计算机辅助拓扑设计简介

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2022.11

目录

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1. Introduction

Brief History

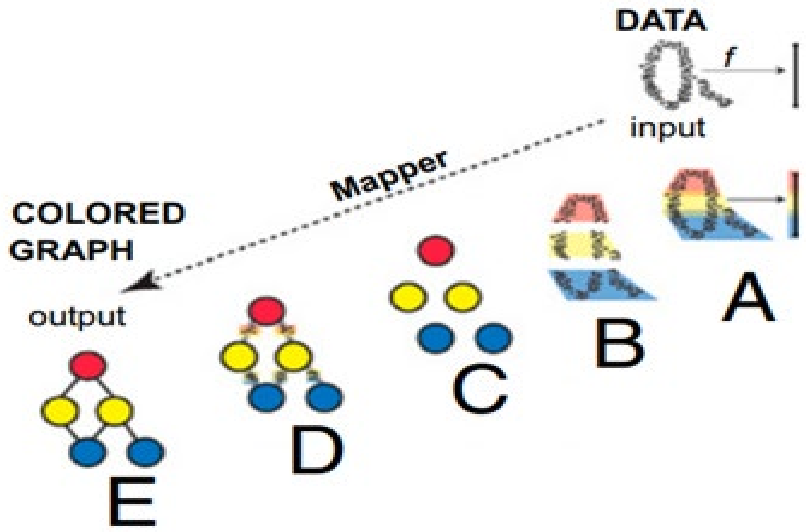
- **Computational Topology** is first presented in:

  Dey T K, Edelsbrunner H, Guha S. Computational topology. Contemporary mathematics, 1999, 223: 109-144.

- **Computational Geometry → Computational Topology**
- **Topological Data Analysis:** Proposed in 2009, a classical application of TDA is published in 2011

Carlsson G. Topology and Data. Bulletin of the American Mathematical Society, 2009, 46(2): 255-308.

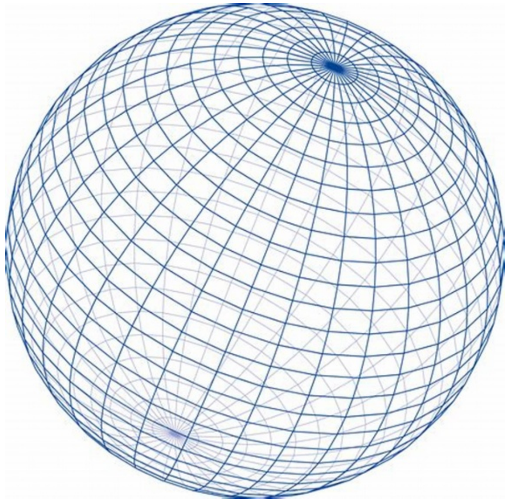
Monica Nicolau, Arnold J. Levine, and Gunnar Carlsson, “Topology-Based Data Analysis Identifies a subgroup of Breast Cancers with a Unique Mutational Profile and Excellent Survival,” Proceedings of the National Academy of Sciences 108, no. 17 (2011): 7265–70.



Topological Structure of a Manifold

Connected components

Loops



A Void (Empty inside a solid)

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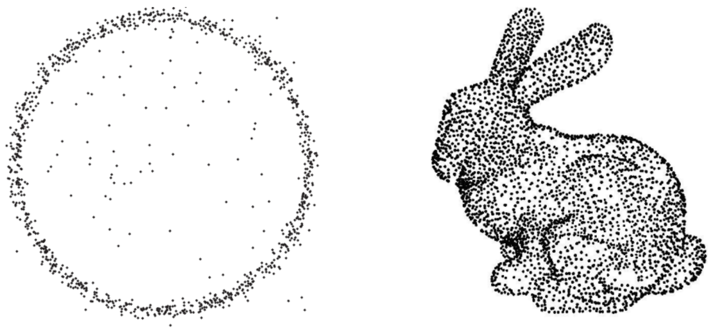
Higher dimensional topological structures

Figure: A continuous deformation doesn’t change the loop in the shape. (Wikipedia: Topology)

Motivation of Topological Data Analysis

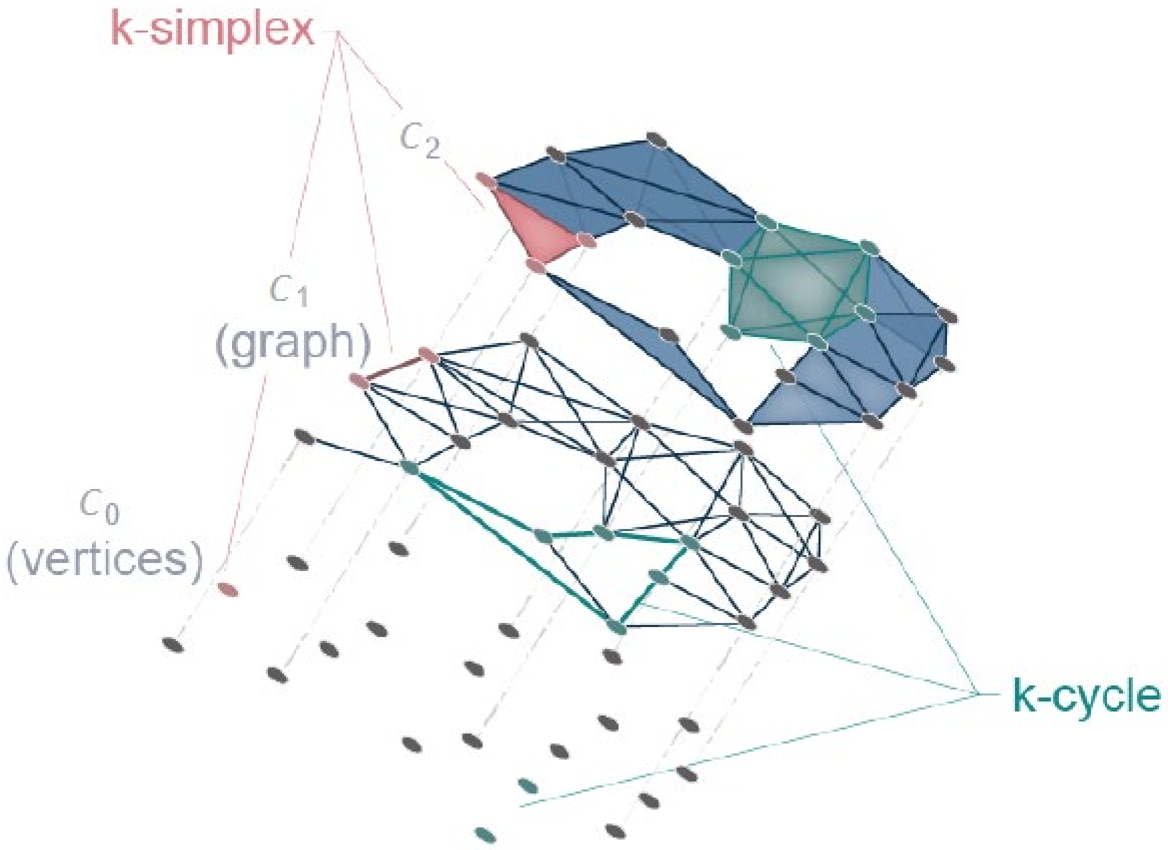


Topological Inference for point cloud data

- How to **infer** the **essential topological features** of a hidden shape?
  - To extract components, loops, voids and higher dimensional features
  - To measure their importance
  - Stable with respect of small perturbations
- Mathematical Background
  - Algebraic Topology (Homology Theory)
  - Statistics
  - Geometry
- A promising bridge between topology and geometry in the view of computation

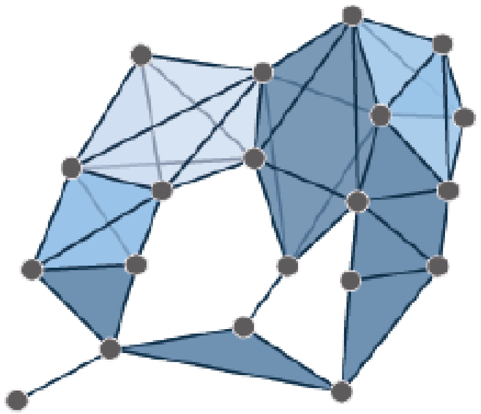
2. Persistent Homology

2.1 Homology



$$\cdots \xrightarrow{\partial_{n+2}} C_{n+1} \xrightarrow{\partial_{n+1}} C_n \xrightarrow{\partial_n} C_{n-1} \xrightarrow{\partial_{n-1}} \cdots$$

$$\partial_n \circ \partial_{n+1} = 0 \qquad H_n = Z_n / B_n$$



Simplicial Complex

k-homology group contains different types of ***k*-holes**.

The rank of homology group is the **Betti numbers.**

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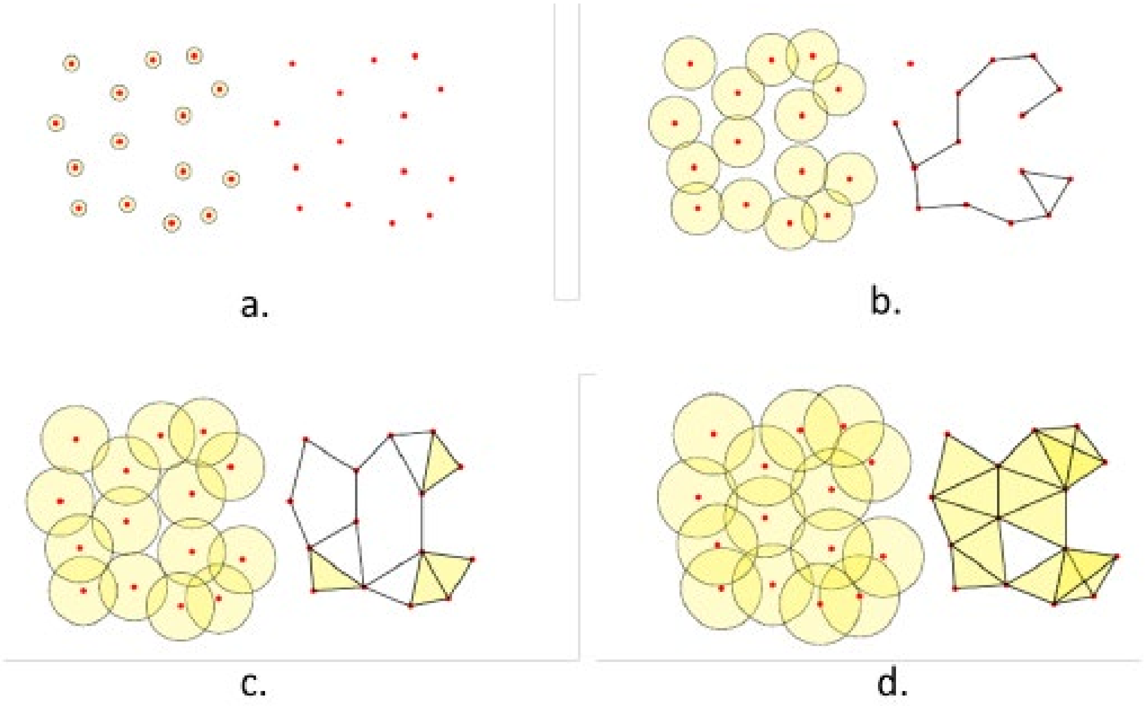
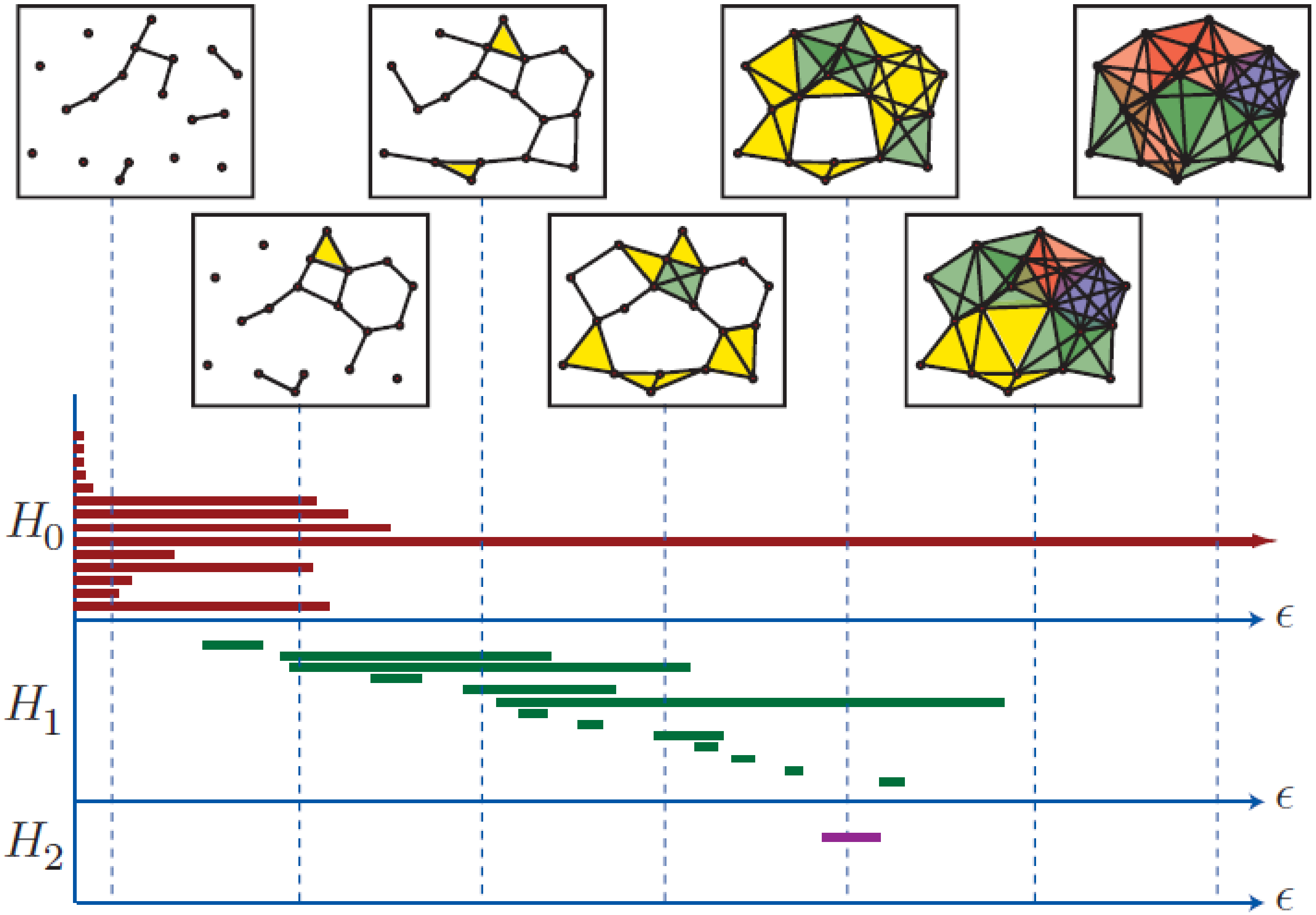


