subspace $\kappa_{\mathcal{L}}(\bar{y})$
 - Be proportional to $f_y(\bar{y})$
 - Ensure $\gamma_r(\bar{y}|\bar{z})$ is a pdf
 - That is, $\int_{\mathcal{S}} \gamma_r(\bar{y}, \bar{z}) d\mu(\bar{y}) = 1$

SUBSPACE-BASED LIGHT SELECTION FUNCTION

$$\text{Optimal } \gamma^*(\bar{y}, \bar{z}) \propto w_t(\bar{y}\bar{z}) f_{yz}(\bar{y}\bar{z}) f_y(\bar{y})$$

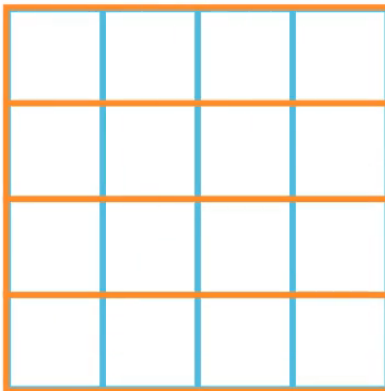
SUBSPACE-BASED LIGHT SELECTION FUNCTION

$$\text{Optimal } \gamma^*(\bar{y}, \bar{z}) \propto w_t(\bar{y}\bar{z}) f_{yz}(\bar{y}\bar{z}) f_y(\bar{y})$$

Requirement: Depends on subspace pair

$$\Gamma(\kappa_{\mathcal{E}}(\bar{z}), \kappa_{\mathcal{L}}(\bar{y}))$$

Subspace Sampling Matrix



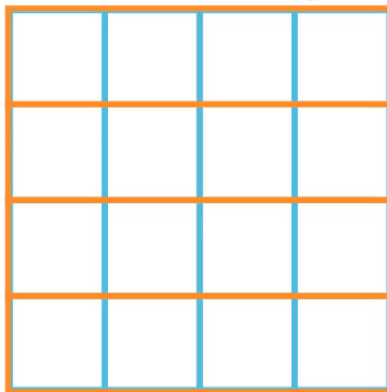
SUBSPACE-BASED LIGHT SELECTION FUNCTION

$$\text{Optimal } \gamma^*(\bar{y}, \bar{z}) \propto w_t(\bar{y}\bar{z}) f_{yz}(\bar{y}\bar{z}) f_y(\bar{y})$$

Requirement: proportional to $f_y(\bar{y})$

$$\Gamma(\kappa_{\mathcal{E}}(\bar{z}), \kappa_{\mathcal{L}}(\bar{y})) f_y(\bar{y})$$

Subspace Sampling Matrix



SUBSPACE-BASED LIGHT SELECTION FUNCTION

Optimal $\gamma^*(\bar{y}, \bar{z}) \propto w_t(\bar{y}\bar{z})f_{yz}(\bar{y}\bar{z})f_y(\bar{y})$

Requirement: normalization

$$\Gamma(\kappa_{\mathcal{E}}(\bar{z}), \kappa_{\mathcal{L}}(\bar{y}))f_y(\bar{y})/Q(\kappa_{\mathcal{L}}(\bar{y}))$$

Subspace Sampling Matrix

Normalization factor Q

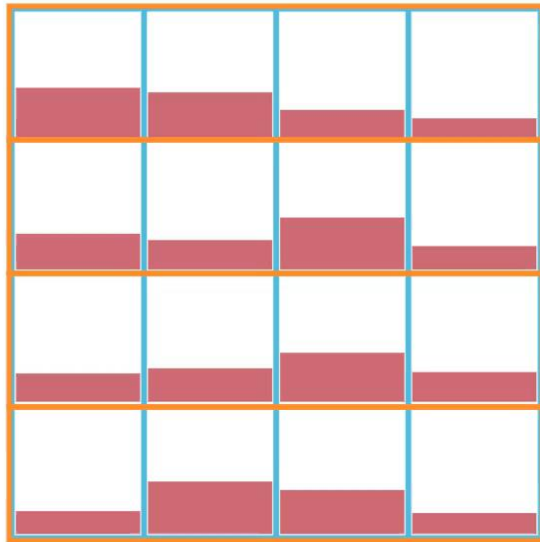
$Q(i)$ is the integral of f_y
for light subspace i

$$Q(i) = \int f_y(\bar{y})(\kappa_{\mathcal{L}}(\bar{y}) = i)d\mu(\bar{y})$$

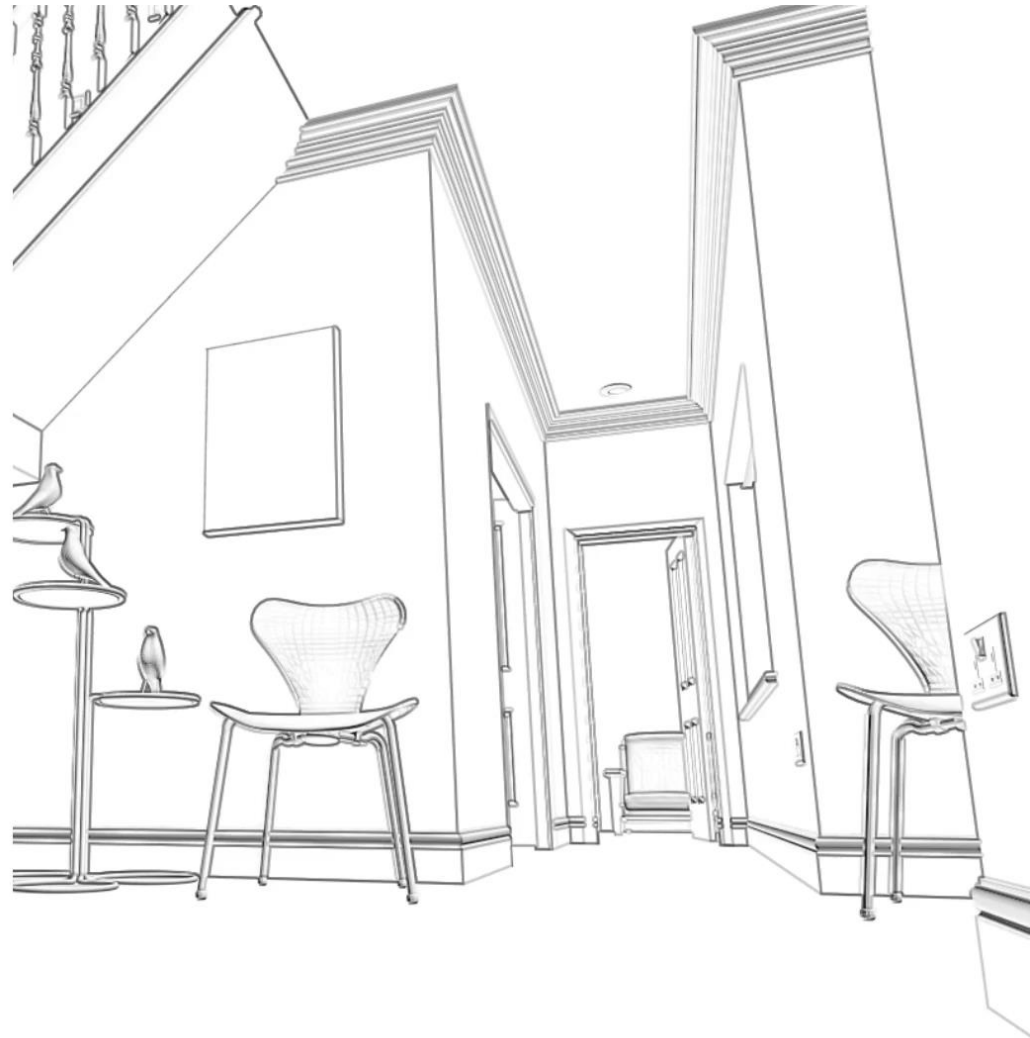
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TWO-STAGE SAMPLING

