Multibody Simulation with Affine Body Dynamics

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Topics Today: Multibody Simulation

I. Background & Challenges

II. Affine Body Dynamics (ABD)

III. Articulation and Restitution

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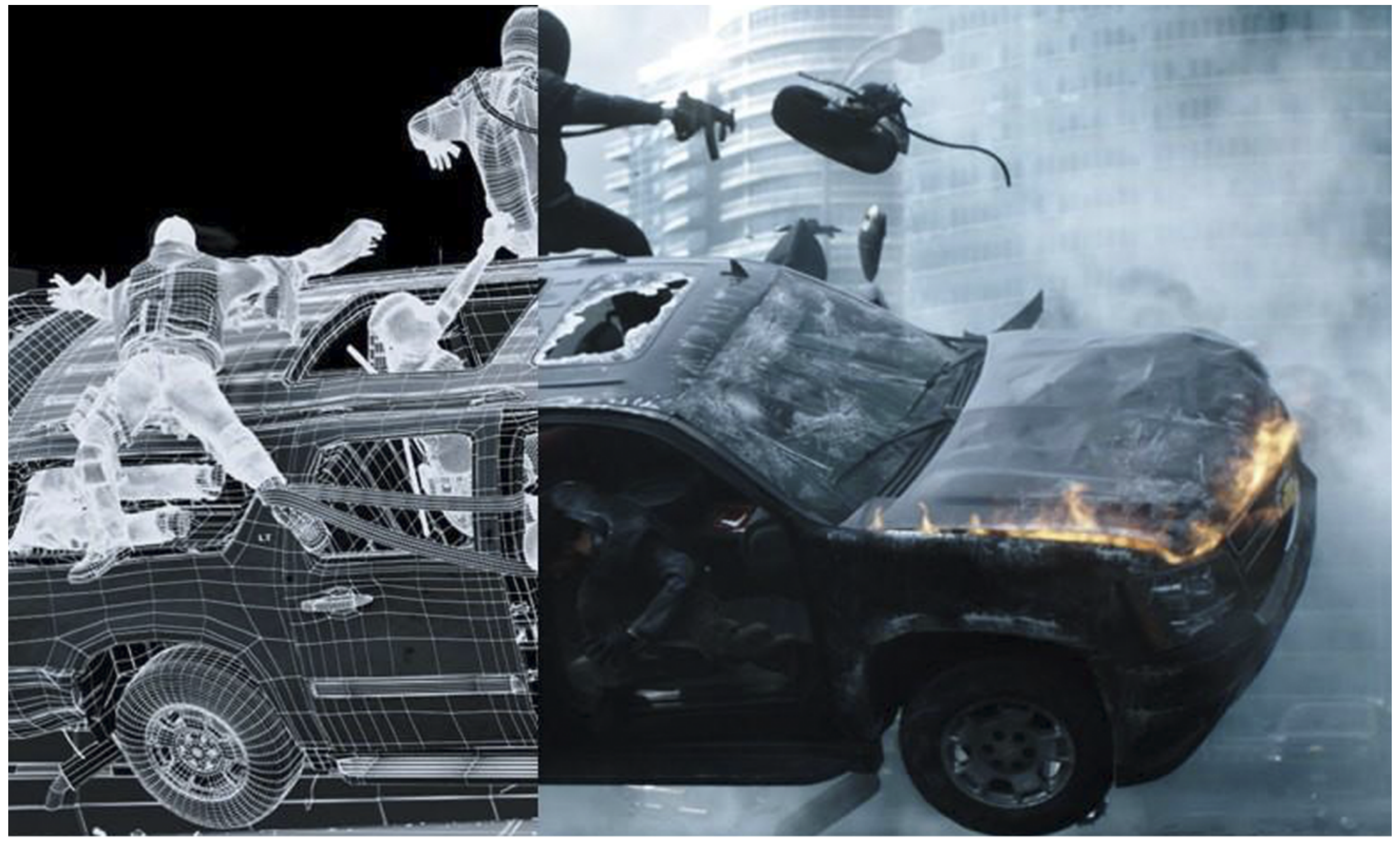
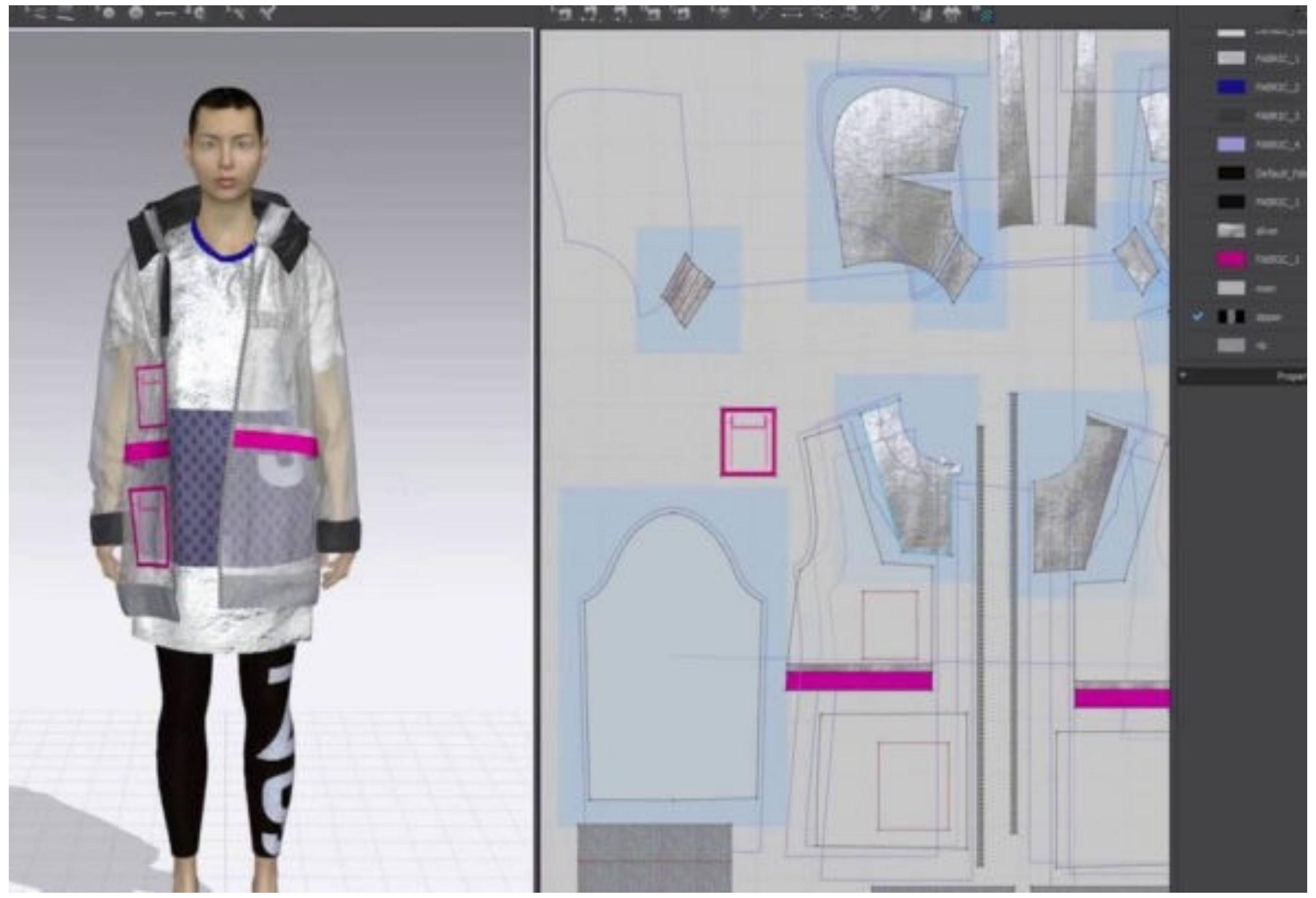
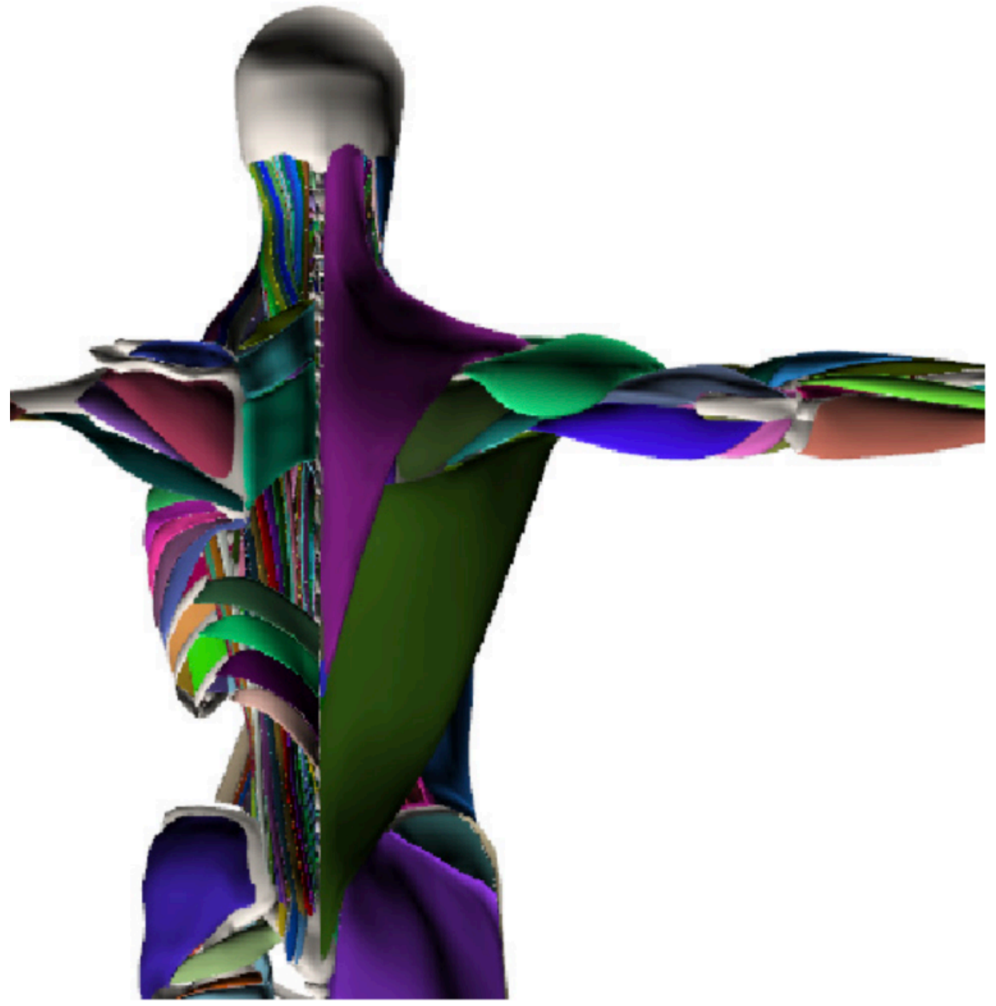
Solids Simulation Applications