Filtration (Čech complex)

As the radii of open balls grow, the **simplicial complex** can be constructed on point cloud data, which gives a nested complex sequence, a **filtration**.

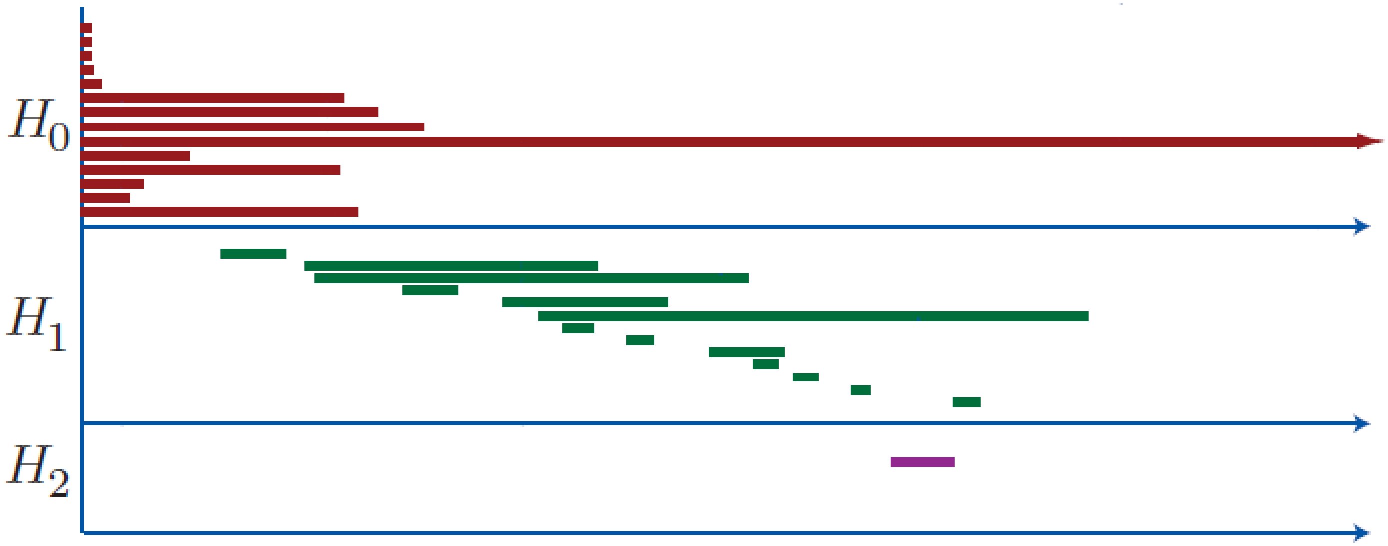
2.3 Persistent Homology



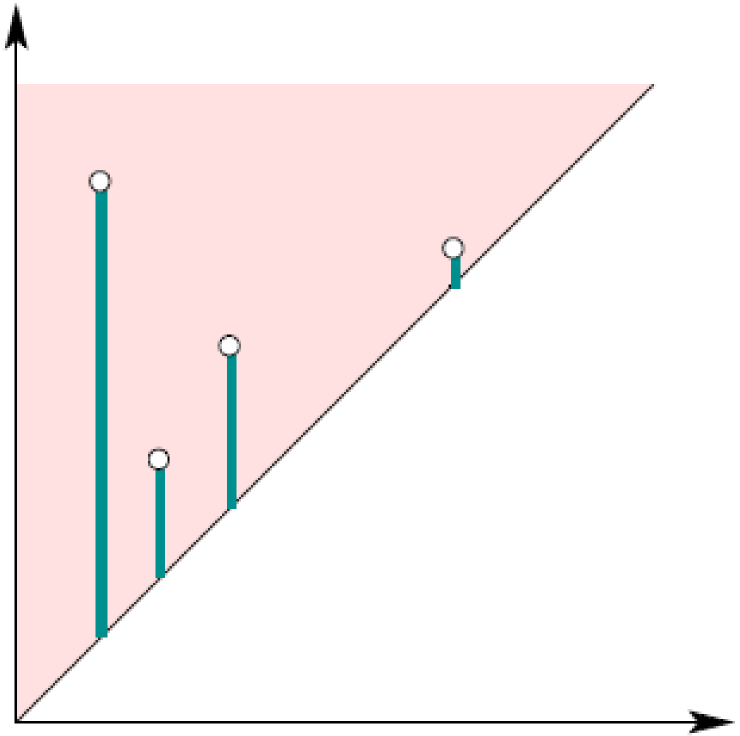
Barcode

- As time goes by, the radii of open balls grow
- New *k*-holes are **born**
- Some *k*-holes are destroyed by higher-dimensional simplex and **die**

2.4 Topological Summaries

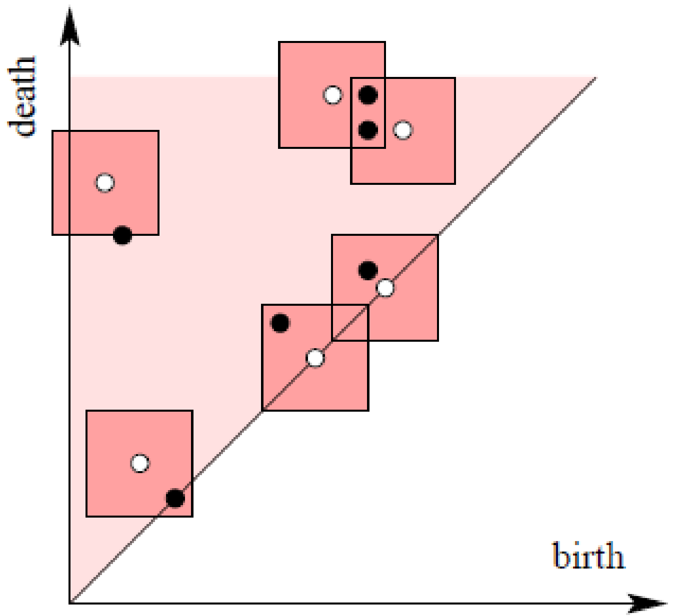


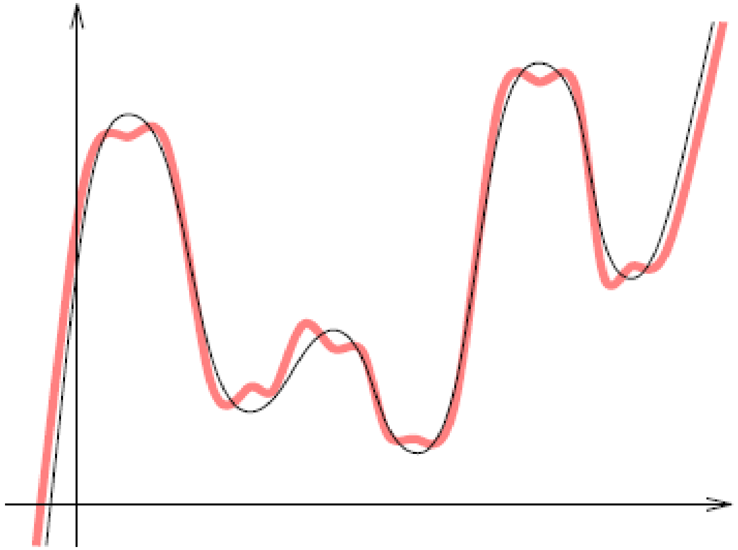
Barcode

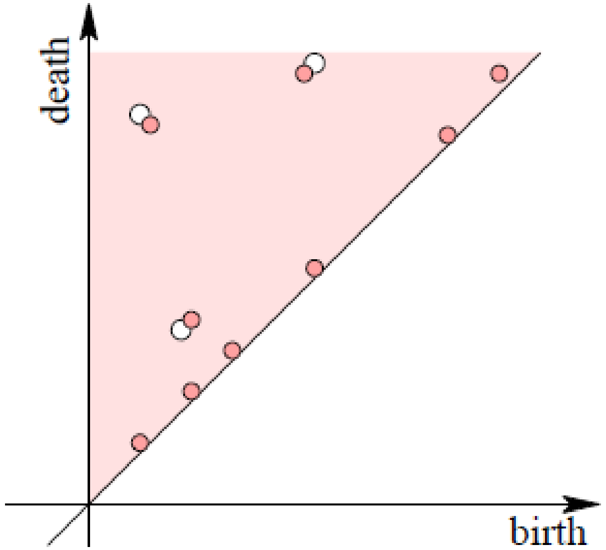


Persistence Diagram

2.5 Distance and Stability







$$W_\infty(X,Y) = \inf_{\eta: X \to Y} \sup_{x \in X} \|x - \eta(x)\|_\infty$$

$$W_\infty(X,Y) \leq \|f - g\|_\infty$$

The ***Wasserstein distance*** of two persistence diagrams is controlled by the small perturbations on the tame function *f*

3. Representation of Persistence Diagrams

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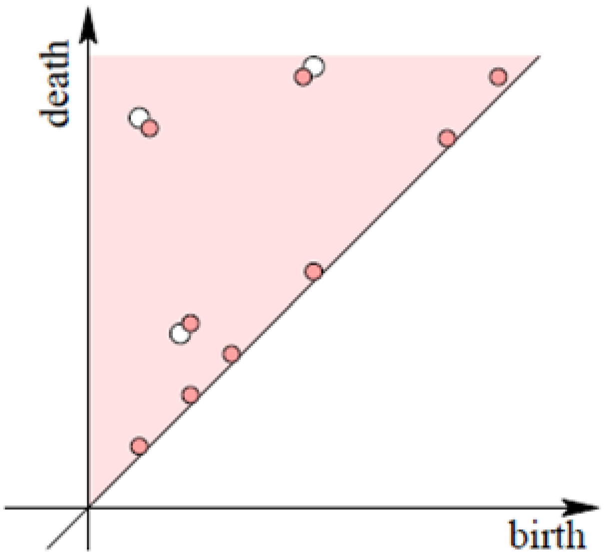
- Analysis and applications of PDs
  - To extract more statistical information from PDs
  - To learn from PDs via machine learning approaches
  - To design feature descriptors using PDs
- Space of PD with metric W_p
  - Irregular points on a PD
  - Inefficient to compute the W_p distance