Subspace Sampling Matrix

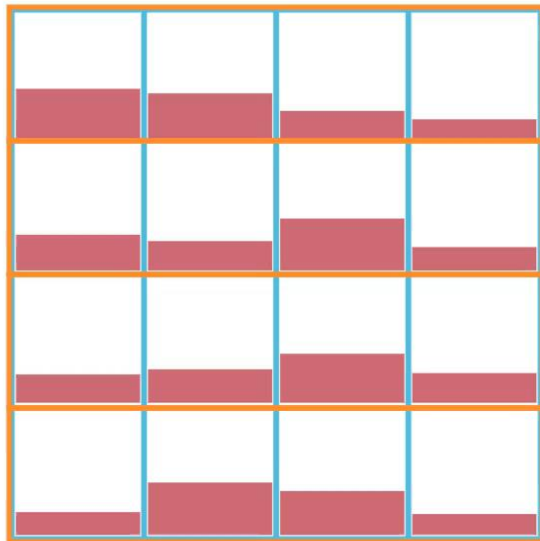


Camera

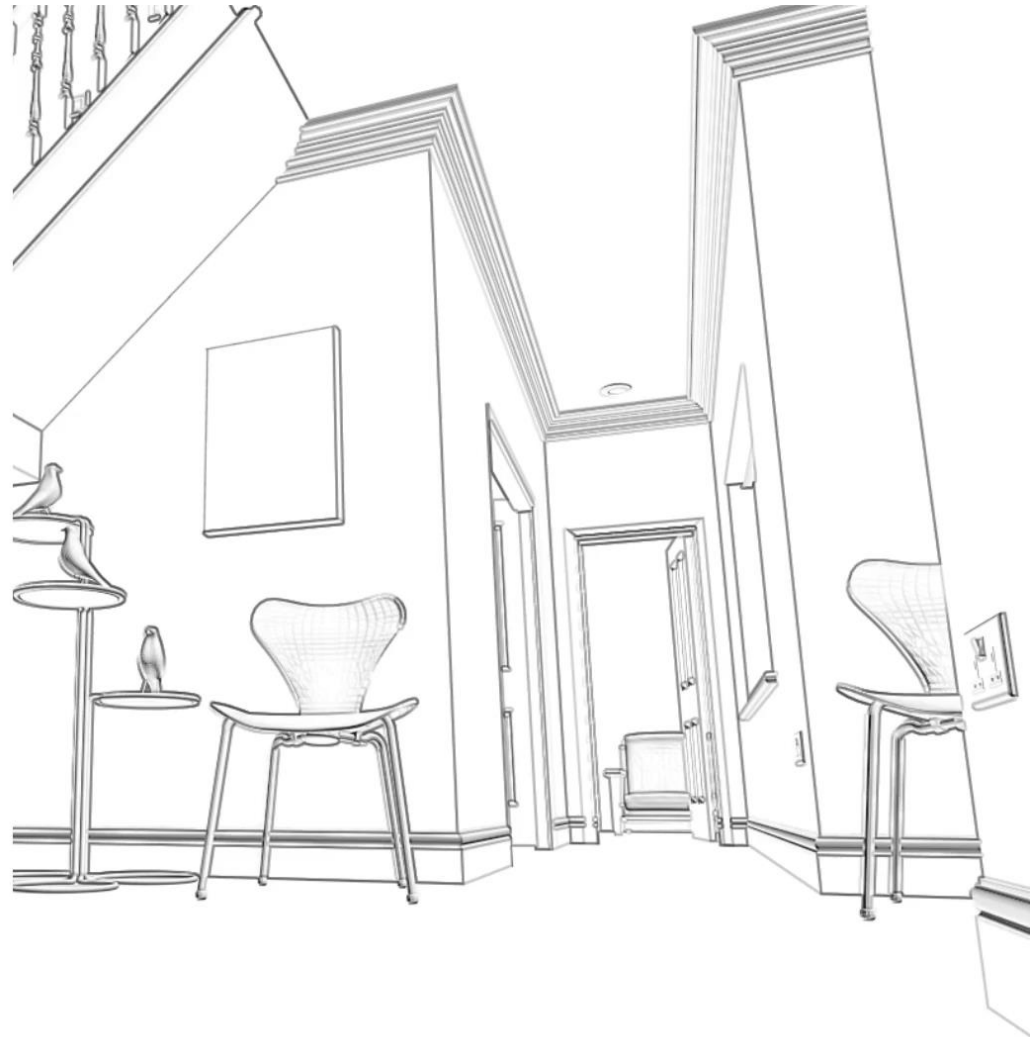


TWO-STAGE SAMPLING

Subspace Sampling Matrix



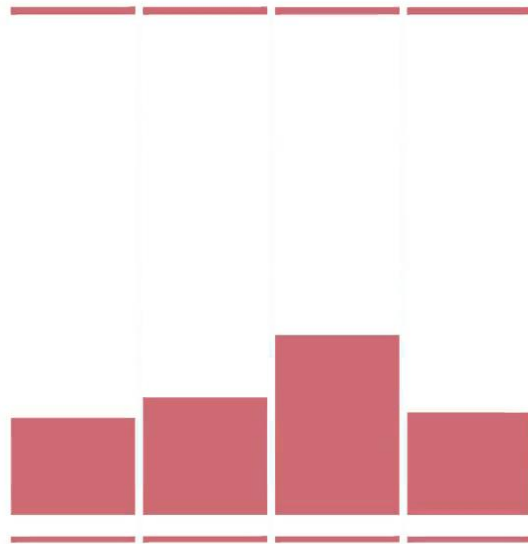
Camera



SUBSPACE-BASED PROBABILISTIC CONNECTIONS

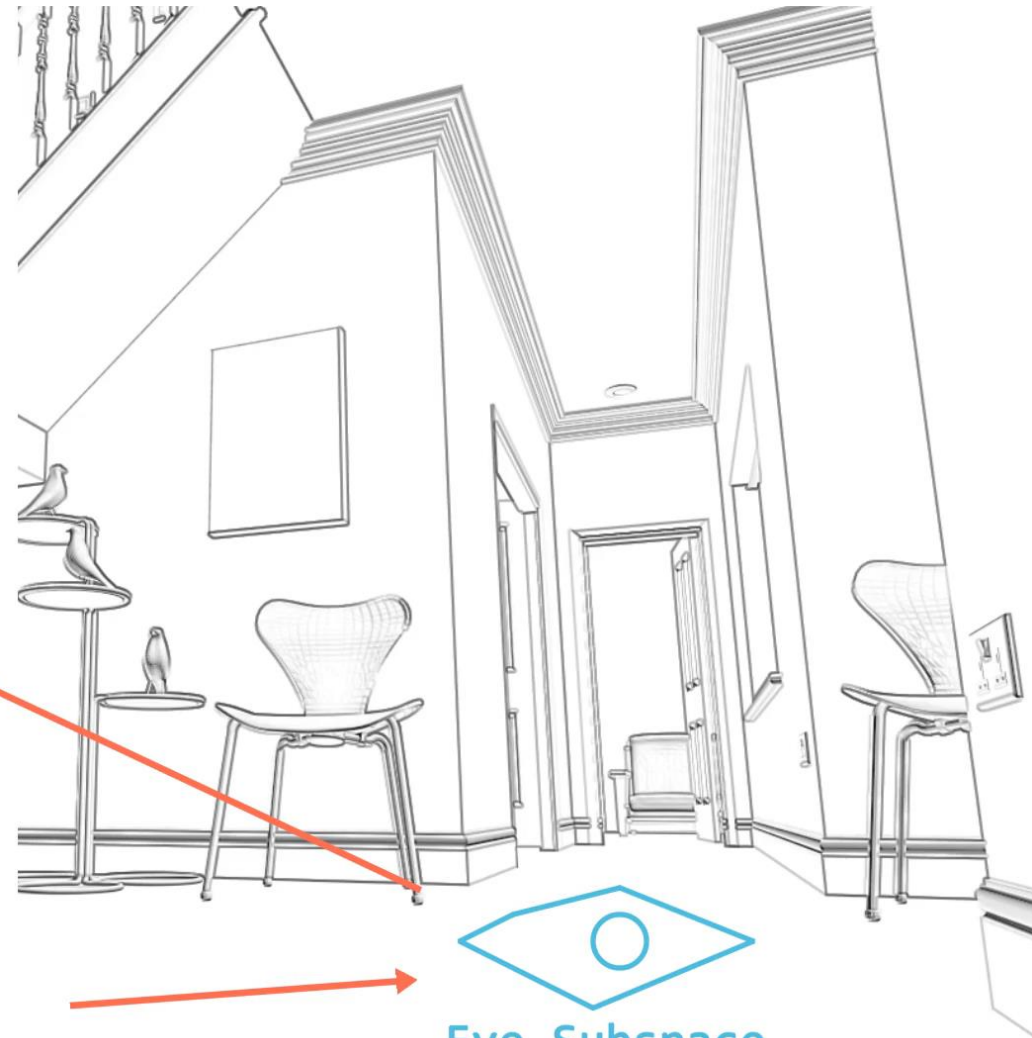
TWO-STAGE SAMPLING

Subspace Sampling Matrix



Sampling Light Subspace

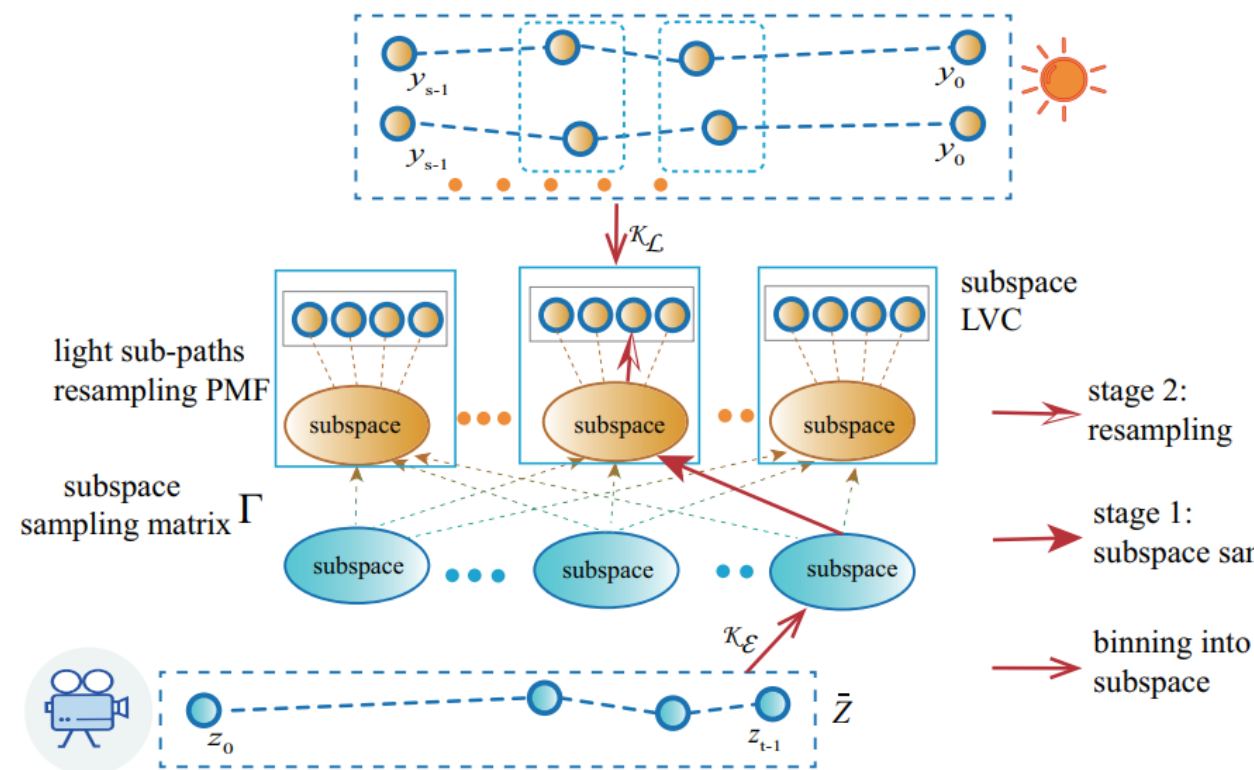
Camera



Eye Subspace

SECOND STAGE SAMPLING: RESAMPLING

- Sampling \bar{y} in pdf f_y/Q
- Resampling method
 - Tracing multiple candidate sub-path $\{\bar{Y}\}$ in pdf $p(\bar{y})$
 - Resample in the candidates in probability proportional to $f_y(\bar{Y})/p(\bar{Y})$
- Resampling pdf \approx target pdf



TWO-STAGE SAMPLING

- Low overhead
 - One subspace indexing for each sub-path
 - Γ is consistently used
- Massive candidate light sub-path
 - $M = 100K$ vs $M = 200$