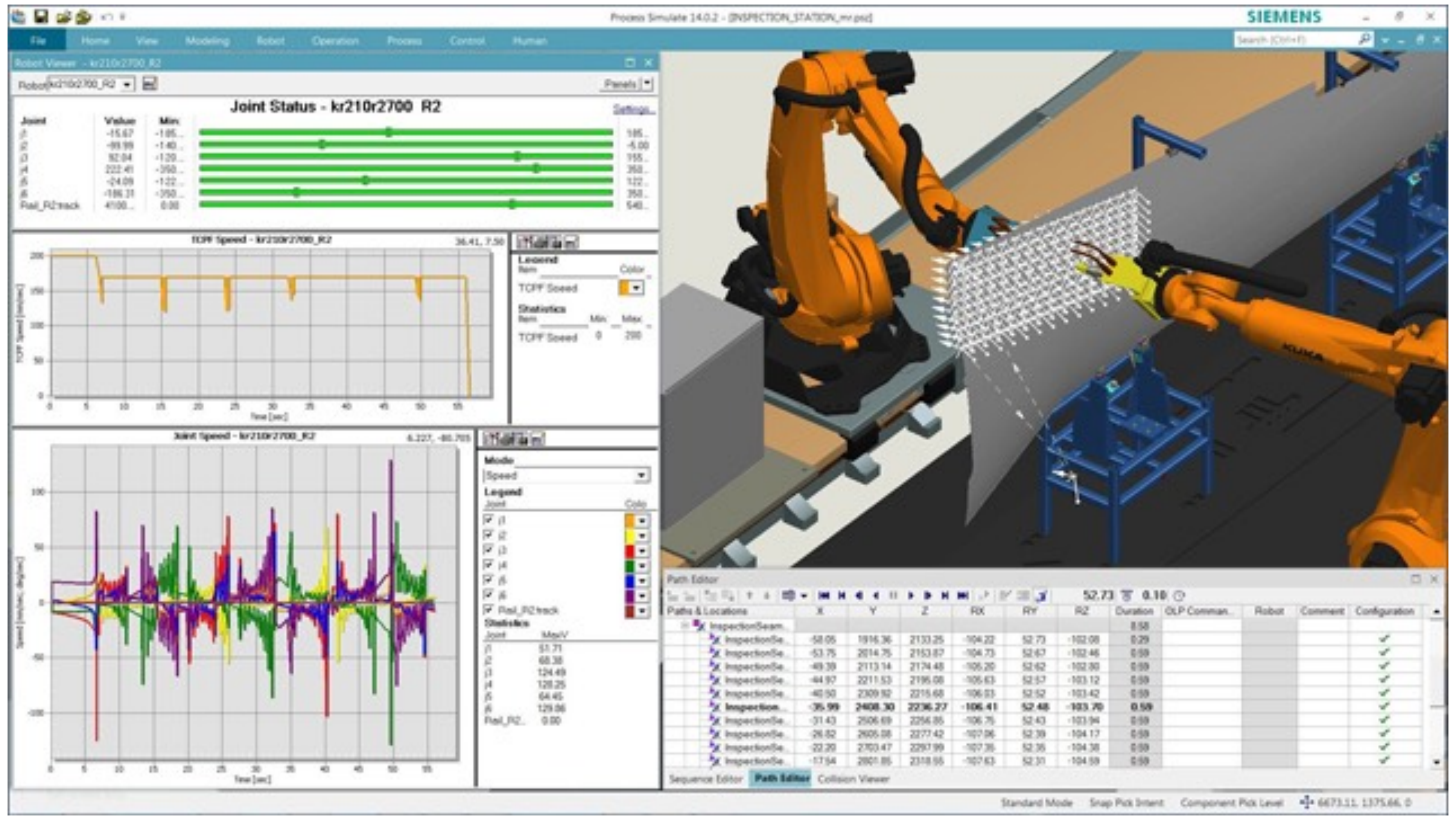
Visual effects & animations



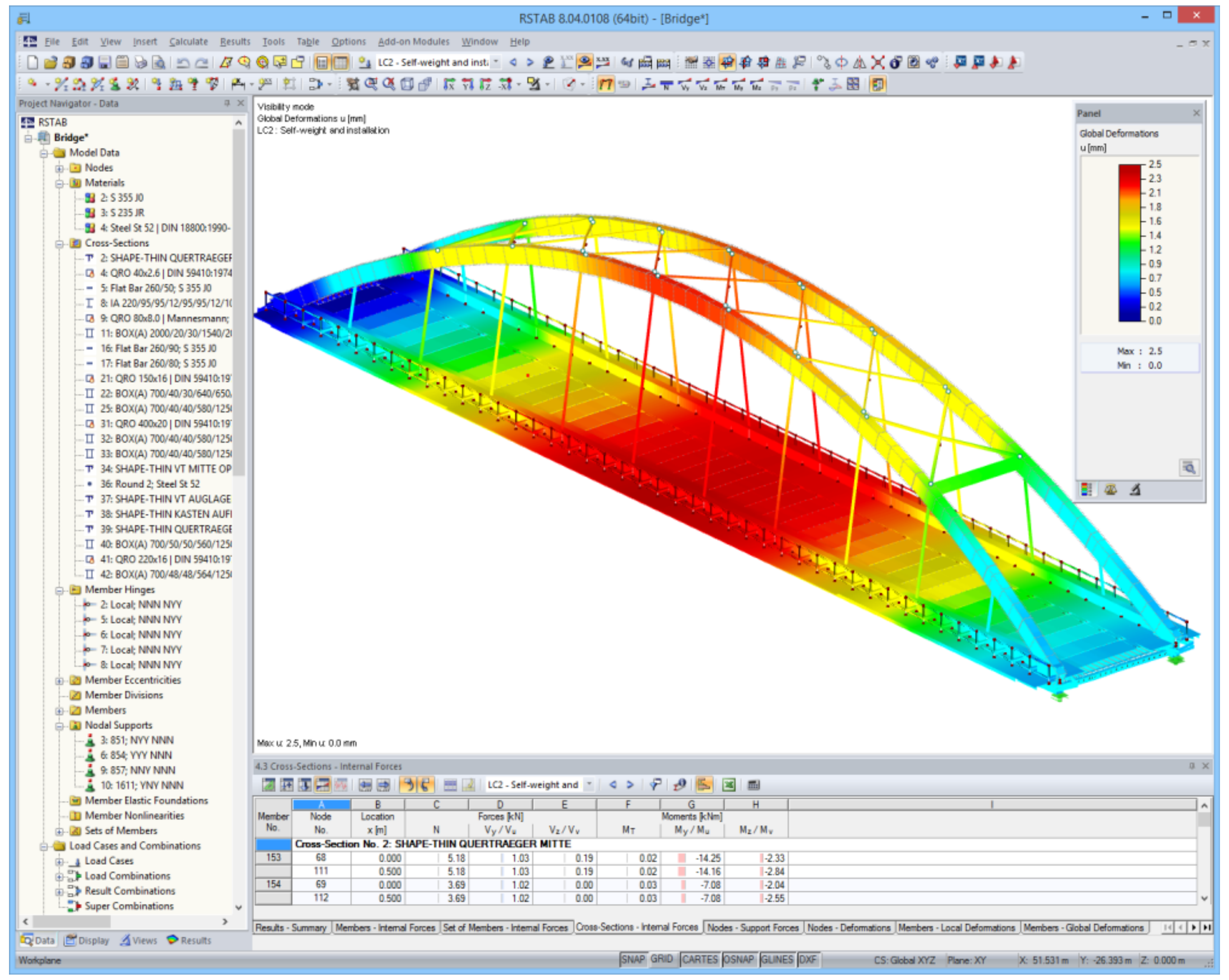
Fashion



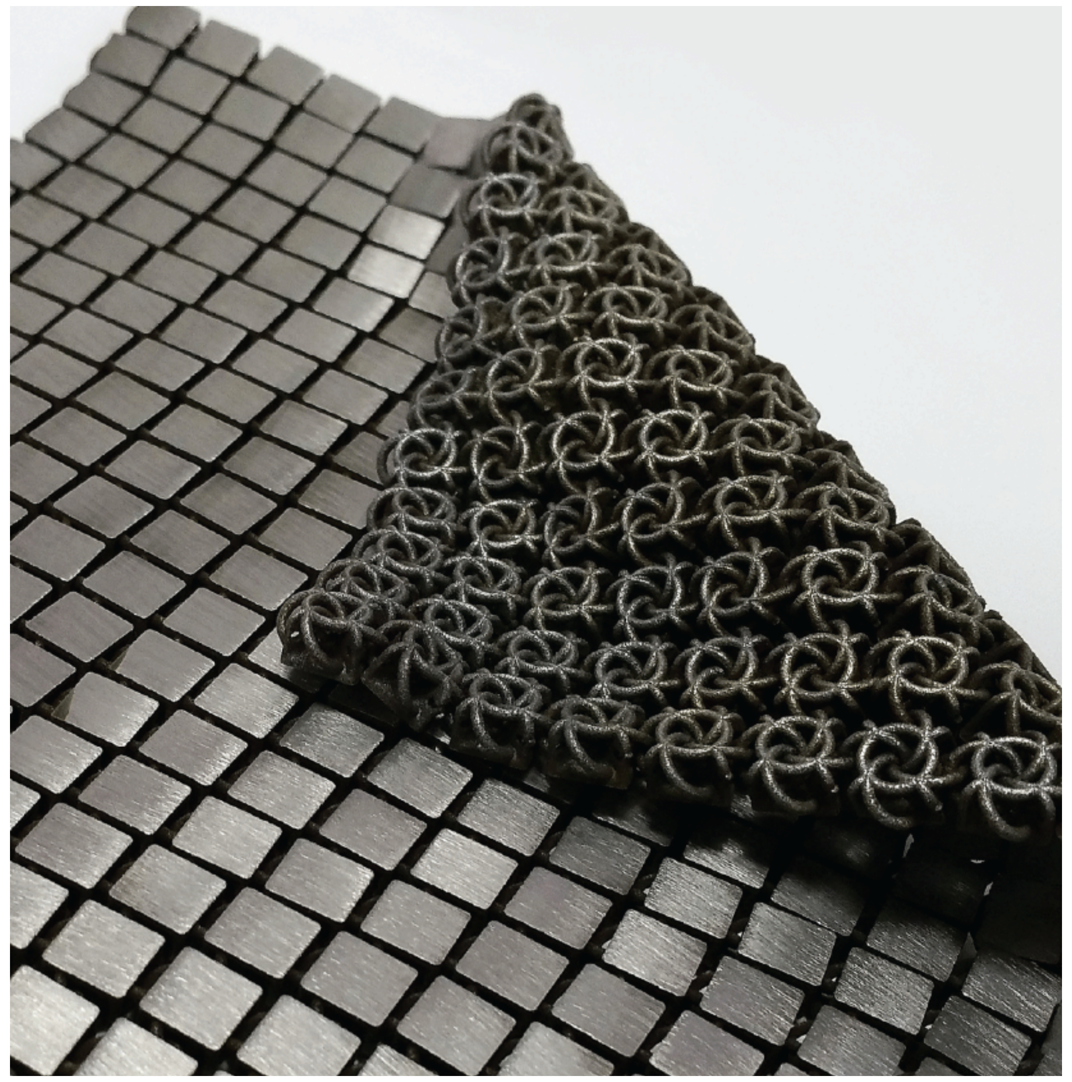
Biomechanics



Robotics

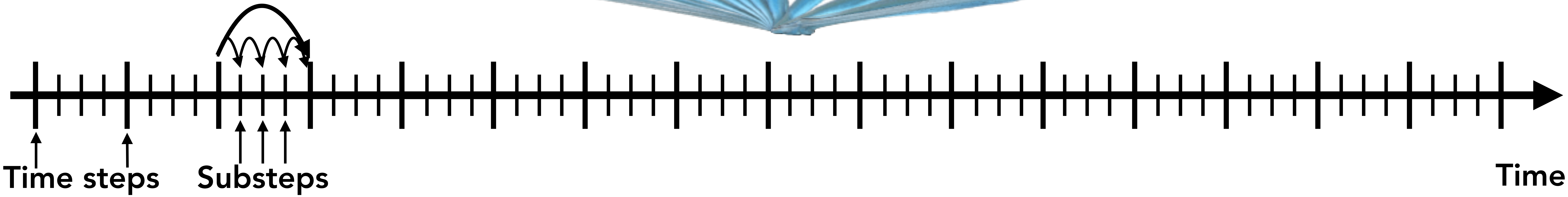


Engineering

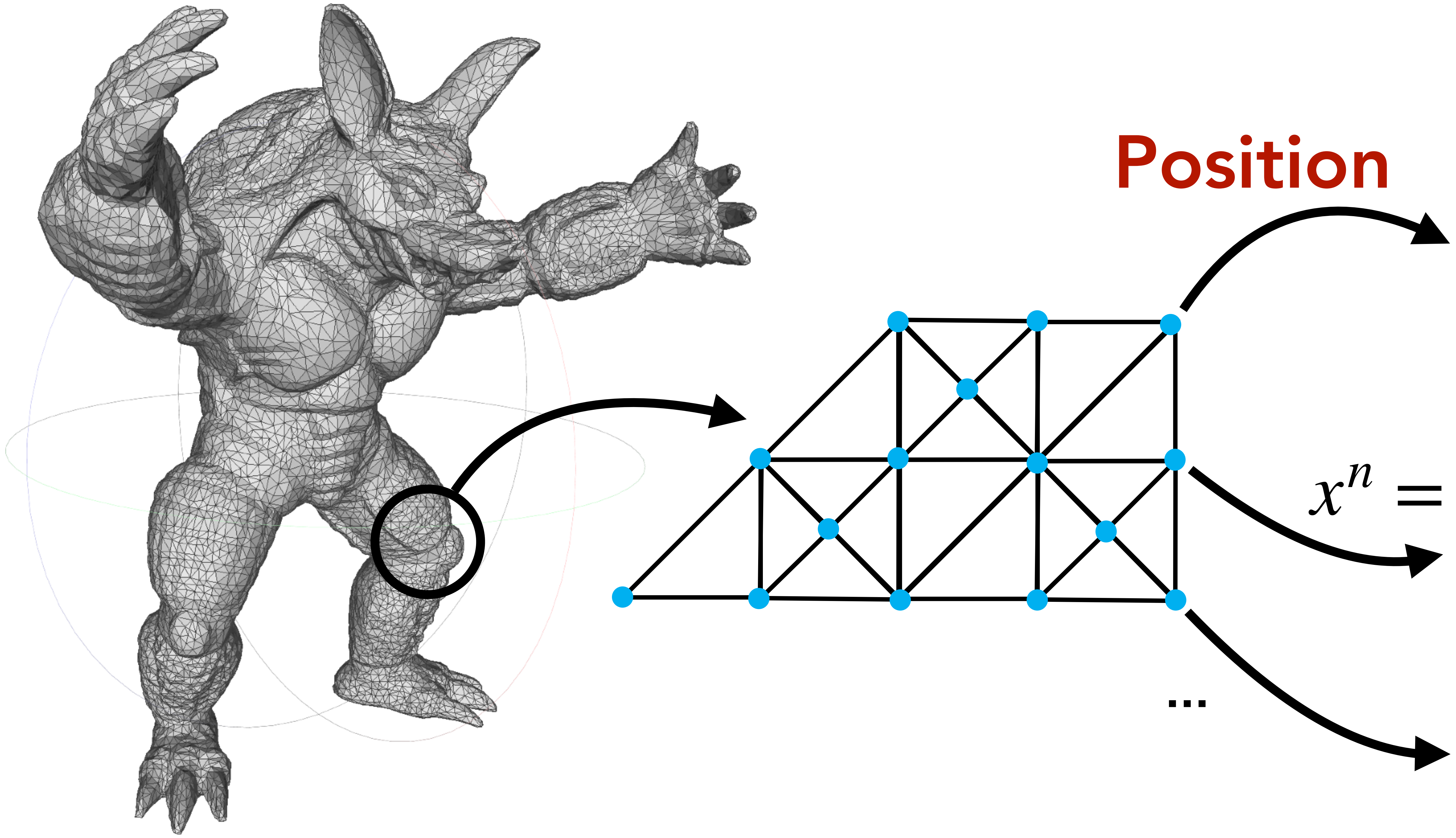


Material science

Time Stepping



DOT [Li et al. 2019]



Position

$$x^n = \begin{bmatrix} x^n_{0x} \\ x^n_{0y} \\ x^n_{0z} \\ x^n_{1x} \\ x^n_{1y} \\ x^n_{1z} \\ \vdots \end{bmatrix} \qquad v^n = \begin{bmatrix} v^n_{0x} \\ v^n_{0y} \\ v^n_{0z} \\ v^n_{1x} \\ v^n_{1y} \\ v^n_{1z} \\ \vdots \end{bmatrix}$$

Velocity

Given x^0, v^0, for time steps $n = 0,1,2,...$

$$\begin{cases} v^{n+1} = v^n + hM^{-1}f(x^{n+1}) \\ x^{n+1} = x^n + hv^{n+1} \end{cases}$$

Velocity update

Position update

$$x^{n+1} - h^2M^{-1}f(x^{n+1}) = x^n + hv^n$$

$$\min_{x^{n+1}} \frac{1}{2}\|x^{n+1} - \tilde{x}^n\|^2_M + h^2\Phi(x^{n+1})$$

$$\begin{cases} f(x) = -\frac{\partial \Phi}{\partial x} \\ \tilde{x}^n = x^n + hv^n \end{cases}$$

Incremental Potential

[Ortiz and Stainier 1999]

$$x^{n+1} = \textbf{argmin}_x \frac{1}{2}\|x - \tilde{x}^n\|_M^2 + \beta h^2 \Phi(x)$$