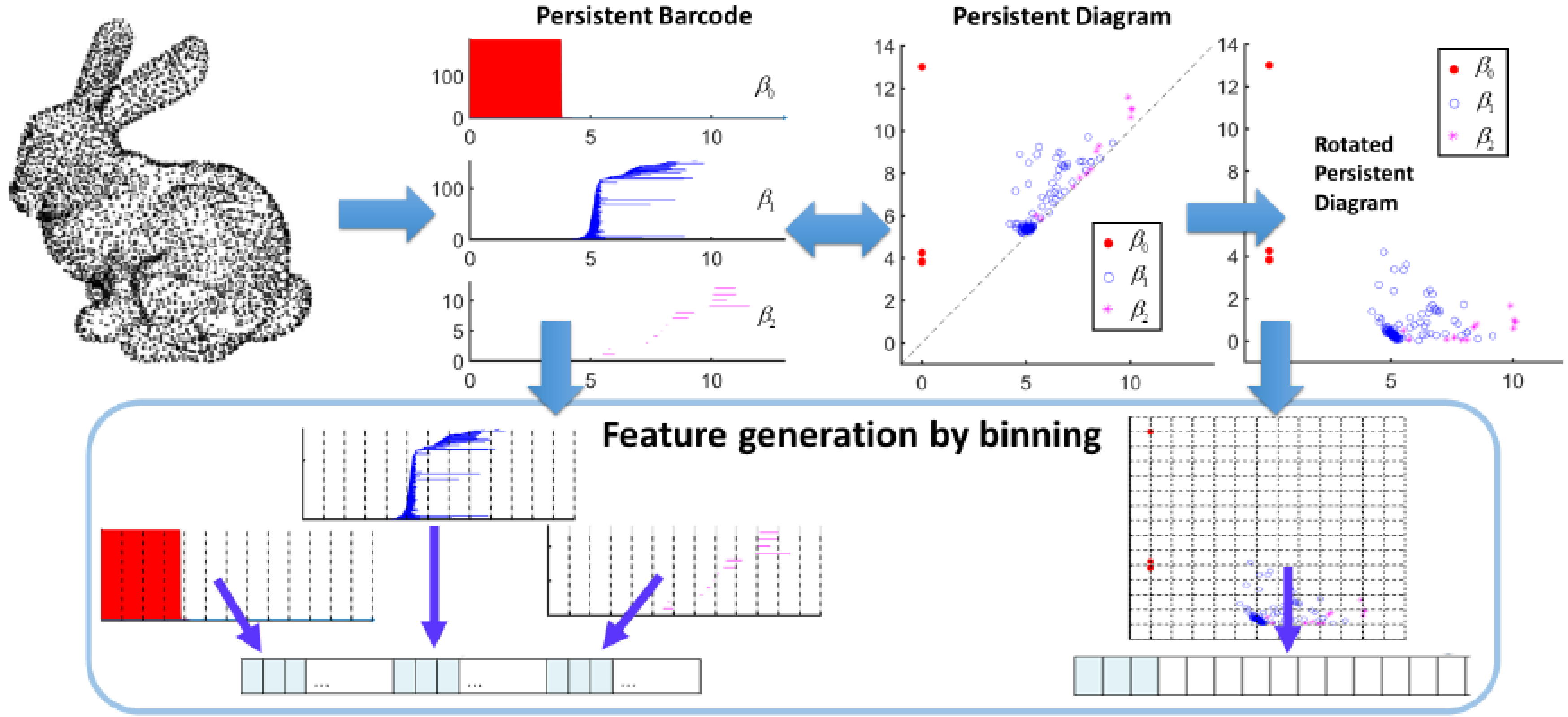


PD should be transformed into a proper representation.

$$W_p(PD^{(1)}, PD^{(2)}) = \inf_b \Big(\sum_{u \in PD^{(1)}} \|u - b(u)\|_\infty^p \Big)^{1/p}$$

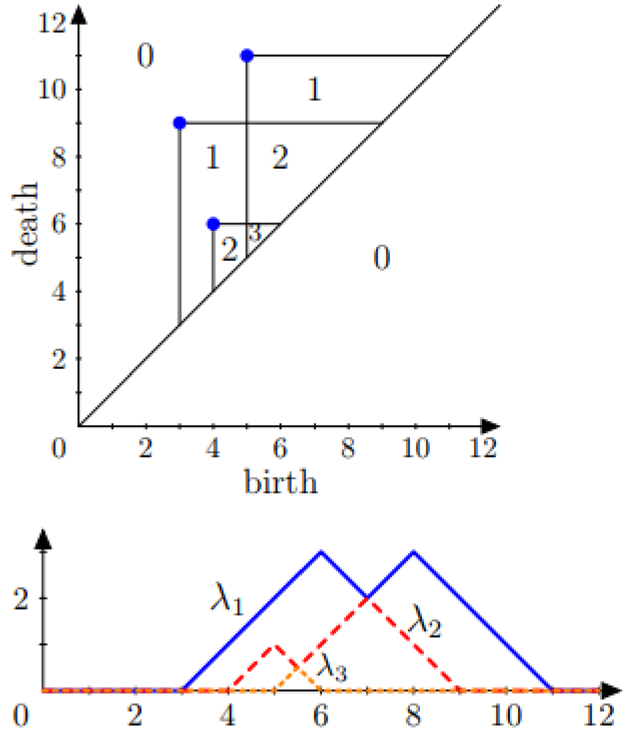
Vectorizing Representation of a PD: to map a PD into a Hilbert space.

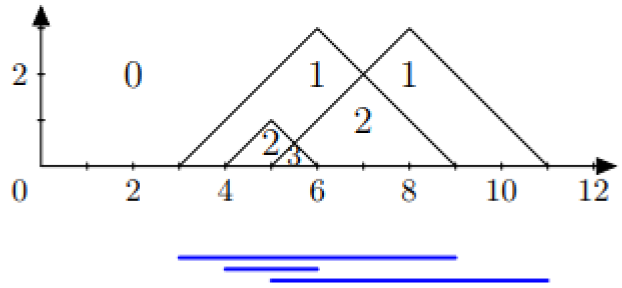
3.2 Vectorizing a PD or a barcode by binning

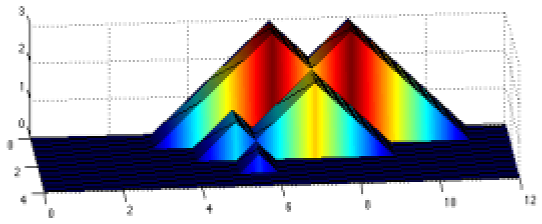


- To construct the filtration (a nested sequence of simplicial complexes) and to compute PD or barcode
- To convert a PD or a barcode into a function or a series of functions (generally, an element of a Hilbert space)
- To generate finite vector representations usually by binning, sampling, etc.

3.3 Persistence landscapes



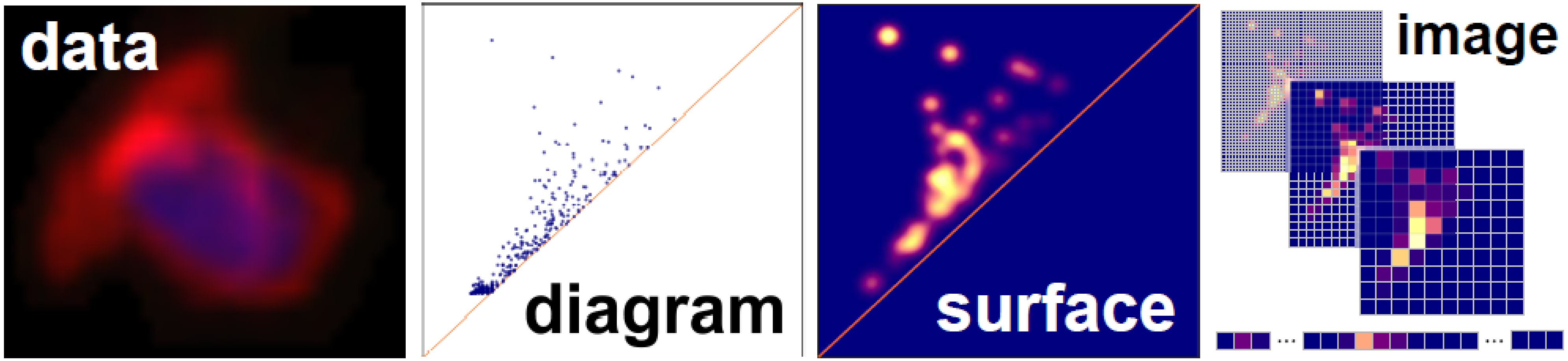




- Persistence landscapes (PL, Bubenik; 2015) provide a topological summary to analyze PDs with statistical approaches (means, confidential interval, distributions)

- PDs or barcodes are transformed into a series of 1-Lipschitz functions (shown in the left-bottom figure). And the stability of PL is guaranteed

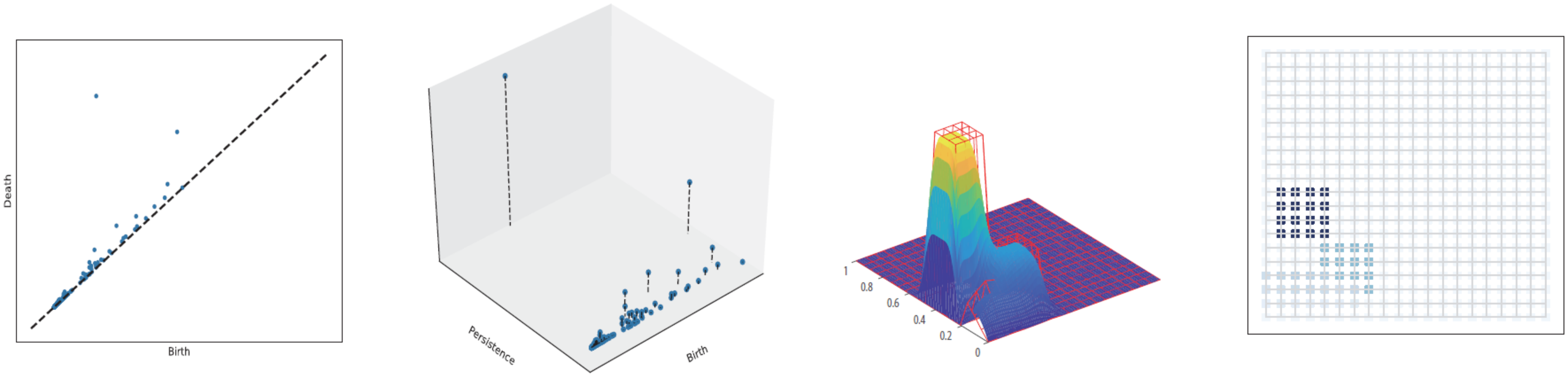
3.4 Persistence Images



- The persistence image (PI) provides a representation of the information in a persistence diagram that allows to transform it into a vector
- The Gaussian function is chosen at each point in the persistence diagram
- It makes the results of persistent homology available in ML algorithms such as linear SVM

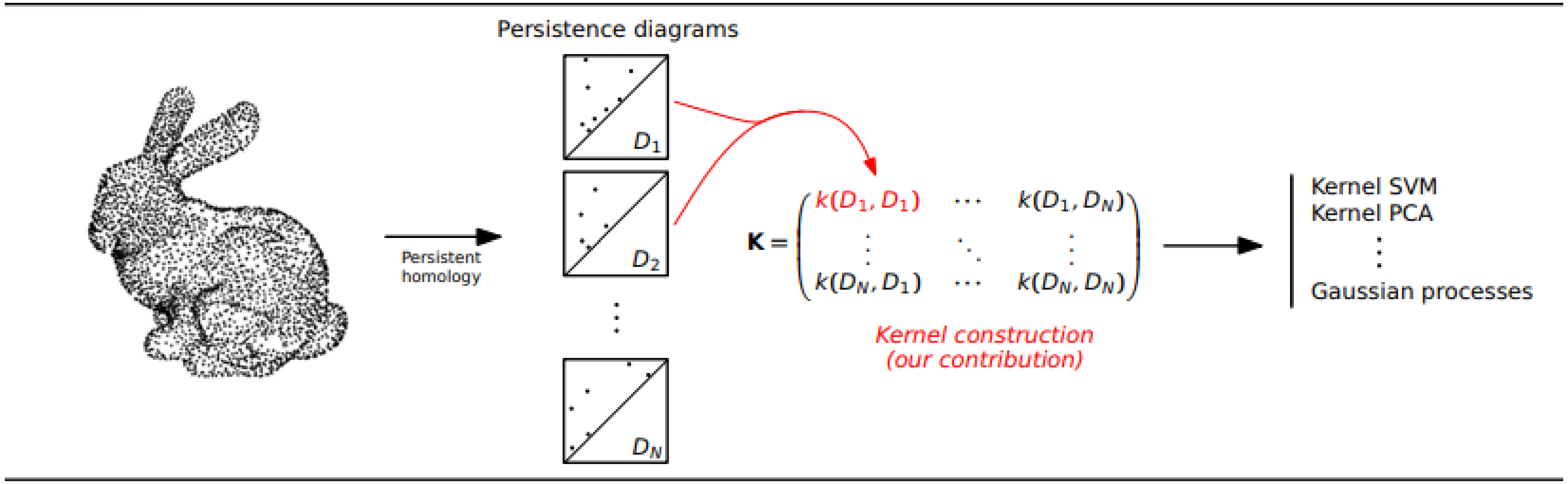
[Adams et al., Persistence Images: A Stable Vector Representation of Persistence Homology, 2017]