SUBSPACE-BASED PROBABILISTIC CONNECTIONS



M=100K

M=200

Reference

OPTIMAL Γ : CHALLENGE

- Based on $w_t(\overline{yz})f_{yz}(\overline{yz})$?
- Circular dependency

OPTIMAL Γ : CHALLENGE

- Based on $w_t(\bar{y}\bar{z})f_{yz}(\bar{y}\bar{z})$?
- Circular dependency

$$w_t(\bar{y}\bar{z})f_{yz}(\bar{y}\bar{z})$$



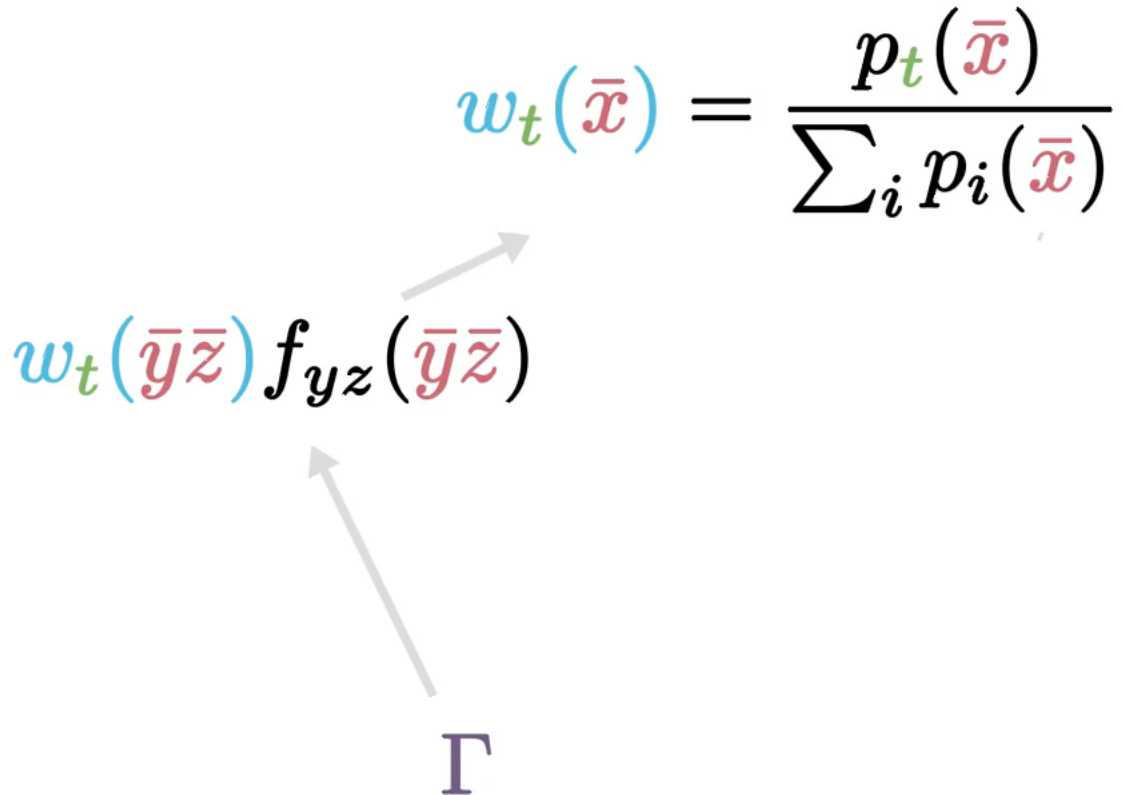
OPTIMAL Γ : CHALLENGE

- Based on $w_t(\overline{yz}) f_{yz}(\overline{yz})$?
- Circular dependency

$$w_t(\overline{x}) = \frac{p_t(\overline{x})}{\sum_i p_i(\overline{x})}$$

$w_t(\overline{yz}) f_{yz}(\overline{yz})$

Γ



SOLUTION

- Find the Γ minimizing the upper bound of rendering variance

$$\sum_{\underline{t}} \int_{\Omega} \frac{w_{\underline{t}}^2(\bar{x}) f^2(\bar{x})}{p_{\underline{t}}(\bar{x})} d\mu(\bar{x})$$

SOLUTION

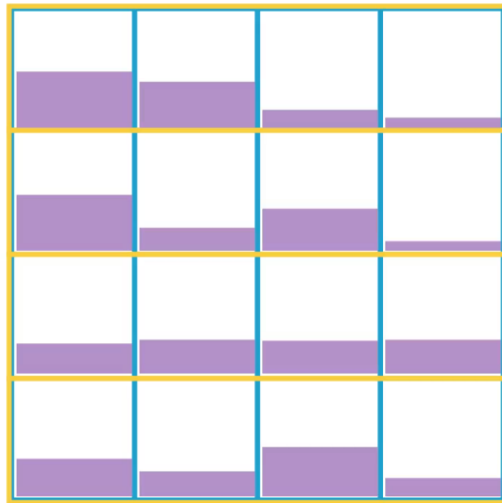
- Find the Γ minimizing the upper bound of rendering variance

$$\int_{\Omega} \frac{f^2(\bar{x})}{\mathcal{F}(\bar{x})} d\mu(\bar{x})$$
$$\mathcal{F}(\bar{x}) = \sum_i p_i(\bar{x})$$