Large Deformation

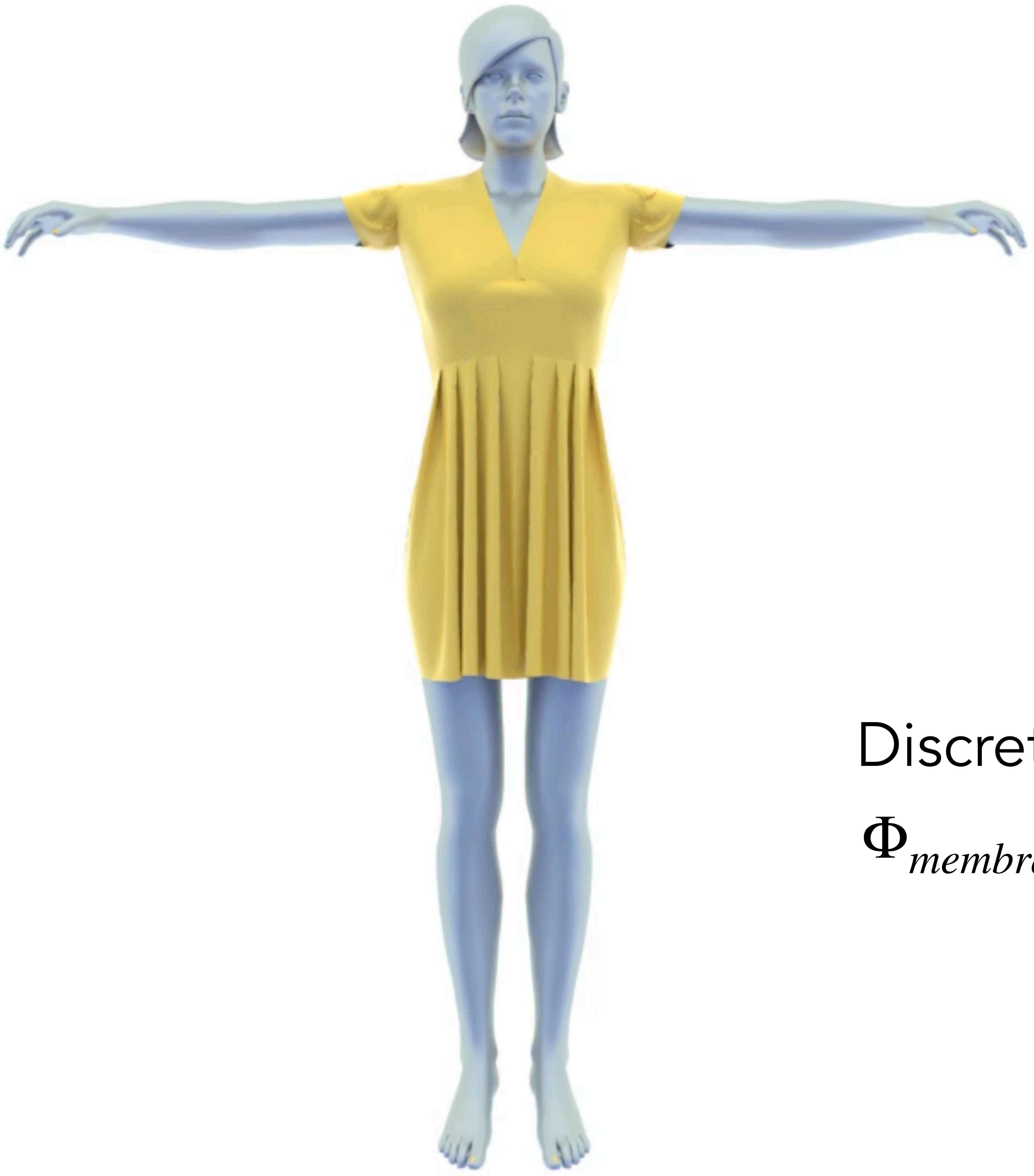
Large Time Step

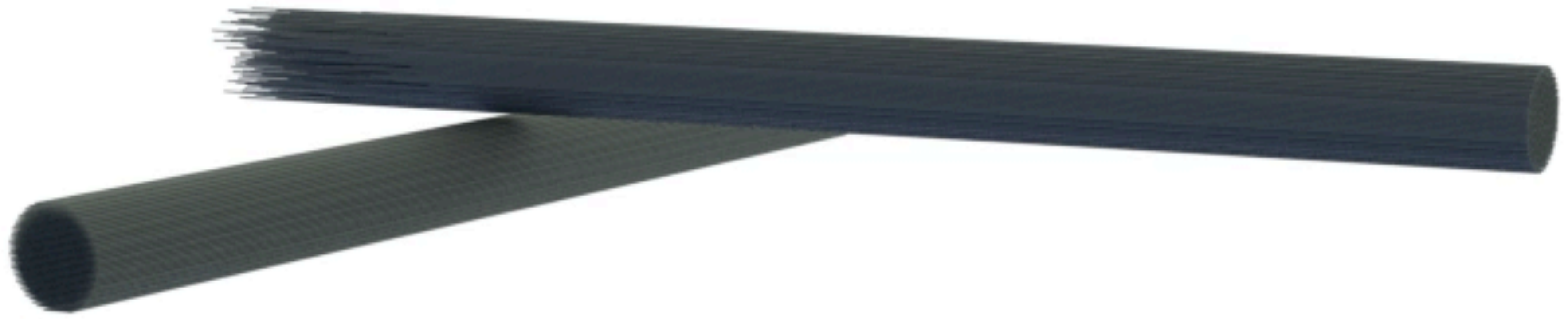
Backward Euler:

$$x^{n+1} = \textbf{argmin}_x \frac{1}{2}\|x - \tilde{x}^n\|_M^2 + h^2 \Phi(x)$$

Elasticity Term

Inertia Term





Discrete Shells:

$$\Phi_{membrane} + \Phi_{bending}$$

Discrete Rods:

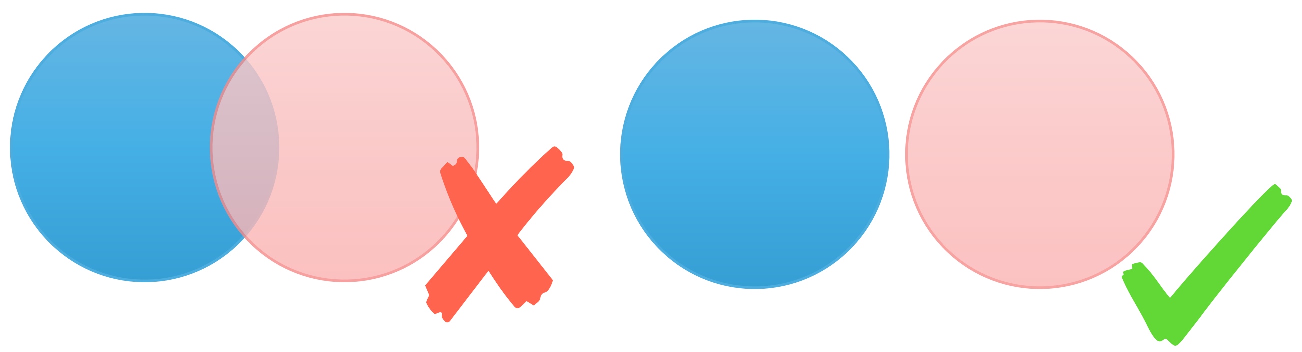
$$\Phi_{stretch} + \Phi_{bending} + \Phi_{twisting}$$