3.5 Persistence B-spline Grid



- Transform a PD into the birth-persistence coordinates and assign a value to each point in a PD to illustrate the importance of the topological features
- Fit the data points in 3D by a cubic uniform B-spline surface using the technique of *progressive and iterative approximation for least squares B-spline surface* (LSPIA)
- Finally, the control grid is obtained to generate a vector by concatenating rows of z-coordinates of the control grid
- **Persistence B-spline Grid** is defined by the matrix formed by z-coordinates of the control points

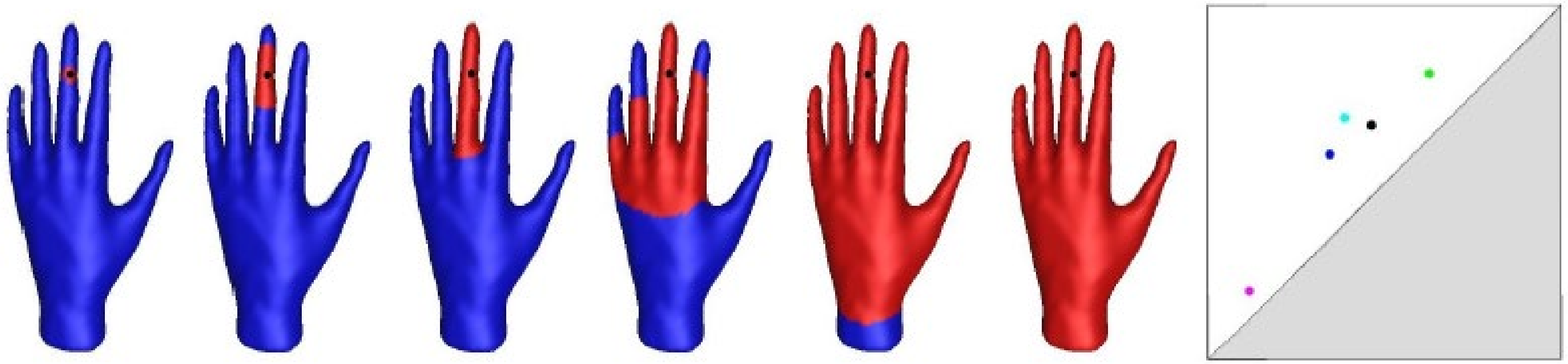
3.6 Kernel methods



- The idea is to define a kernel for persistence diagrams to enable a theoretically sound use of these summary representations in the framework of kernel-based learning techniques

- They include, persistence scale space kernel (PSSK, 2015; Reininghaus et al.), Persistent weighted Gaussian kernel (PWGK, 2016; Kusano et al.), Persistence Fisher Kernel (2018; Le and Yamada), etc.

4. Applications in Geometric Design and Related Areas



- The persistence diagram corresponding to this family is shown in the right
- The pink, blue, light blue, black and green points correspond to the middle, index, ring, pinky and thumb respectively

[Carrière et al, Stable Topological Signatures for Points on 3D Shapes, 2015]

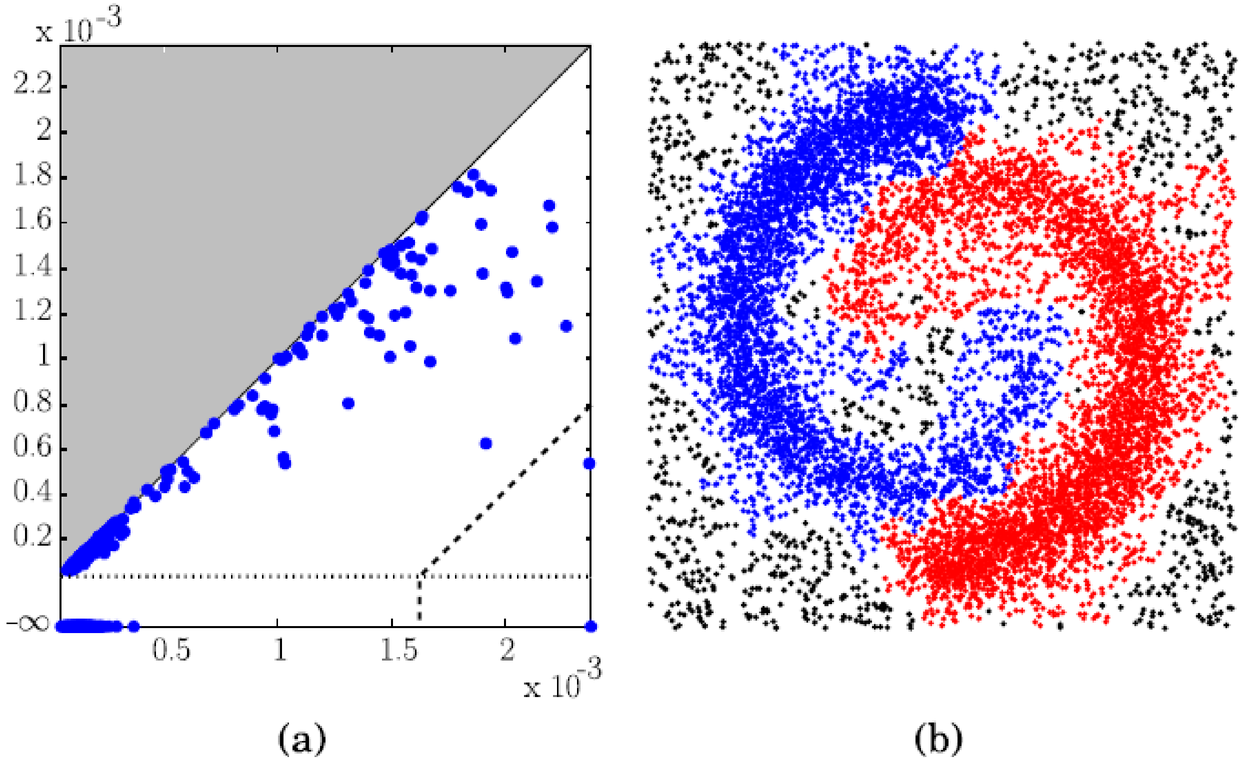
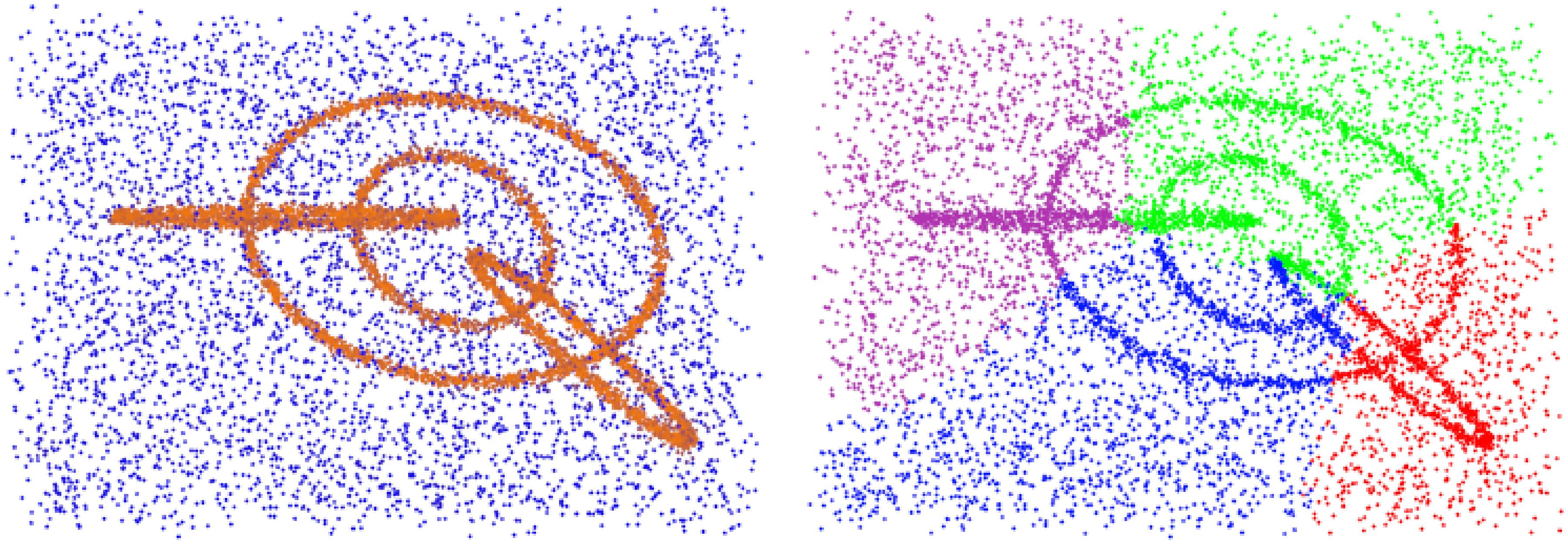


Fig. 6. The twin spirals data set from Figure 2, processed using a smaller Rips parameter: (a) the persistence diagram; (b) the final clustering with late appearing connected components filtered out (in black).

[Chazal et al. Persistence-Based Clustering in Riemannian Manifolds, 2013]

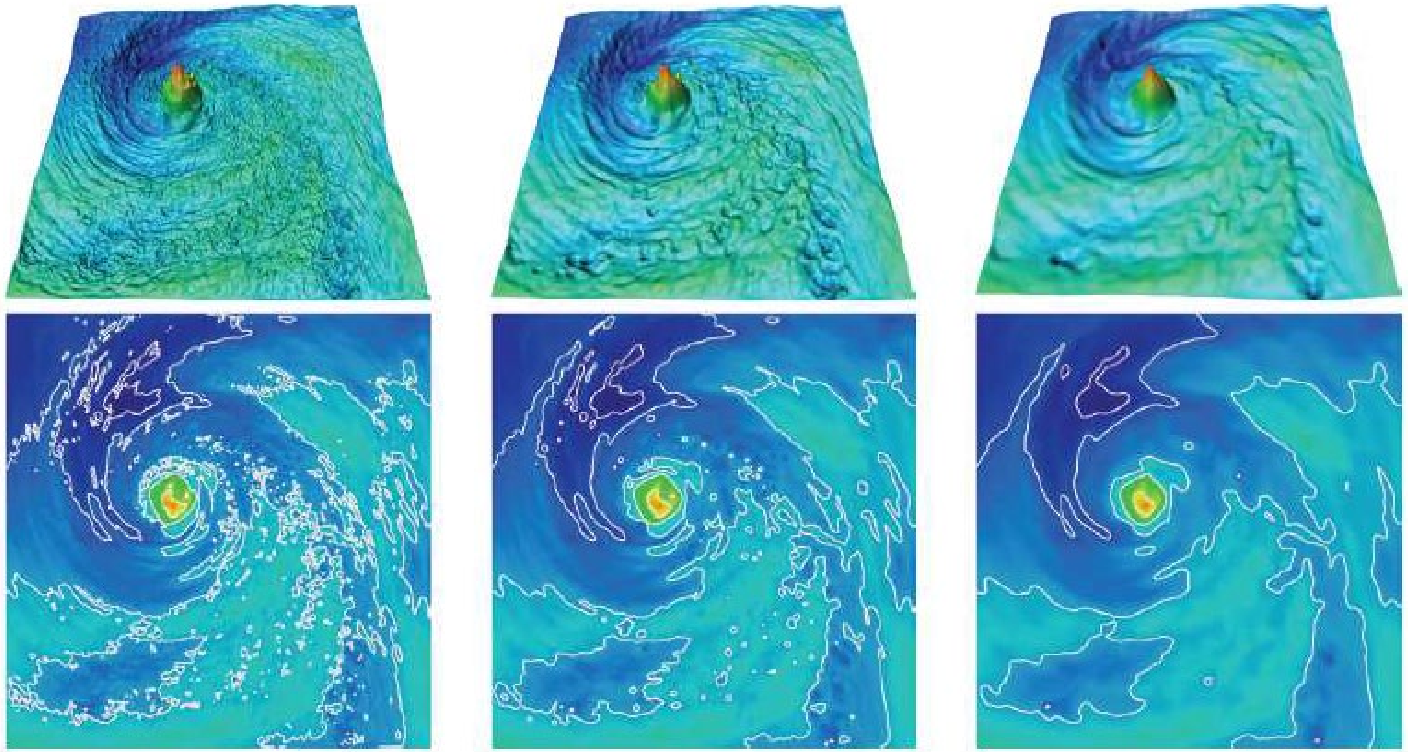
4.2 Clustering of Point Clouds



- The rings are detected by the clustering method with persistence (left)
- Compared with the result obtained by spectral clustering (right)

Chazal et al. Persistence-Based Clustering in Riemannian Manifolds, 2013

4.3 Topological Denoising



(a) Original data. 14492 extrema.

(b) Result for $P = 0.4$. 279 extrema.