OPTIMAL Γ

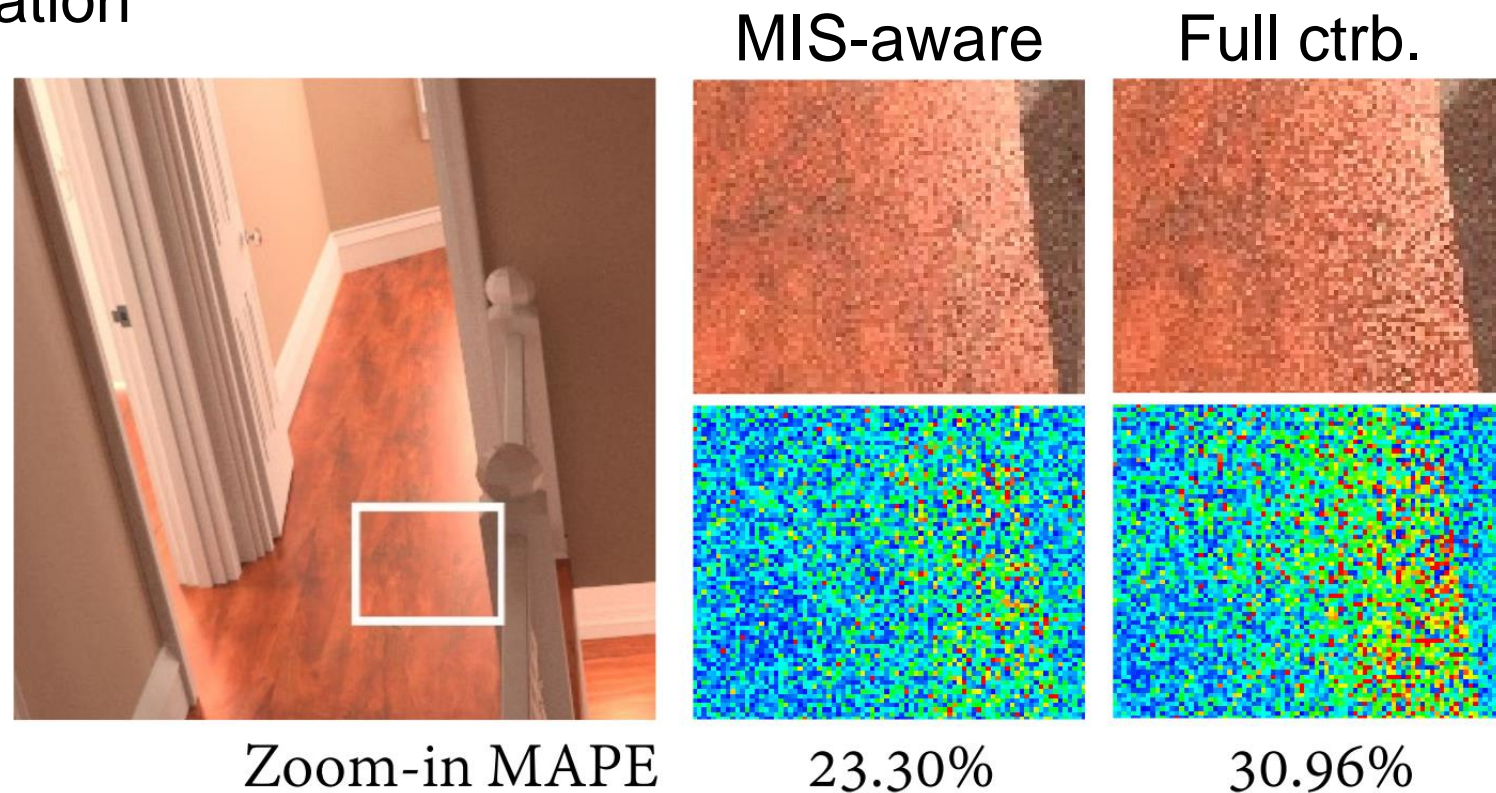
- Find the best mixture to approximate the path contribution
- Convex optimization

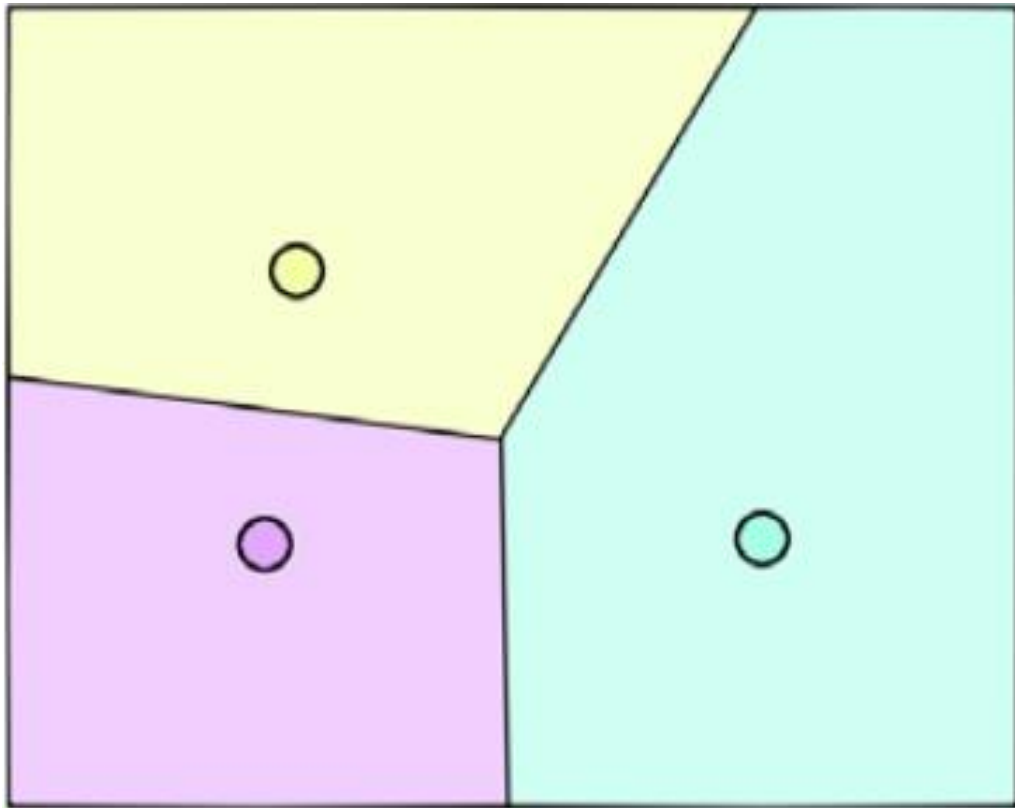
Subspace Sampling Matrix



MIS-AWARE LIGHT SELECTION

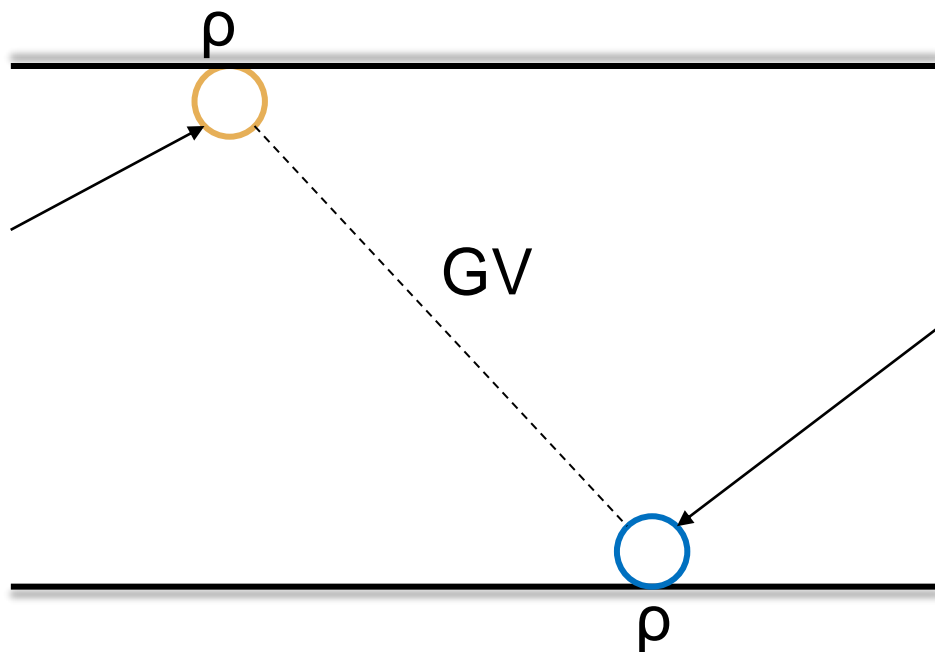
- Trace a set of full paths and minimize their variance upper bound
- Take MIS into consideration