$$\min_{x^{n+1}} \frac{1}{2}\|x^{n+1} - \tilde{x}^n\|_M^2 + h^2\Phi(x^{n+1})$$

Multibody Contact

$$\min_{x^{n+1}} \frac{1}{2}\|x^{n+1} - \tilde{x}^n\|_M^2 + h^2 \sum_i^{\#body} \Phi(x^{n+1})$$

$$s.t. \quad g(x^{n+1}) \geq 0$$



Interpenetration-free

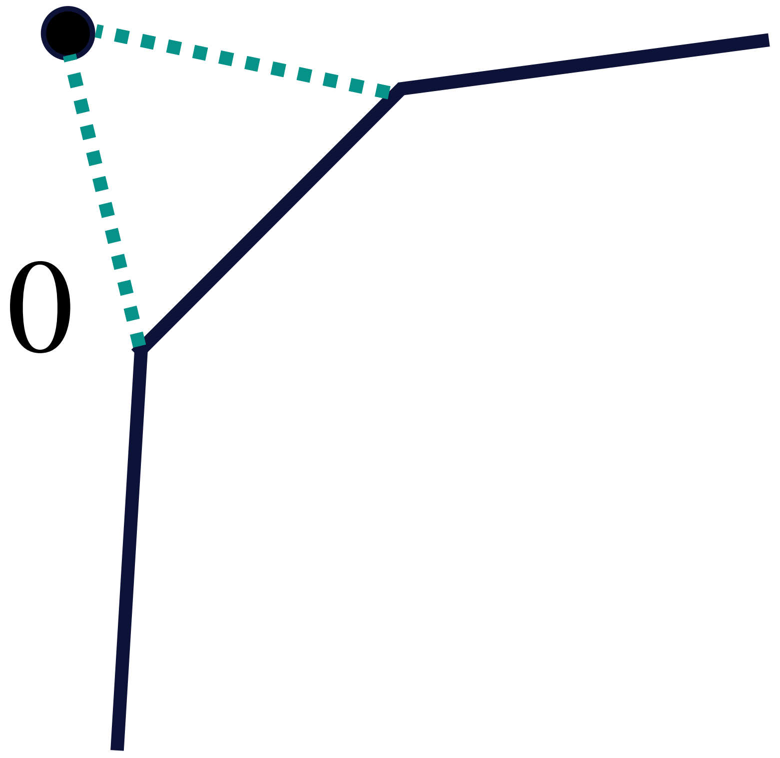
Non-linear and non-smooth!

Traditional Contact Models

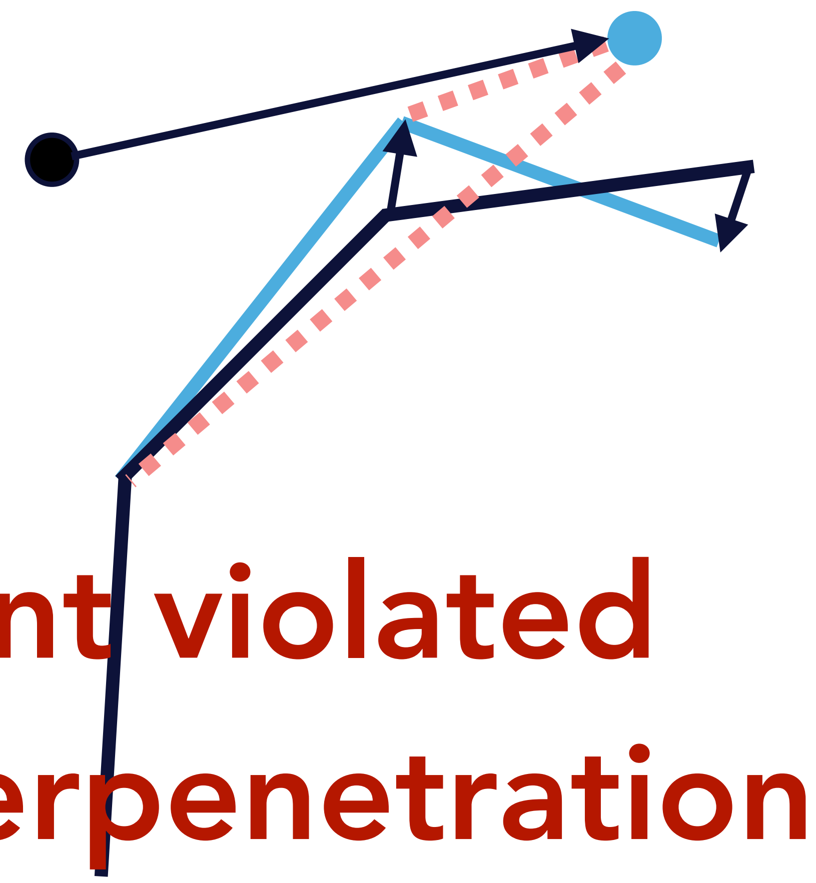
Constraint definition:

Approximated constraints:

$$g(x) = V > 0$$

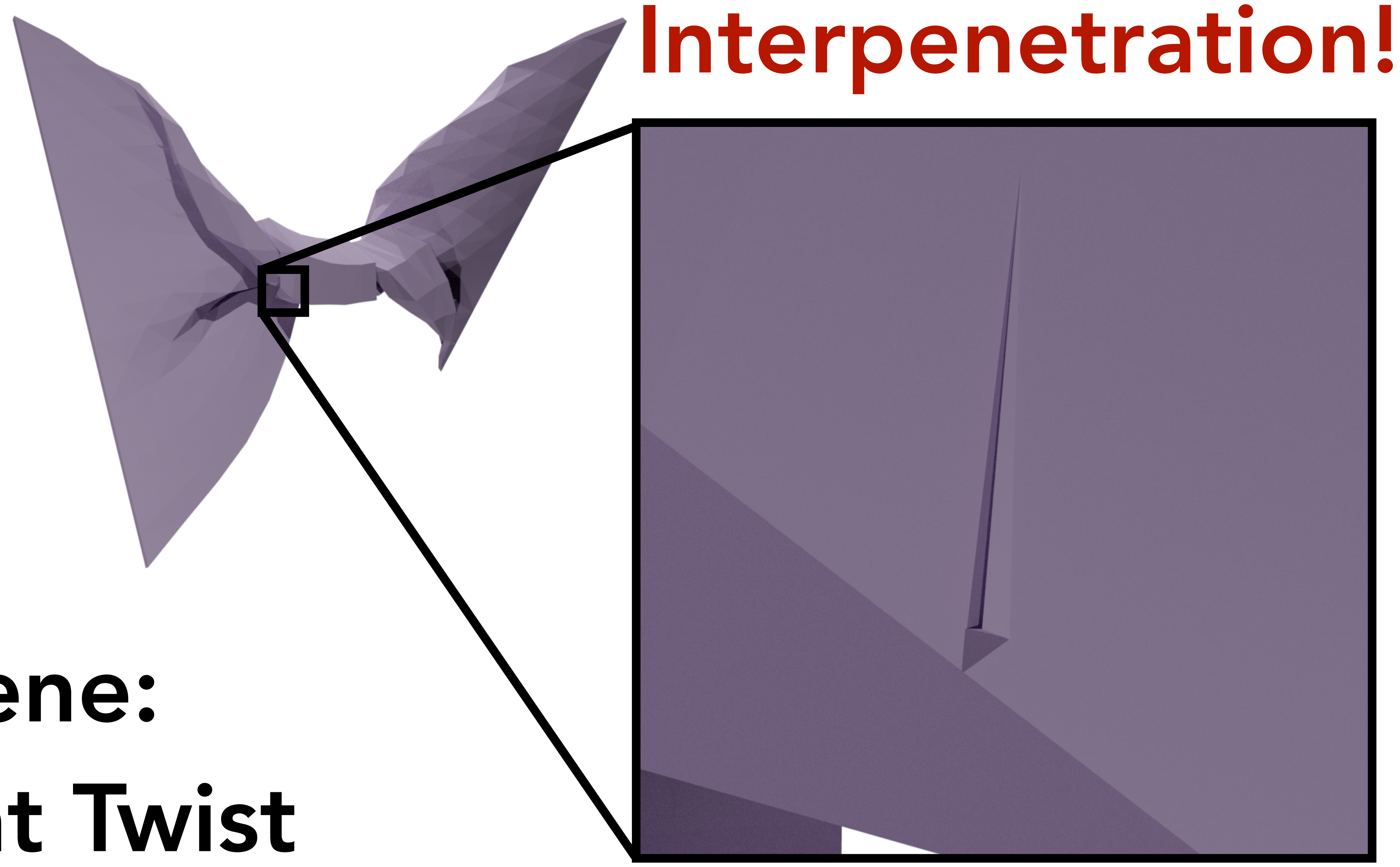


Large displacement:

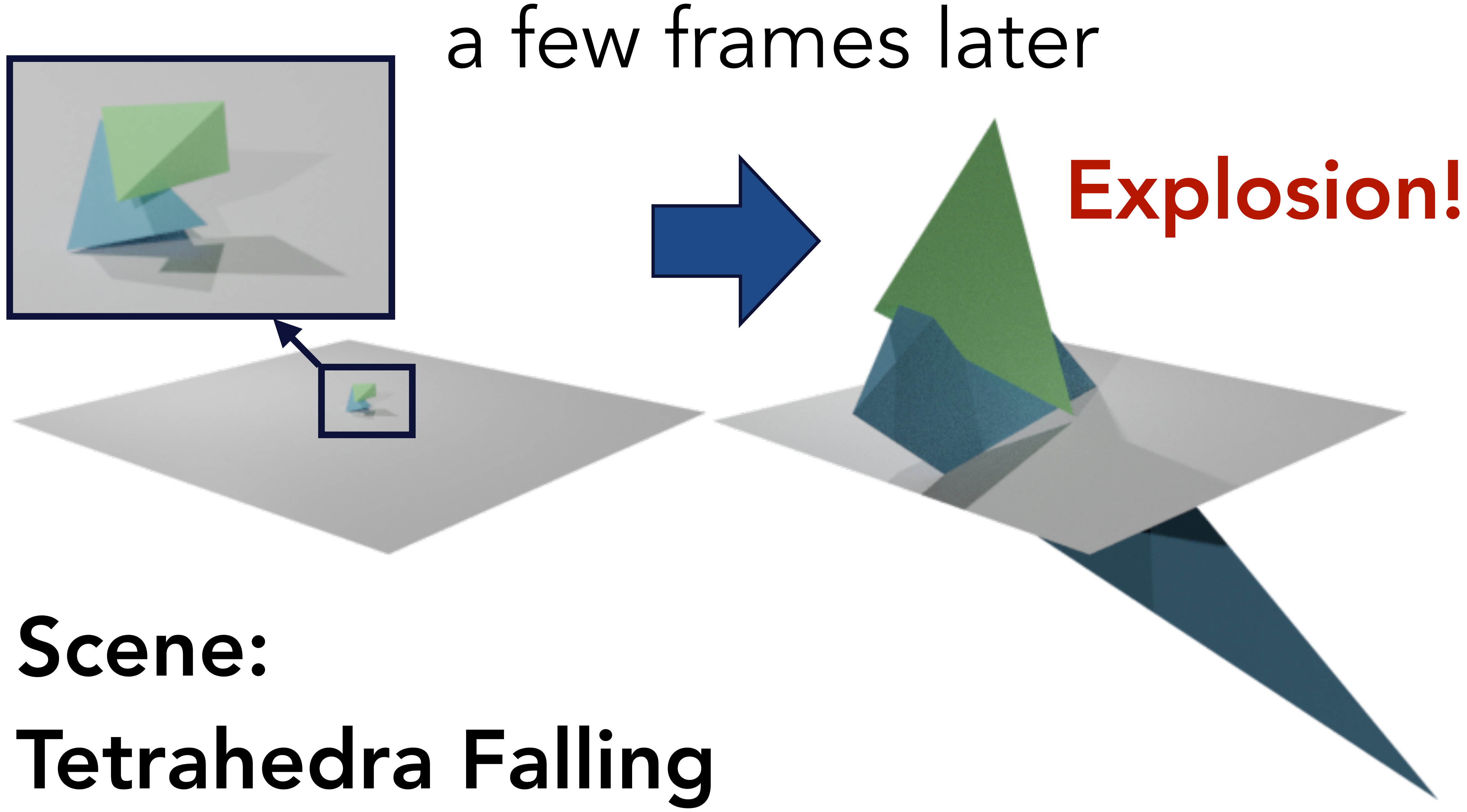


Constraint violated without interpenetration!