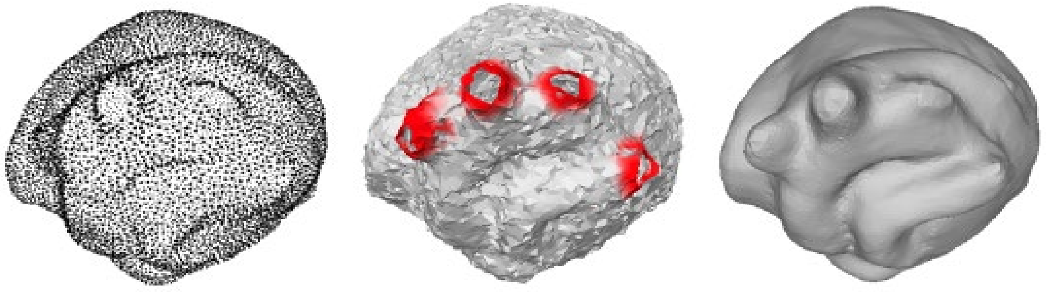
(c) Result for $P = 1.5$. 12 extrema.

Figure 10. Temperature in the Hurricane Isabel data set (slice $z = 20$). Using persistence-based filtering, we create a hierarchy of scalar fields: with increasing persistence P, our method creates increasingly smoother versions of the data.

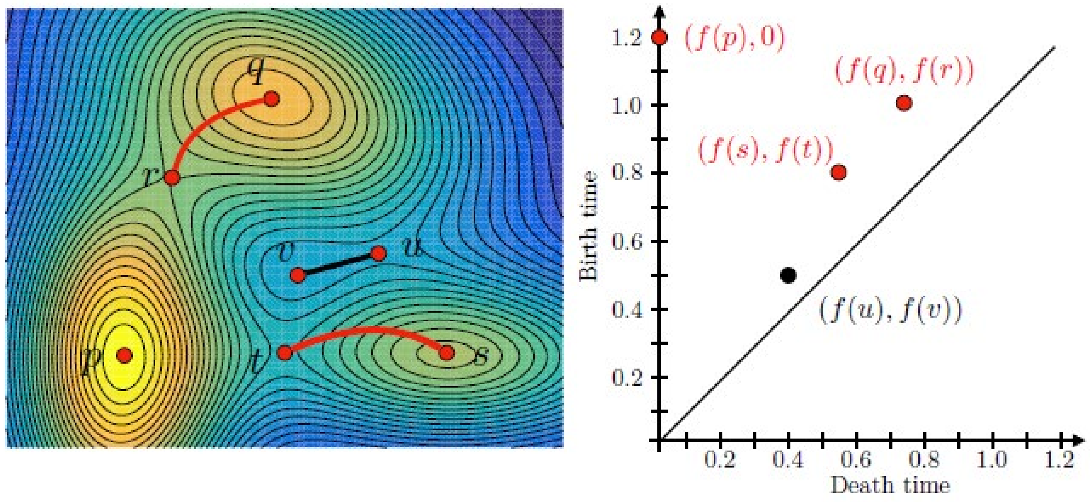
- Keep the salient features while using a denosing method

Günther et al., Fast and Memory-Efficient Topological Denoising of 2D and 3D Scalar Fields, 2014

4.4 Topology-aware Reconstruction



(a) (b) (c)



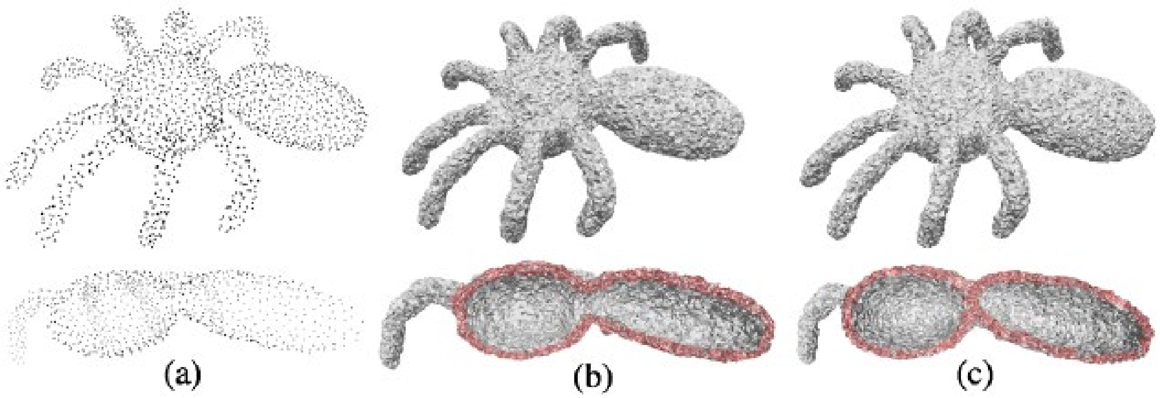
(a) (b)

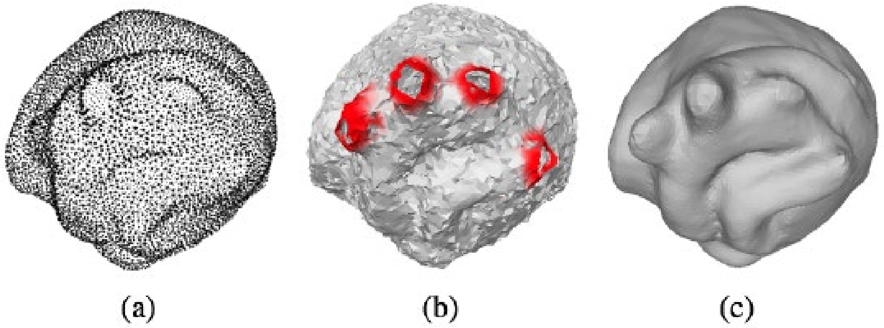
- Brüel-Gabrielsson, Rickard, et al. "Topology-Aware Surface Reconstruction for Point Clouds." *Computer Graphics Forum*. Vol. 39. No. 5. 2020.