CENTROID-DISTANCE STYLE

- Centroid sub-path set $\{C\}$
 - Sampled from the prefix/suffix of the pre-traced full paths
- Distance function d
- $\kappa(\bar{y}) = \arg \min_i d(\bar{y}, C_i)$



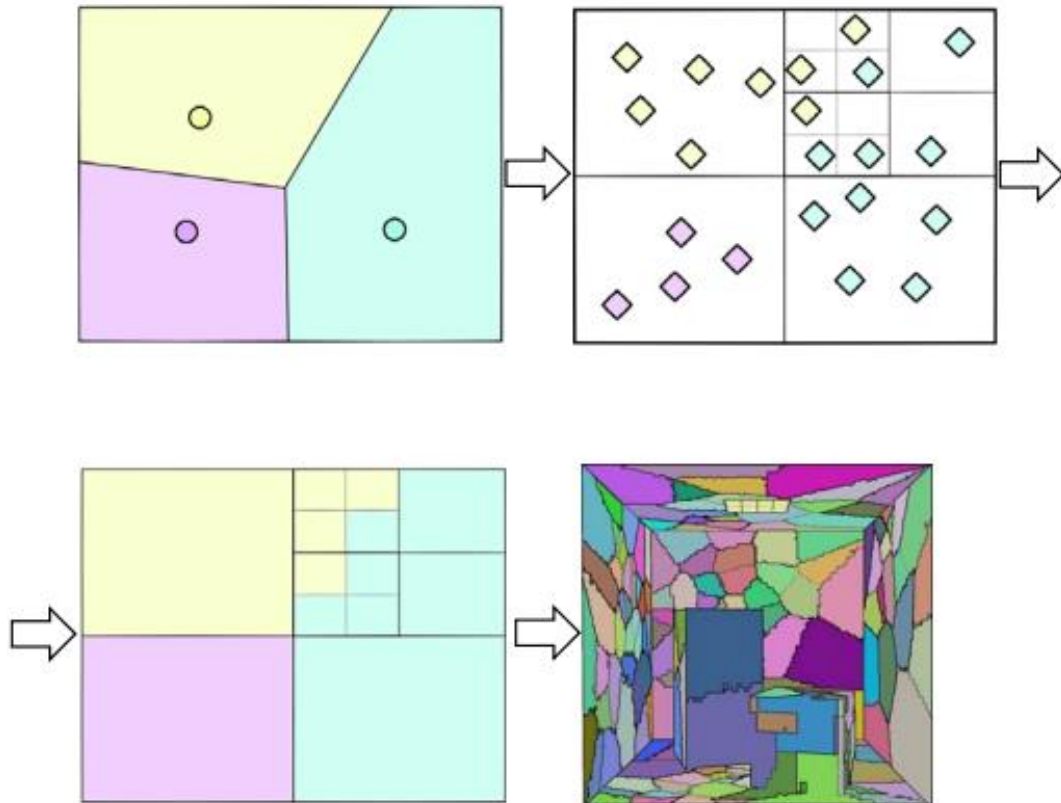
f_{YZ} DEPENDS ON

- Visibility V
 - position
- BSDF term ρ
 - position, normal, incident
- Geometry term G
 - position, normal

DISTANCE FUNCTION

$$d = d_s^2 + \sigma_s^2(1 - \cos \theta_n) + k_d \sigma_s^2(1 - \cos \theta_d)$$

- d_s : spatial distance
- θ_n : angle between normal
- θ_d : angle between incidents
- σ_s : spatial scale of the scene
- k_d : incident weight



SPEED UP BY DECISION TREE

- Centroid-Distance style κ
 - $O(N)$ indexing overhead
- Approximate by decision tree



RESULT

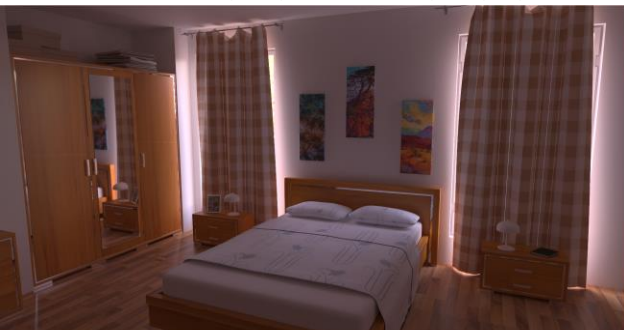
PRE-PROCESSING OVERHEAD

- Pre-traced path dataset
 - 1,000,000 full paths
 - Up to 20 seconds
- Determine Γ, κ, Q
 - Less than 2 seconds
- Pre-processing time has been included in the time-cost

RESULT

TEST SCENES AND PRE-PROCESSING TIME

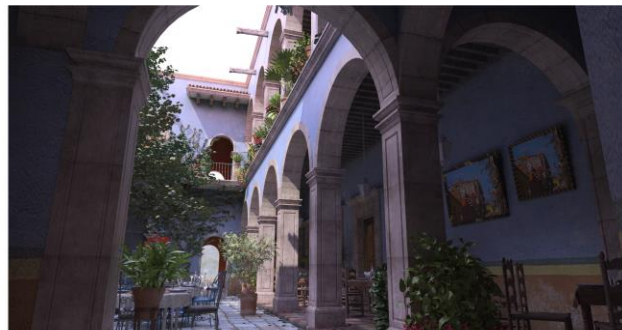
BEDROOM 16.58s



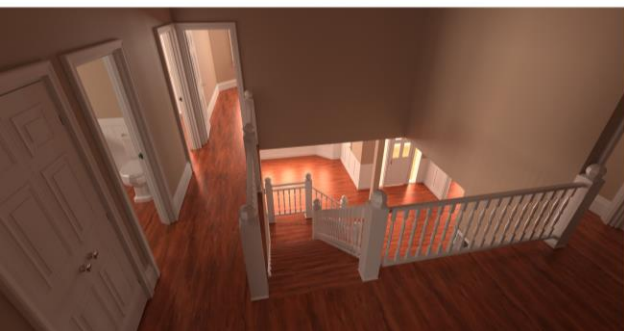
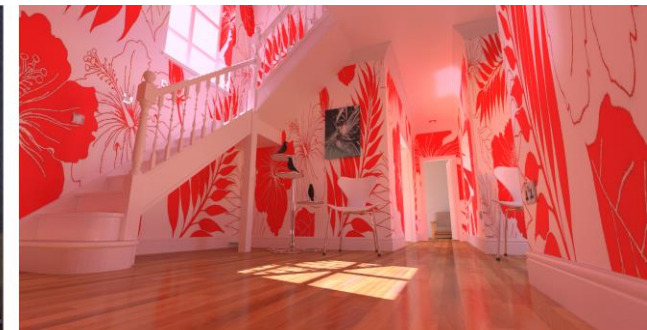
DOOR 3.41s



GARDEN 6.90s



HALLWAY 6.49s



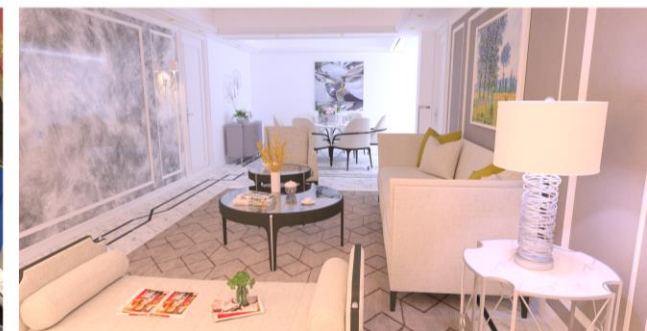
HOUSE 15.50s



KITCHEN 3.03s



SPONZA 3.21s



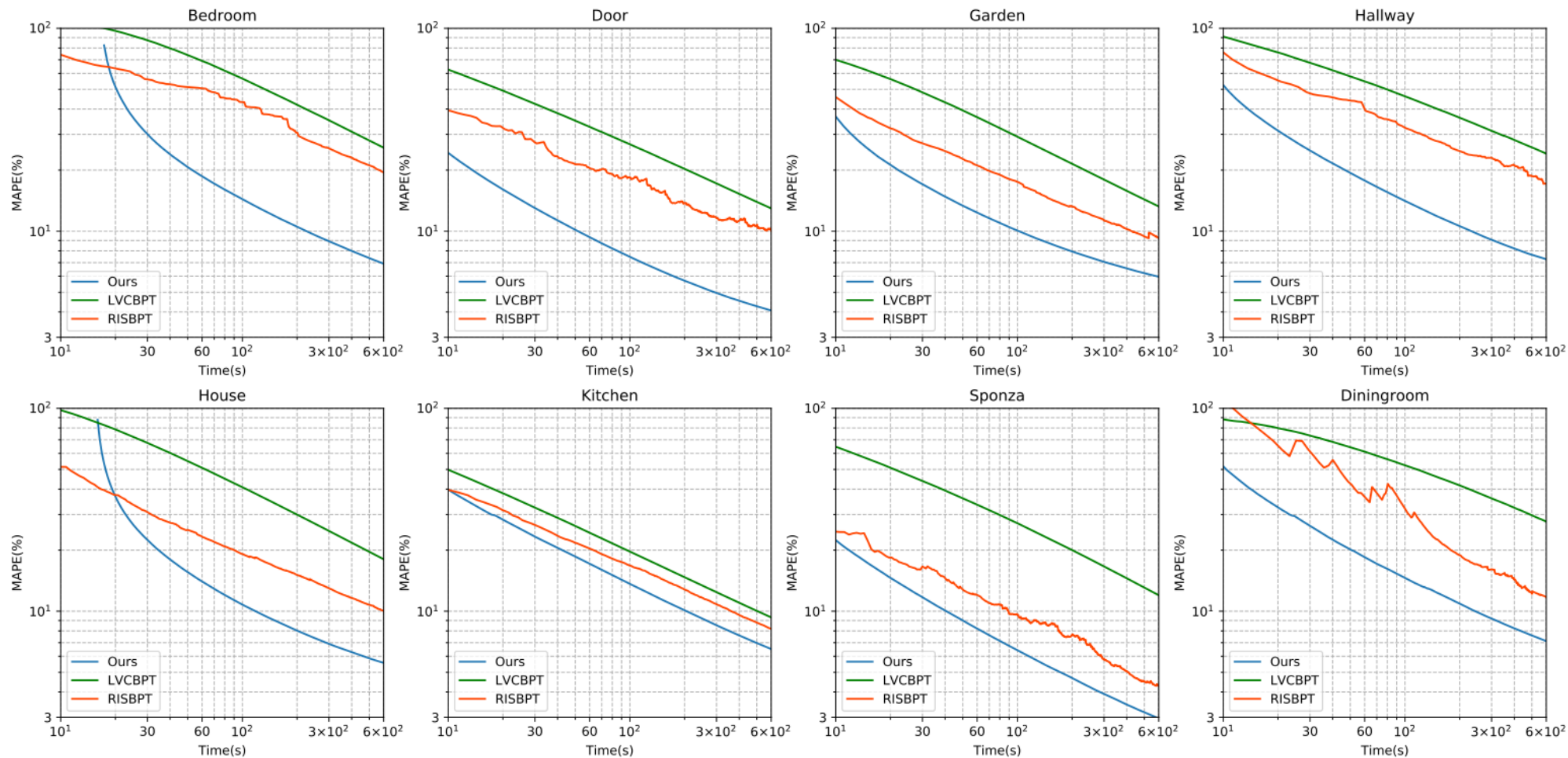
DININGROOM 5.43s

TO ACHIEVE MAPE = 12%

Scene	Bedroom		Door		Garden		Hallway		House		Kitchen		Sponza		Diningroom	
	Time	Itr.	Time	Itr.	Time	Itr.	Time	Itr.	Time	Itr.	Time	Itr.	Time	Itr.	Time	Itr.
LVCBPT	3182.7	6075	724.2	2328	756.0	2023	3295.8	4498	1471.0	4741	325.6	562	599.6	1852	4207.5	7068
RISBPT	2136.8	1407	273.4	269	255.7	357	1983.8	766	364.2	550	232.9	165	60.7	94	569.8	268
Ours	147.48	171	35.6	59	64.5	124	146.6	132	80.6	150	134.5	150	28.8	68	160.9	187