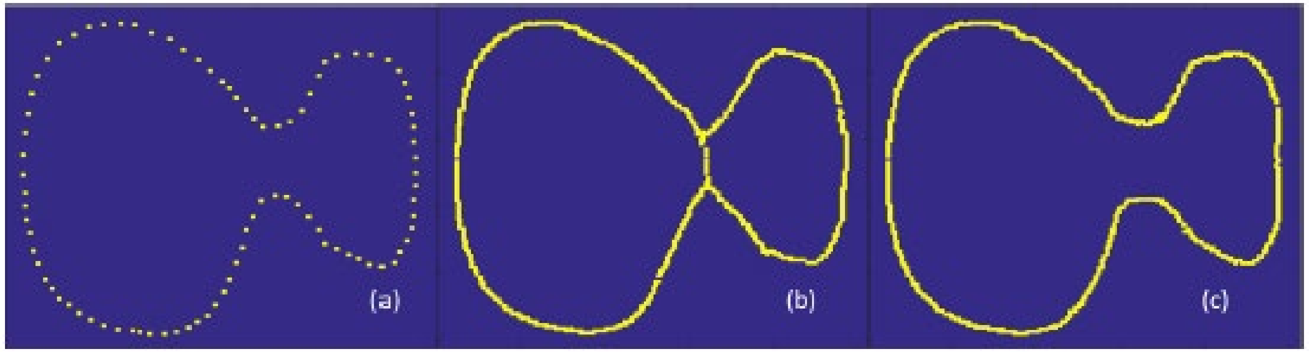
- **Objective Function:**

$$-((d_1-b_1)^2-(d_2-b_2)^2)$$

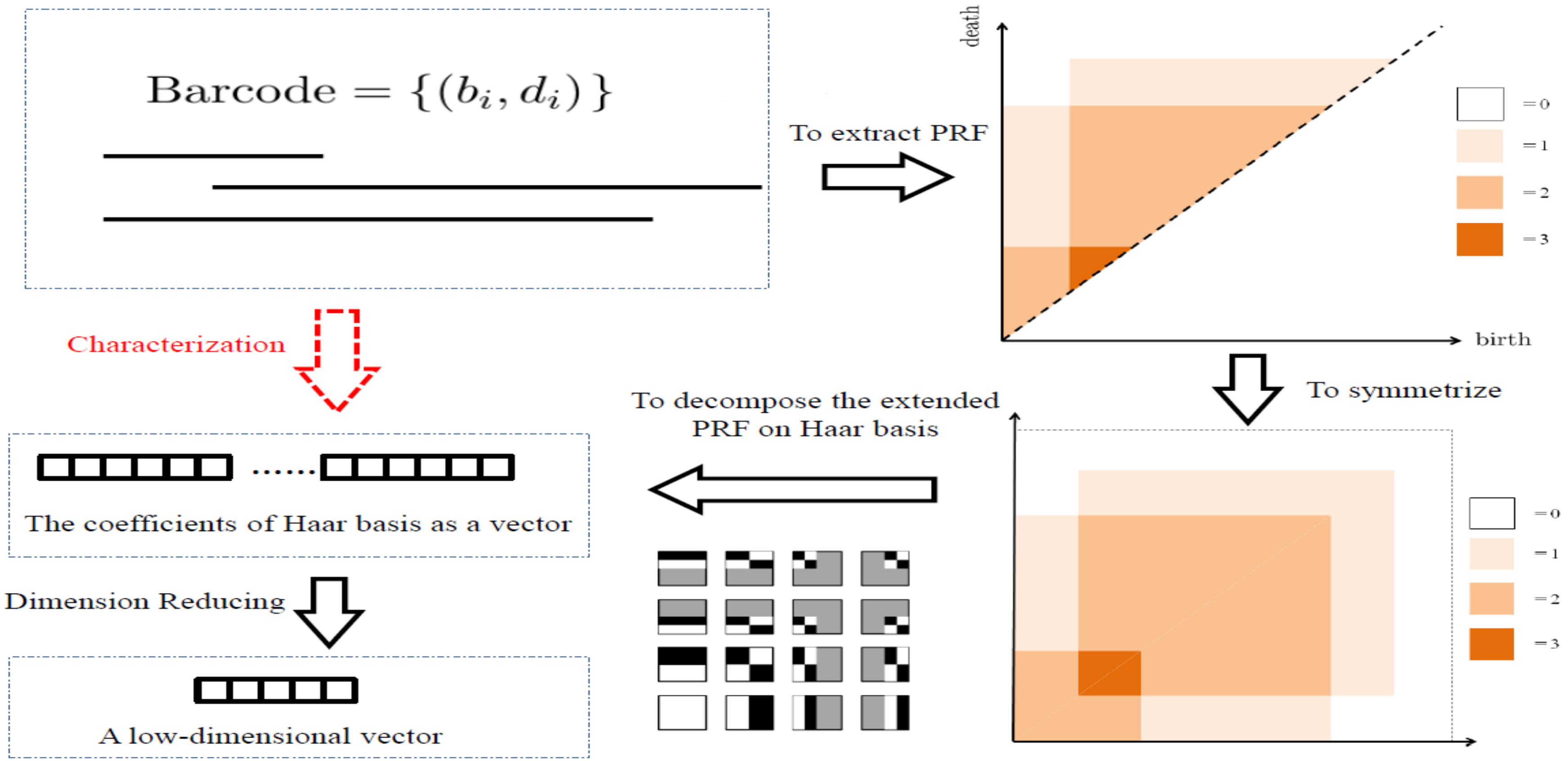
$$-((d_2-b_2)^2-(d_3-b_3)^2)$$





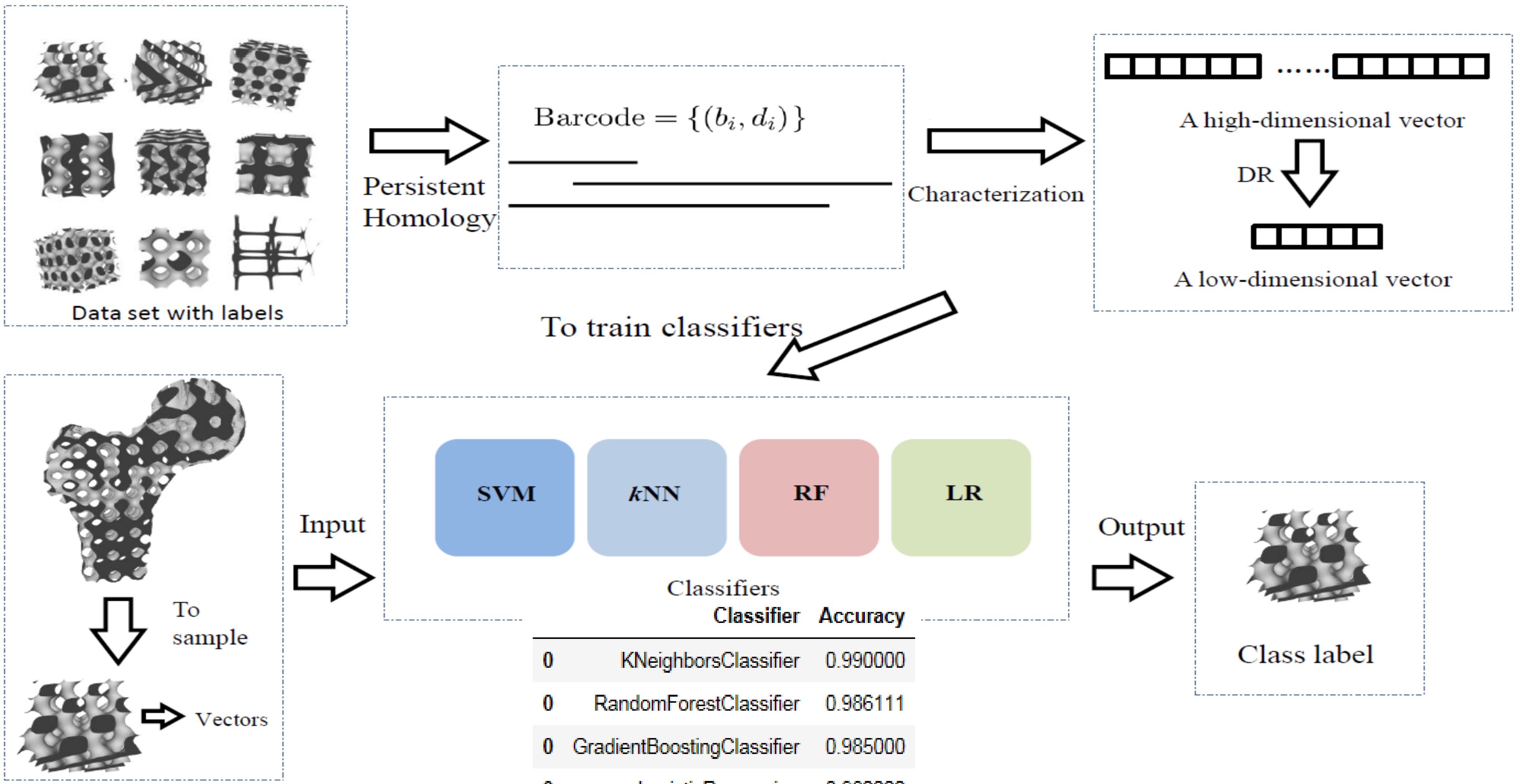


- Brüel-Gabrielsson, Rickard, et al. "Topology-Aware Surface Reconstruction for Point Clouds." *Computer Graphics Forum*. Vol. 39. No. 5. 2020.



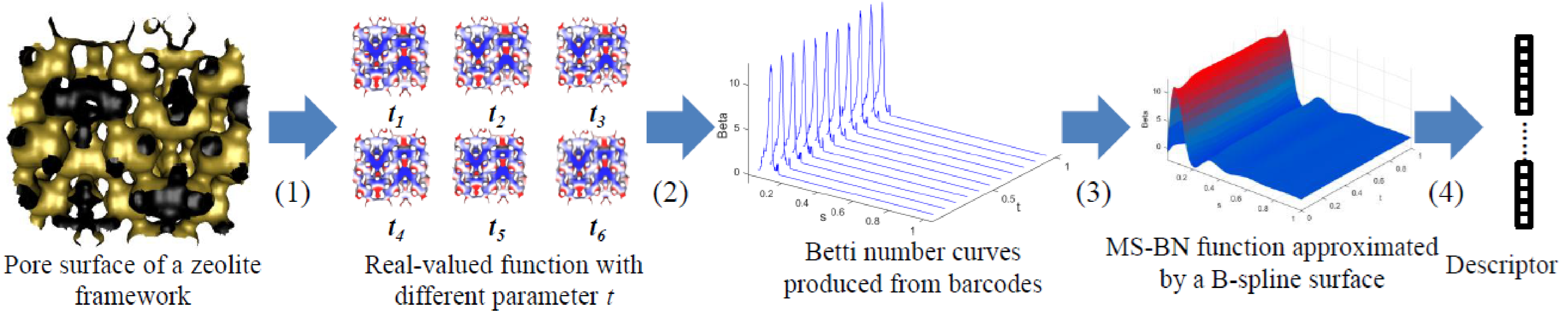
Zhetong Dong, Chuanfeng Hu, Chi Zhou, Hongwei Lin, Vectorization of persistence barcode with applications in pattern classification of porous structures , Computers & Graphics, Volume 90, 2020,Pages 182-192

4.5 Characterization of Persistence Barcode by Haar Basis



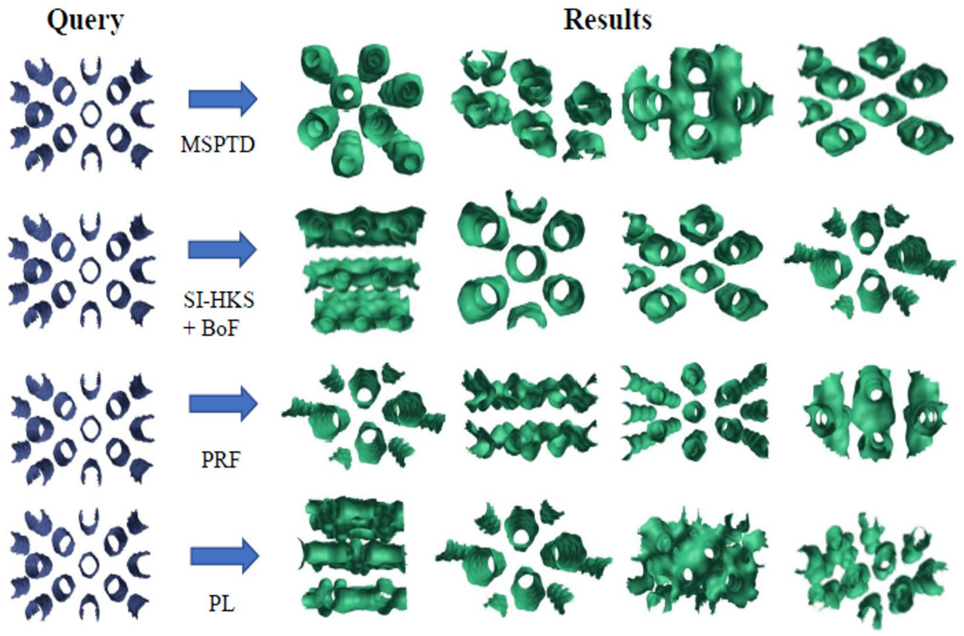
	Classifier	Accuracy
0	KNeighborsClassifier	0.990000
0	RandomForestClassifier	0.986111
0	GradientBoostingClassifier	0.985000
0	LogisticRegression	0.982222
0	LinearSVC	0.981667
0	SVC	0.990000

- Pipeline for generating multiscale persistent topological descriptor



Zhetong Dong, Junyu Pu, Hongwei Lin. Multiscale Persistent Topological Descriptor for Porous Structure Retrieval. SIAM-GD 2021, Computer Aided Geometric Design, 88: 102004 (2021)

- Retrieval results of the channel surfaces of DON



Retrieval performance on the synthetic porous data set

Methods/Accuracy (%)	NN	FT	ST	DCG
SI-HKS with histogram	90.1	73.3	91.6	92.7
SI-HKS with BoF	81.3	76.1	91.0	91.8
PL descriptor	68.1	59.3	75.0	83.4
PI descriptor	81.5	71.8	86.7	90.1
PRF descriptor	81.8	74.1	87.5	90.6
MSPTD (ours)	**94.9**	**78.4**	**94.5**	**95.5**

4.7 Topology Editing for Point Cloud

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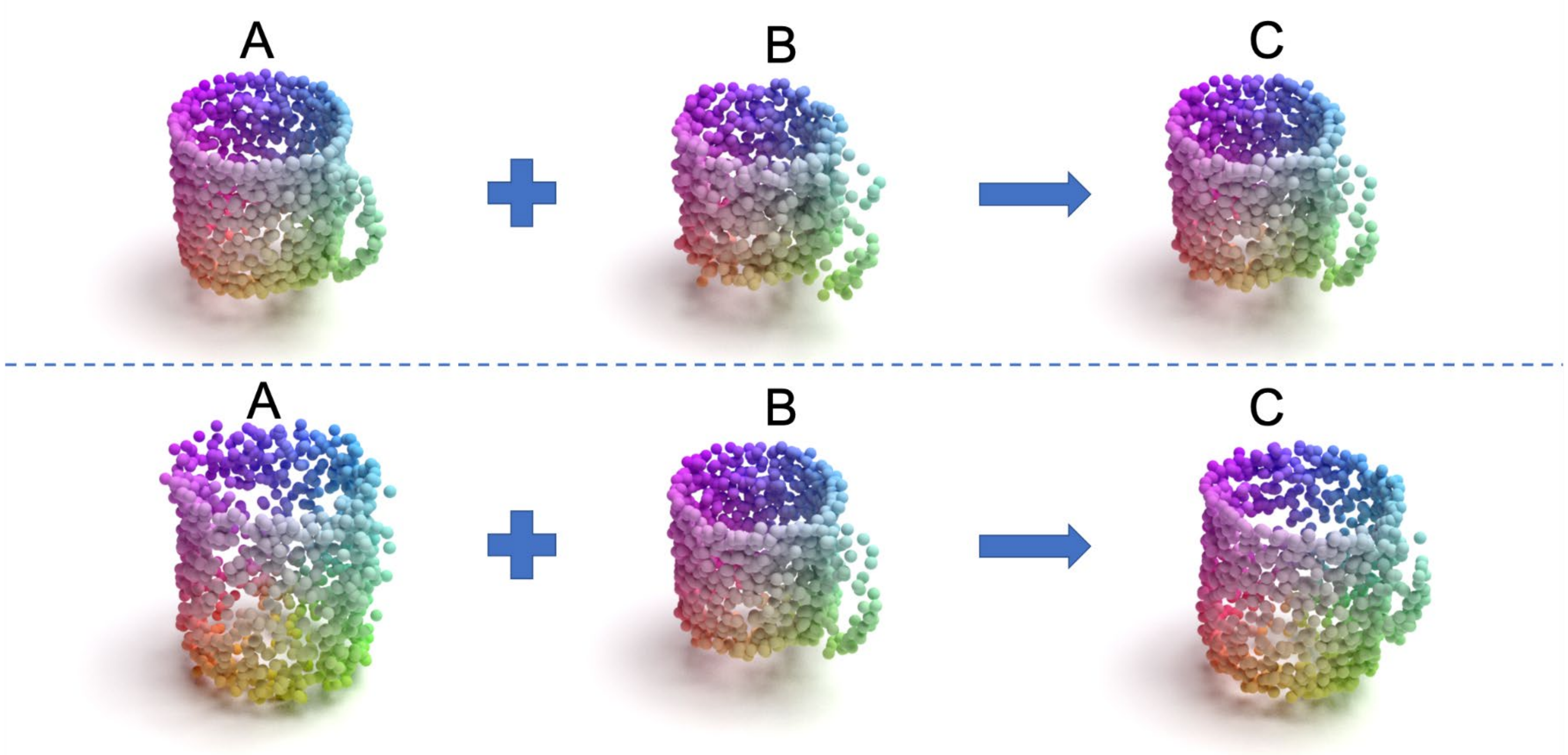
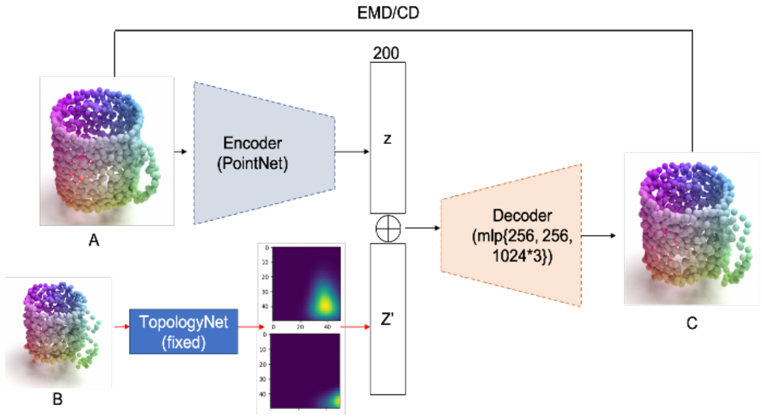
Topology Editing Results:

- Cup with arc-shaped handle to cup with 3-shaped handle
- Cup without handle to cup with handle