- Our method
 - Less iterations and time
 - Strong in difficult visibility

MAPE-TIME CONVERGENCE

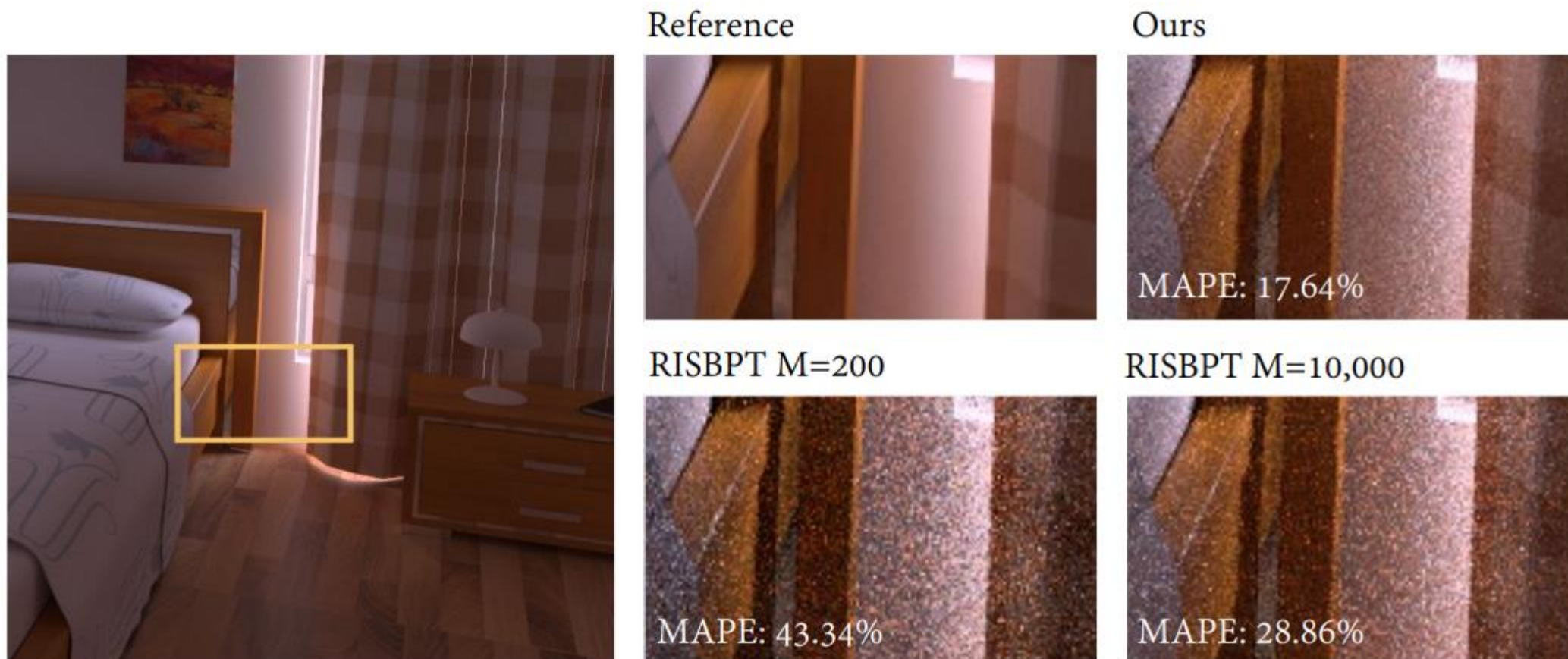


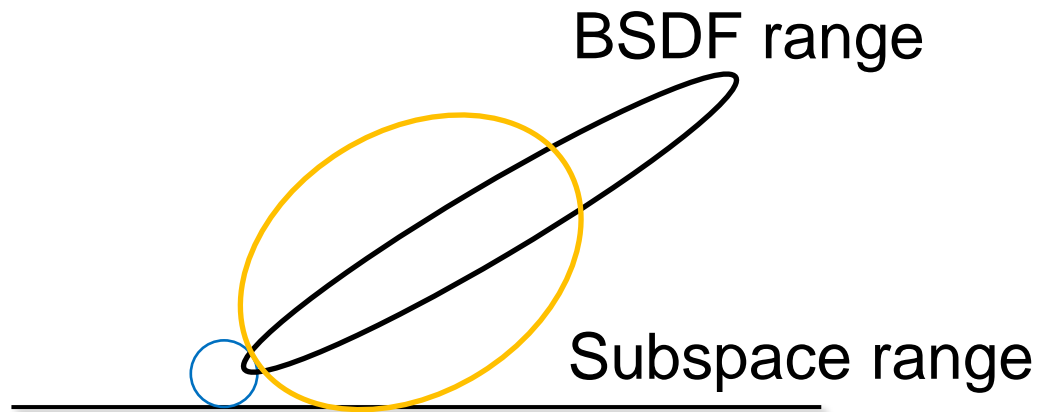
100 ITERATIONS COMPARISON WITH RECORD-BASED METHOD

Scene \ Method	RISBPT		Ours	
	Time (s)	MAPE	Time (s)	MAPE
Bedroom	149.43	36.95%	95.12	14.92%
Door	100.89	18.34%	58.34	9.47%
Garden	70.99	19.72%	53.18	13.01%
Hallway	252.94	23.45%	112.27	13.42%
House	65.83	22.47%	58.70	14.19%
Kitchen	140.63	14.87%	90.98	14.23%
Sponza	64.54	11.48%	41.05	9.94%
Diningroom	211.68	18.18%	88.44	15.48%

- Our method
 - Less time-cost
 - MIS-aware light selection
 - More candidates for resampling

64 ITERATIONS COMPARISON WITH RECORD-BASED METHOD OF MORE CANDIDATES





HIGHLY-GLOSSY SCENE

- For highly glossy material
 - The connection range of subspace can't capture the shape of BSDF

RESULT

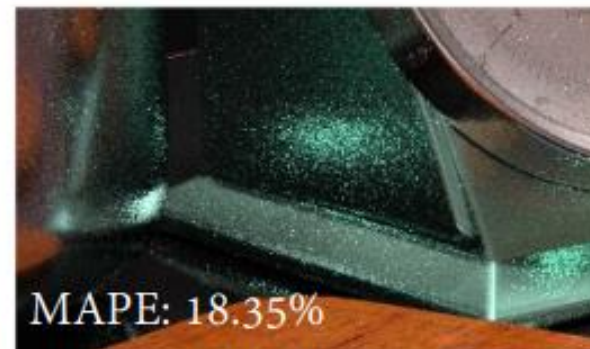
HIGHLY-GLOSSY SCENE



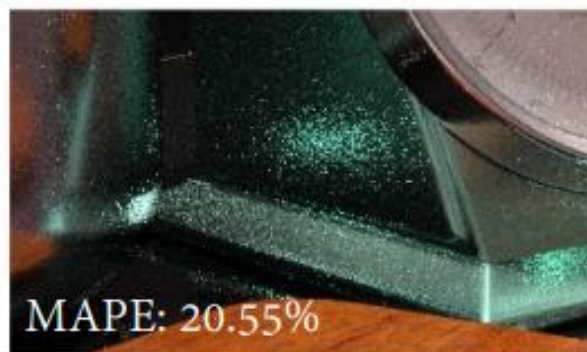
Reference



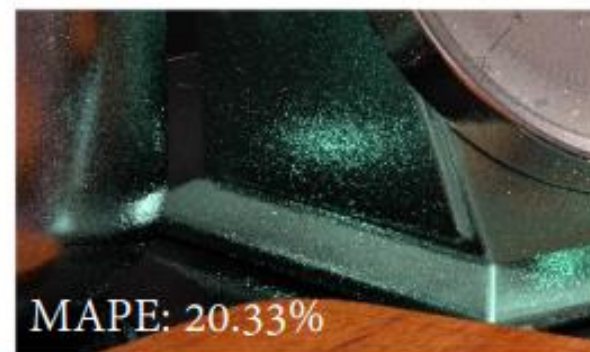
Ours



LVCBPT



RISBPT



DIVIDE THE PATH SPACE

- By introducing subspace, we
 - Select the sub-path for connection based on subspaces
 - Approximate the path contribution function using millions of sampling distribution
 - Fast, robust and MIS-aware light selection

HOWEVER

- We need pre-tracing to get the knowledge of path space
 - Expensive in scenes with difficult visibility
 - May be solved by online learning or combining with other adaptive sampling methods
- Subspace is too large to capture the shape of highly glossy BSDF
 - May be solved by adaptive coarse-to-fine sub-path space division



THANKS!