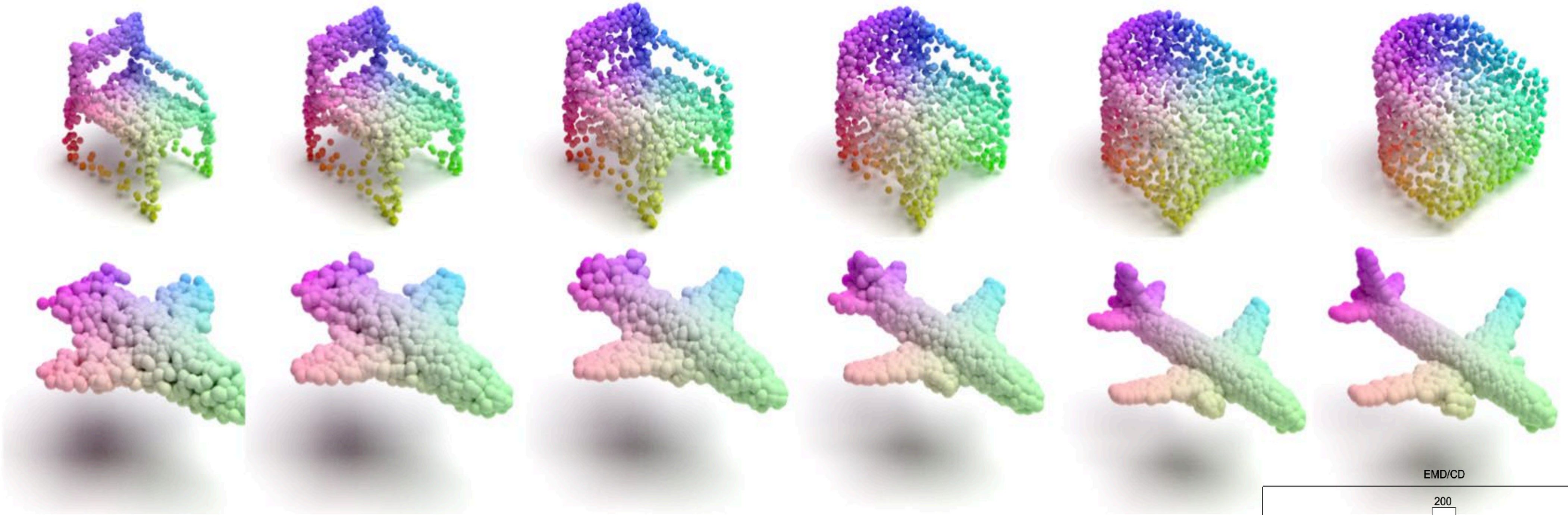


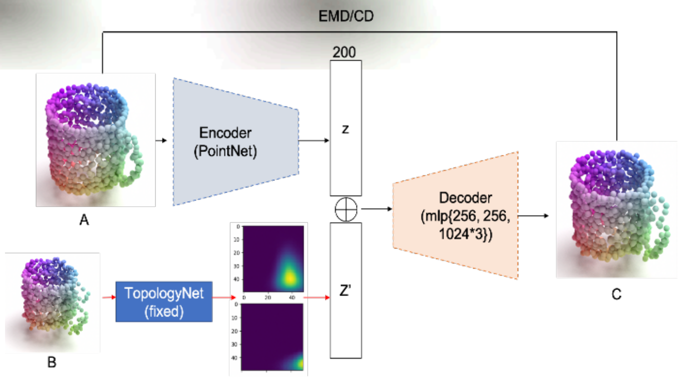


Chi Zhou, Zhetong Dong, Hongwei Lin. Learning persistent homology of 3D point clouds. Computers & Graphics, 102: 269-279 (2022)

Interpolation Results:

- The left most and right most are the input point clouds
- The middle point clouds are generated by interpolation using topological implicit vector





Chi Zhou, Zhetong Dong, Hongwei Lin. Learning persistent homology of 3D point clouds. Computers & Graphics, 102: 269-279 (2022)

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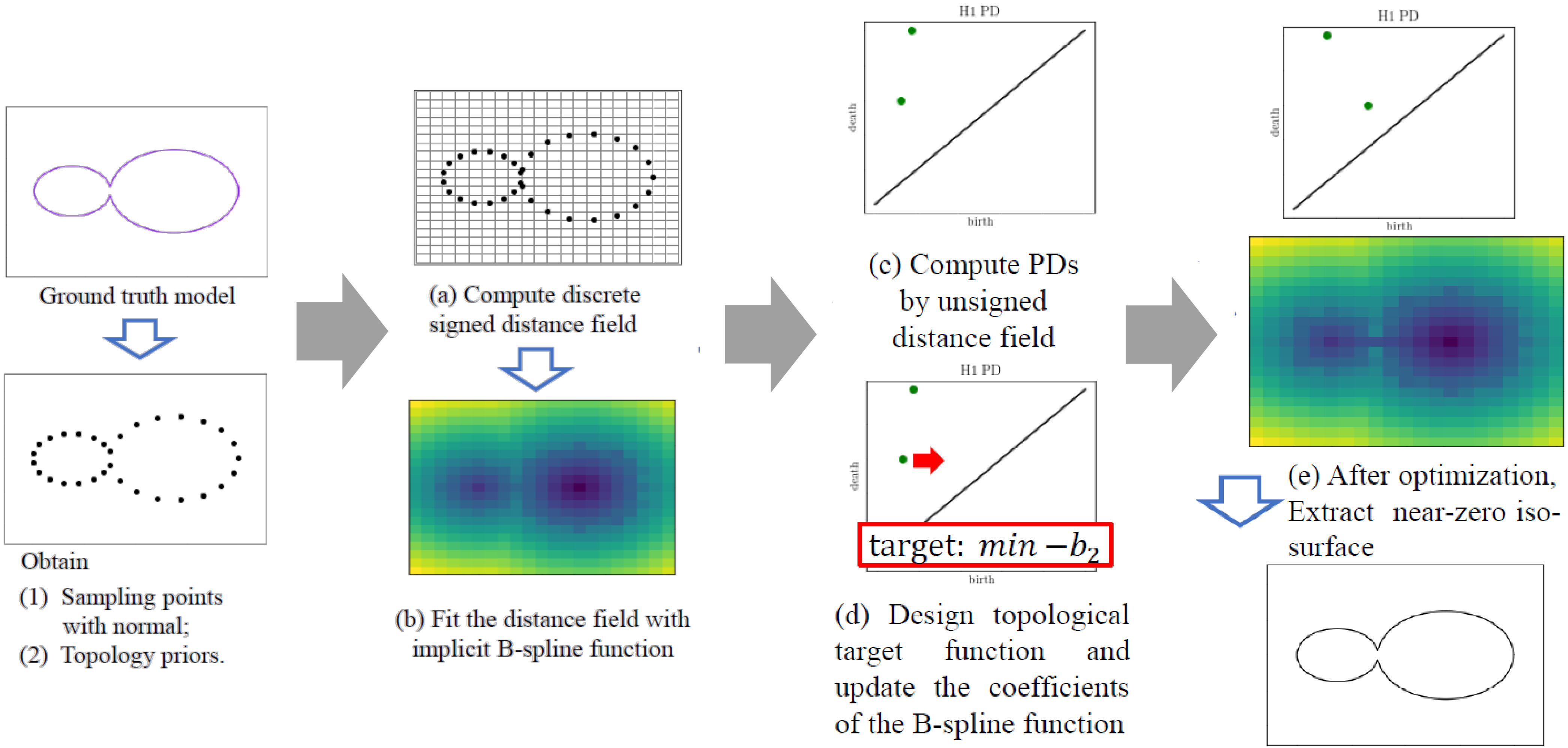
4.9 Topology-controllable implicit surface reconstruction

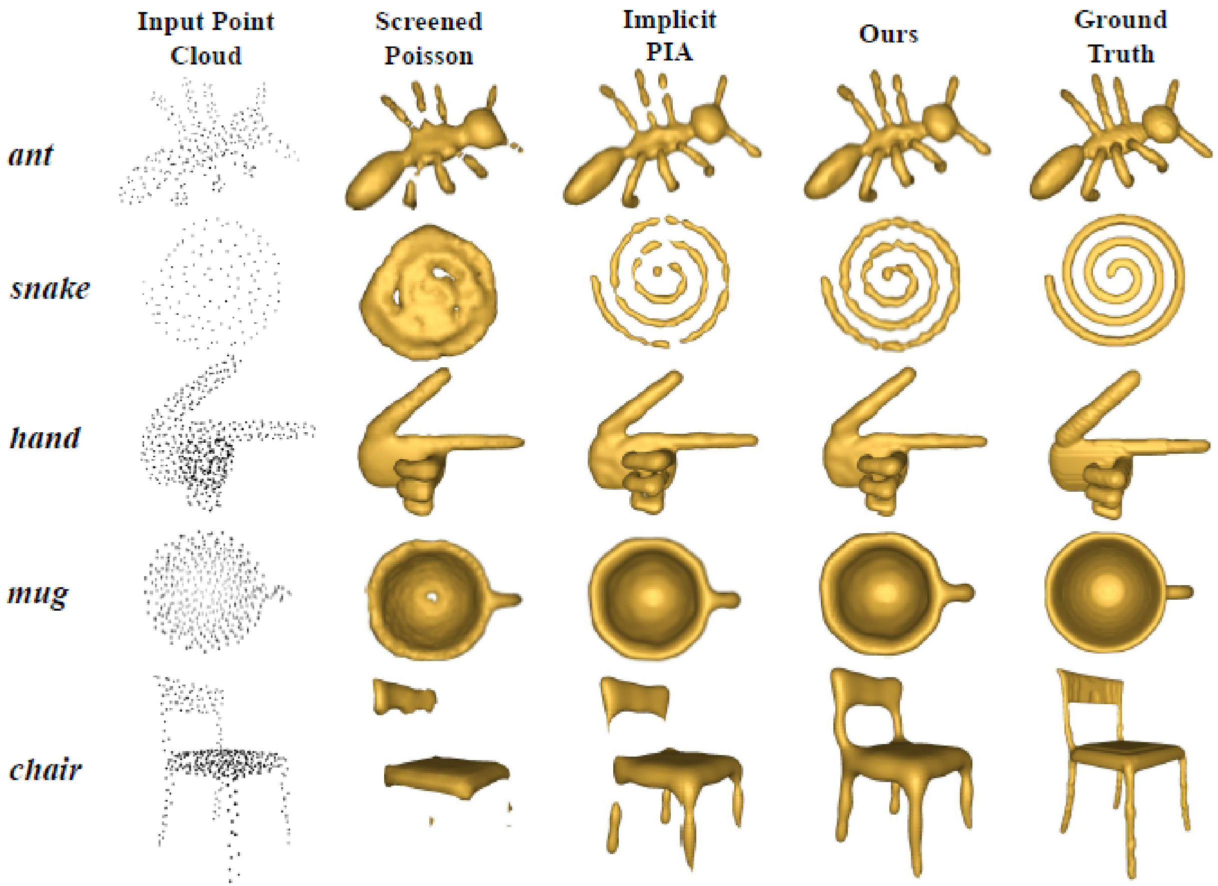
Pipeline



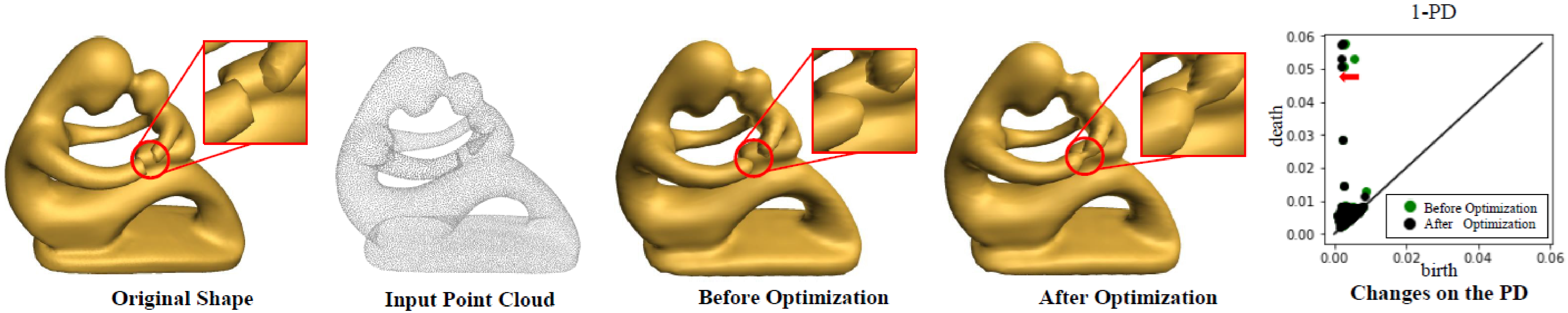
Zhetong Dong, Jinhao Chen, Hongwei Lin. Topology-controllable Implicit Surface Reconstruction Based on Persistent Homology. Computer-Aided Design,150: 103308 (2022)

Surface reconstruction in different sampling cases

➢ Sparse Sampling



(a) Results from *sparse* and *uniform* sampling point clouds.



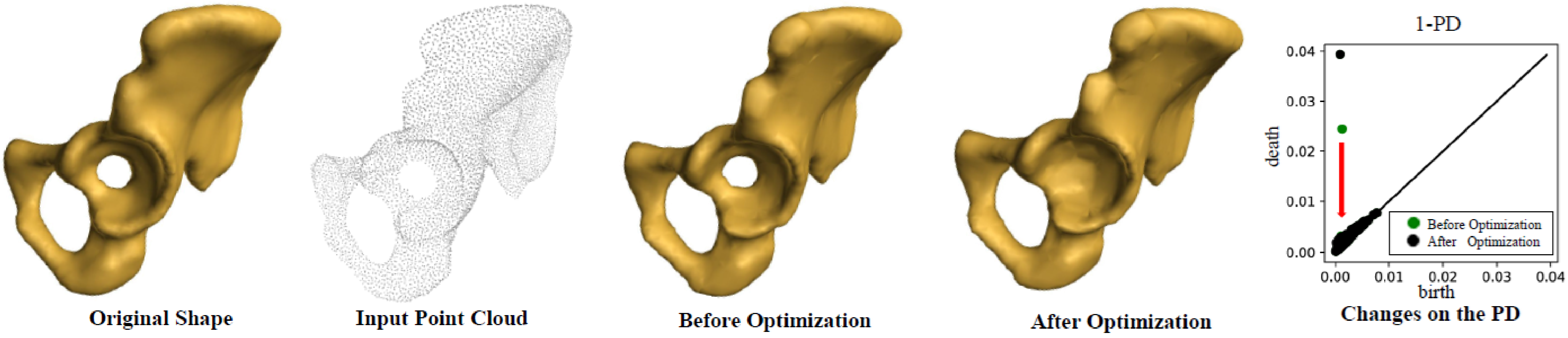
(a) Topology repair.

Application: Topology repair

Motivation: To repair these topological errors through topology-controllable optimization.

Topology-controllable optimization: To minimize $L^{(1)} = \sum_{i=1}^{4} b_i$, where b_i's denote the birth values of the four most persistent 1D features, representing four loop structures in the shape.

Result: The broken loop appeared on the extracted surface after topology-controllable optimization.



(b) Hole removal.

Application: Hole removal

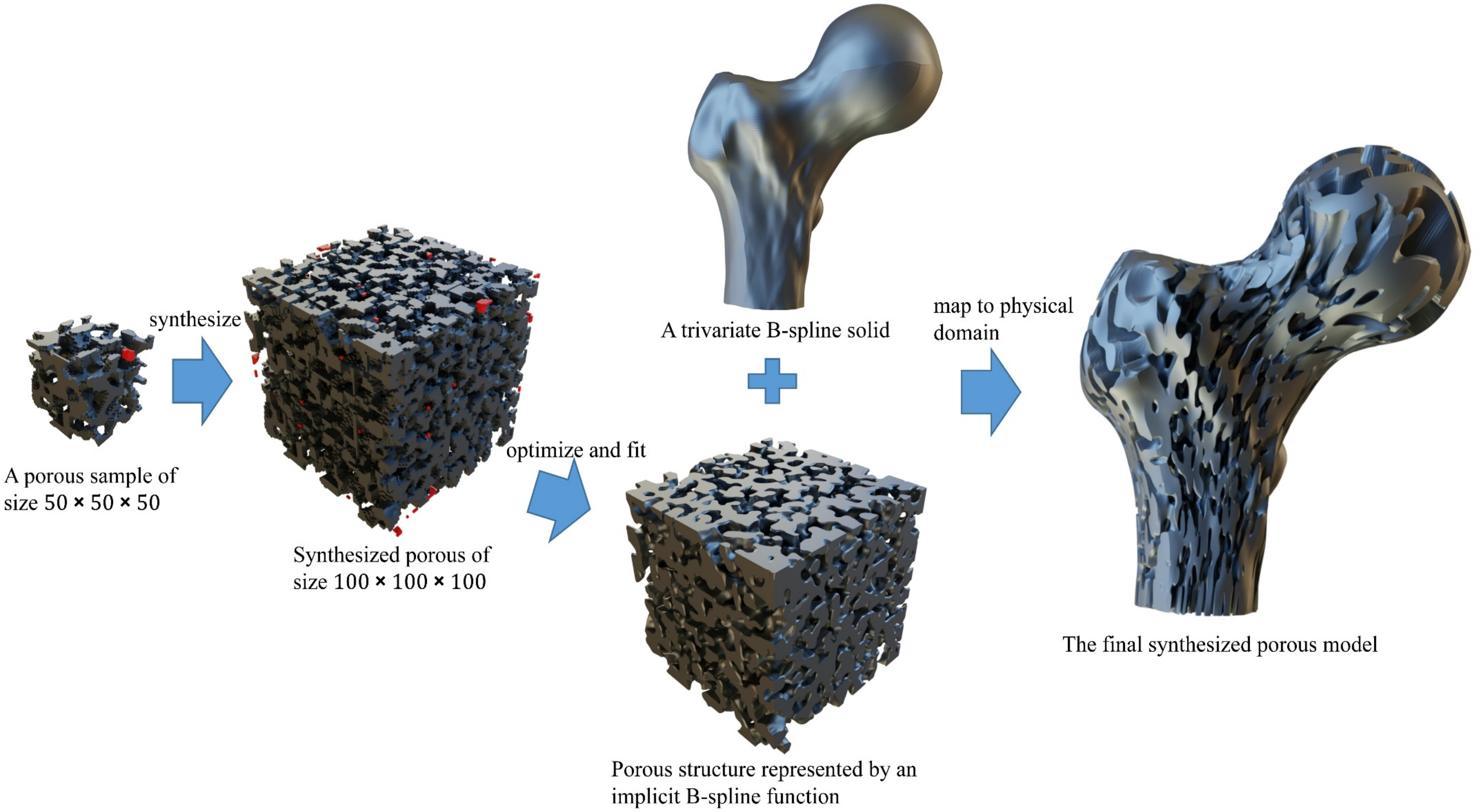
Motivation: To remove the 1D holes (1D homology classes) on the shape.

Topology-controllable optimization: To minimize $L^{(1)} = d_2 - b_2$, where (b_2, d_2) represents the persistence pair of the second most persistent feature.

Result: The extracted surface was obtained with the removed small-scale handle loop.

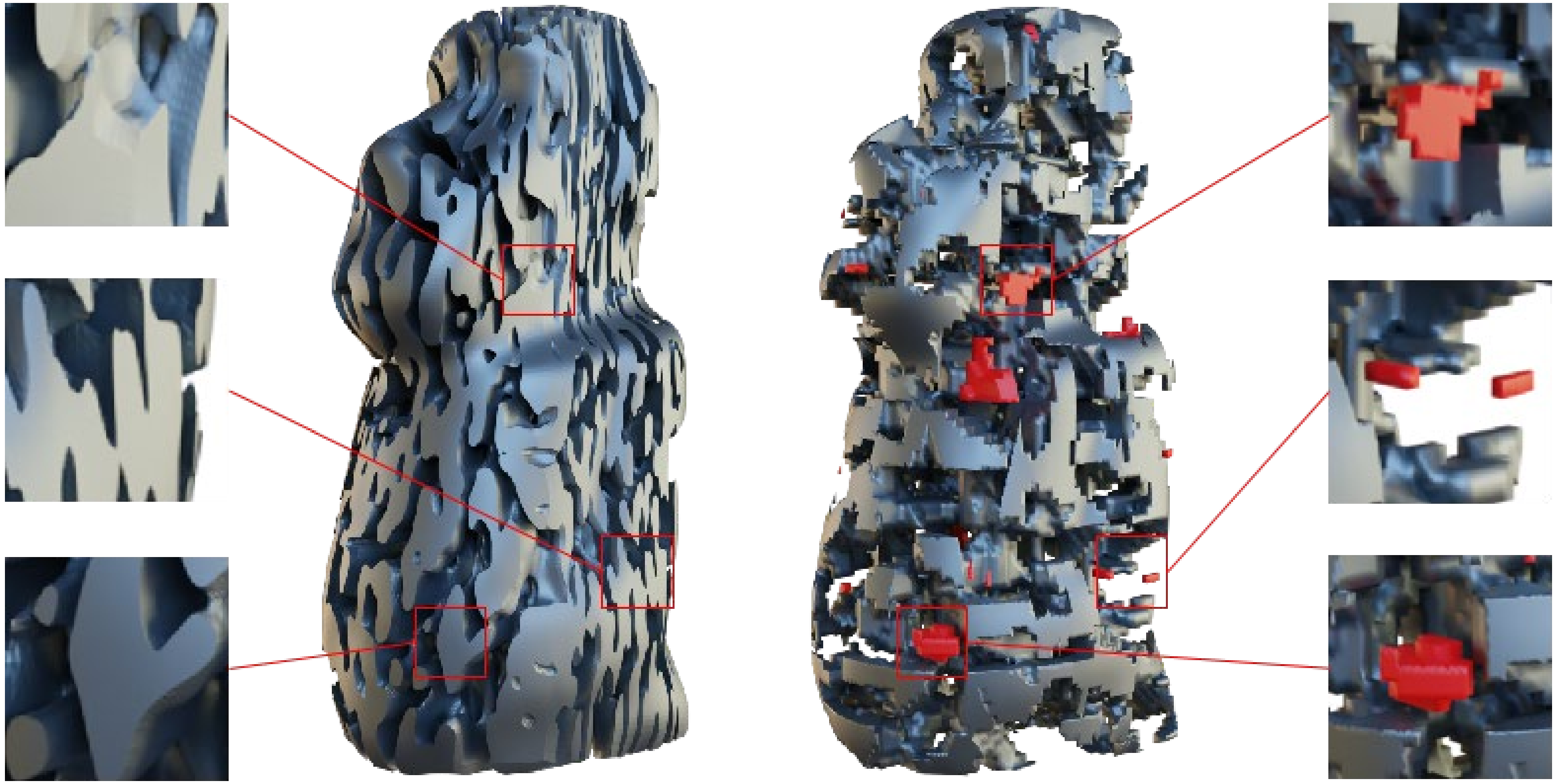
4.10 Connectivity-guaranteed porous synthesis

- Workflow：



Depeng Gao, Jinhao Chen, Zhetong Dong, Hongwei Lin. Connectivity-guaranteed porous synthesis in free form model by persistent homology. Computers & Graphics, 106: 33-44 (2022)

- Comparison with Boolean intersection：



(a)

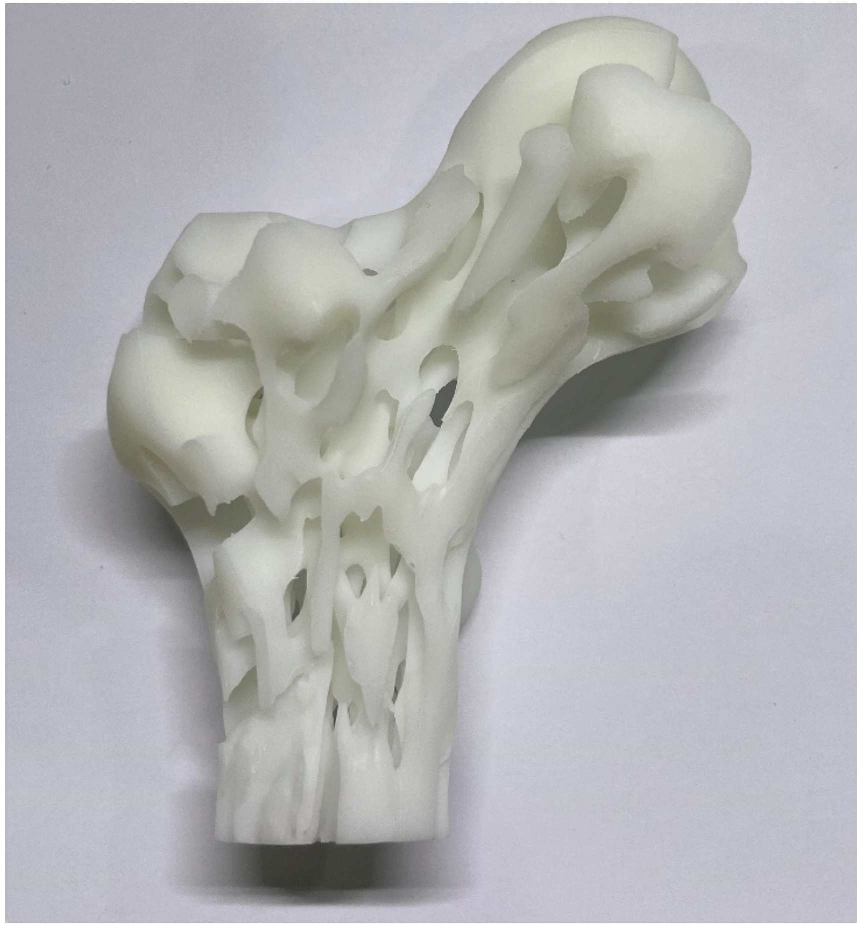
Porous model obtained by the proposed method

(b)

Porous model obtained from Boolean operation

4.10 Connectivity-guaranteed porous synthesis

- Porous scaffold manufactured



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5. Conclusion

5 Conclusion

- Computational Geometry → Computational Topology
- Computer Aided Geometric Design →Computer Aided Topologic Design

- 计算机辅助拓扑设计：以持续同调为主要数学工具，系统解决几何设计和几何处理中的拓扑问题

谢　谢！