Common failures:



Scene:
Mat Twist



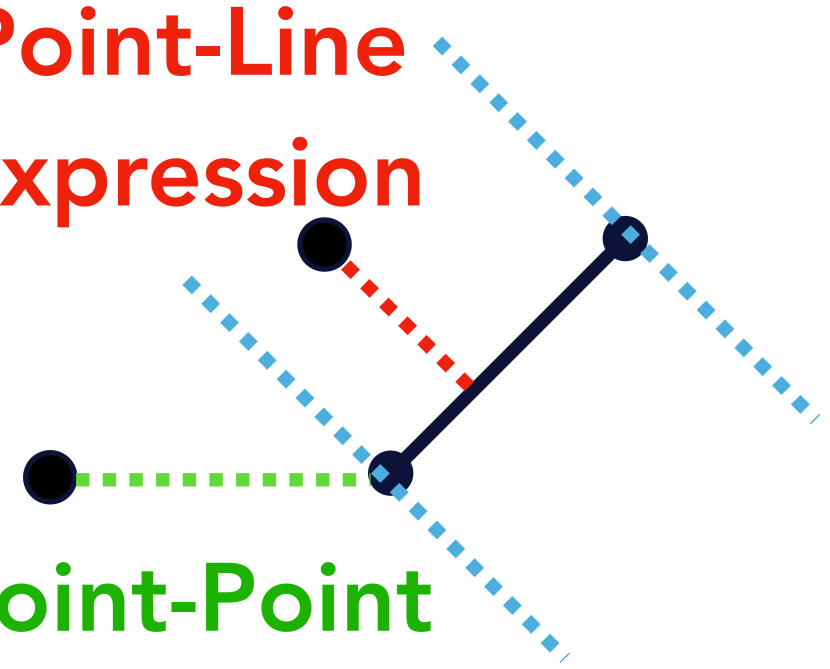
Scene:
Tetrahedra Falling

Mat Twist (100s)
45K nodes
133K tets
h: 0.04s
4x playback speed

[Li et al. 2020]

Always compute the precise distance D

Use $g(x) = D(x) > 0$

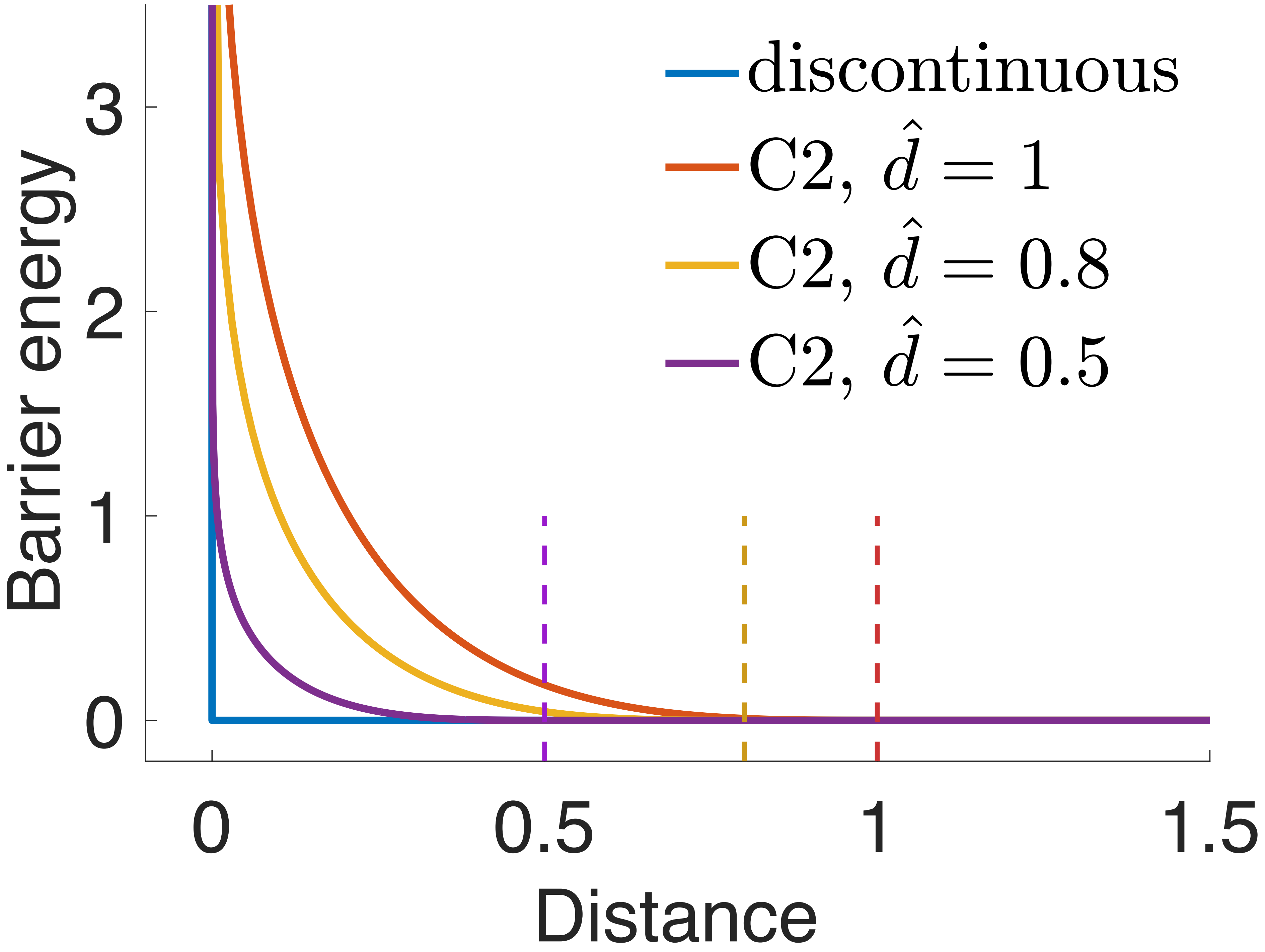


$$\min_{x^{n+1}} \frac{1}{2}\|x^{n+1} - \tilde{x}^n\|_M^2 + h^2\left(\sum_i^{\#body} \Phi(x^{n+1}) + B(x^{n+1})\right)$$

Solve with line search method!

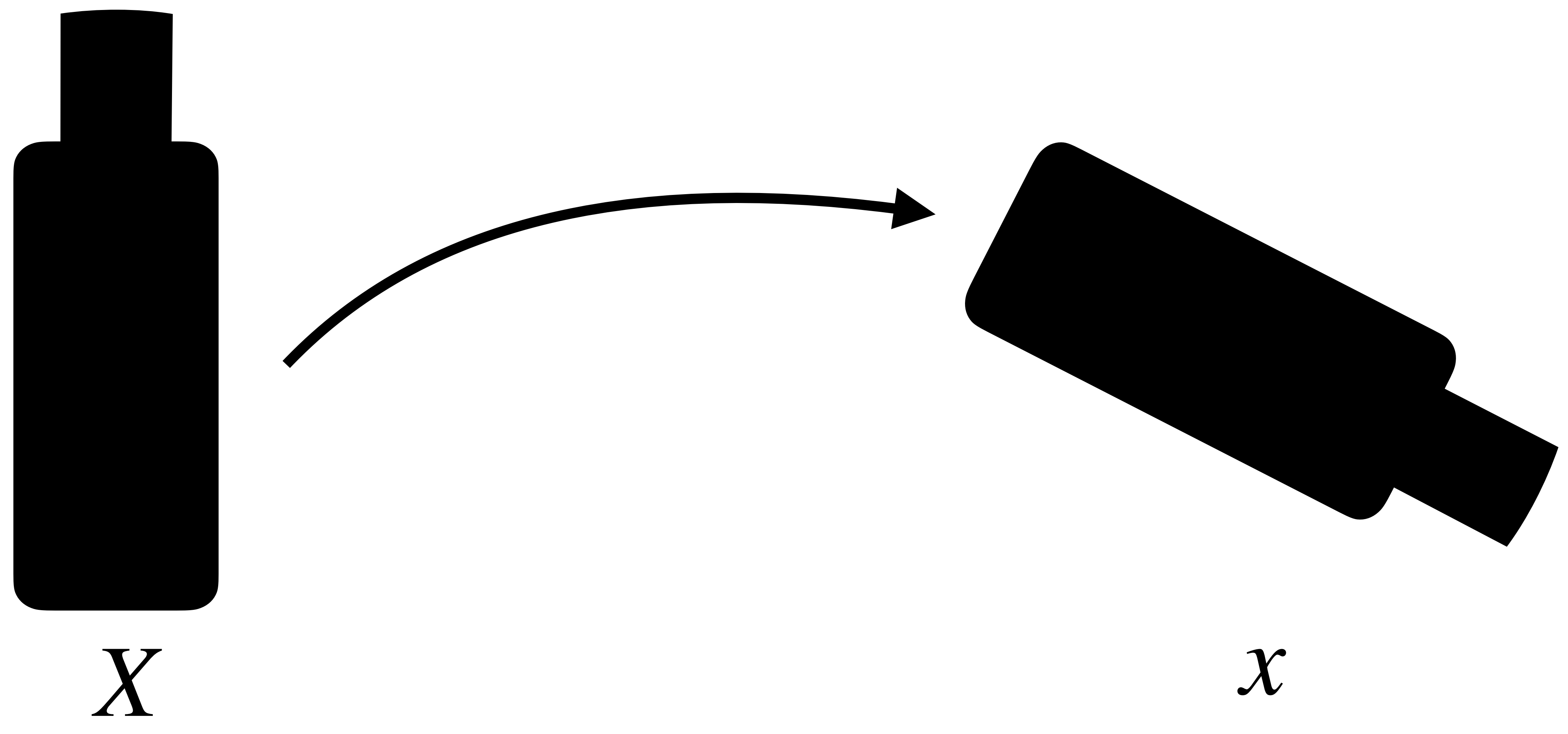
IPC's C2 clamping:

$$b(D, \hat{d}) = \begin{cases} -(D-\hat{d})^2 \log(D/\hat{d}) & \text{if } D < \hat{d} \\ 0 & \text{if } D \geq \hat{d} \end{cases}$$



Stiff Objects

SE(3) Rigid Bodies:



$$x_i - x_0 = R(X_i - X_0)$$

$$x_0 = X_0 + a$$

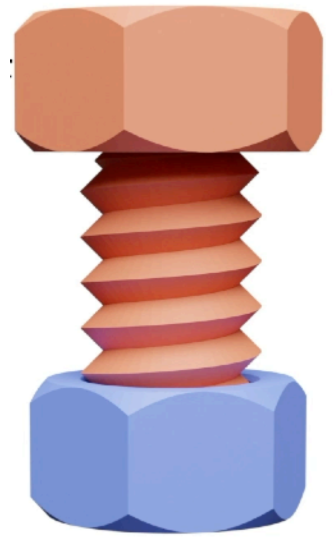
R: Rotation matrix — **3 DOFs**

a: Translation vector — **3 DOFs**

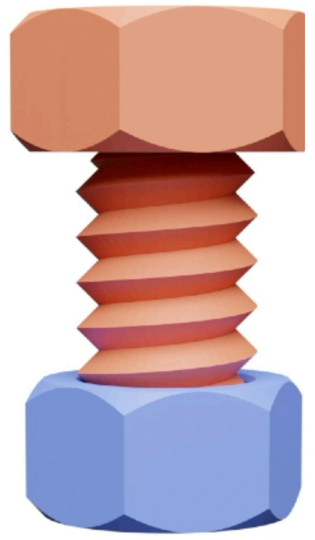
Same for the whole body!

Related Work: Bullet Physics

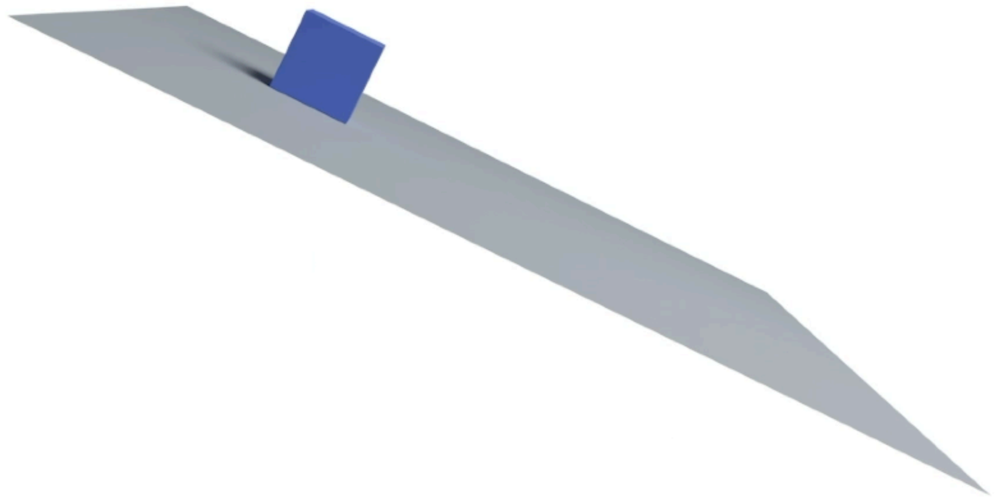
Related Works on SE(3) Rigid Body



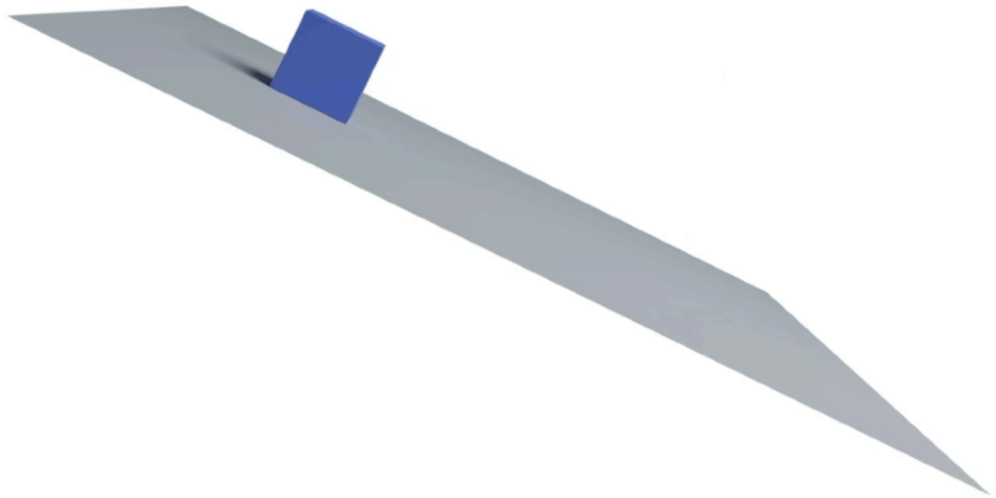
Houdini



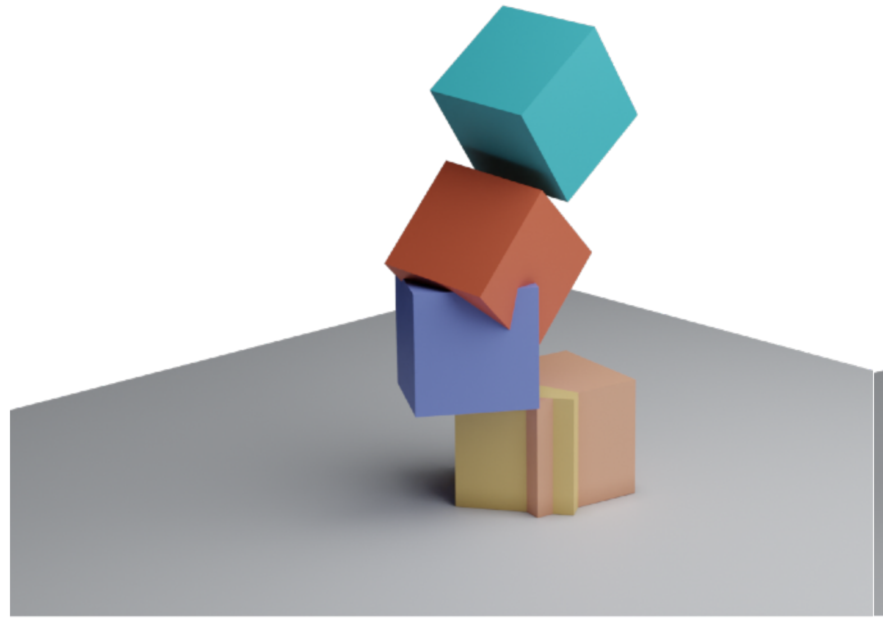
Solution



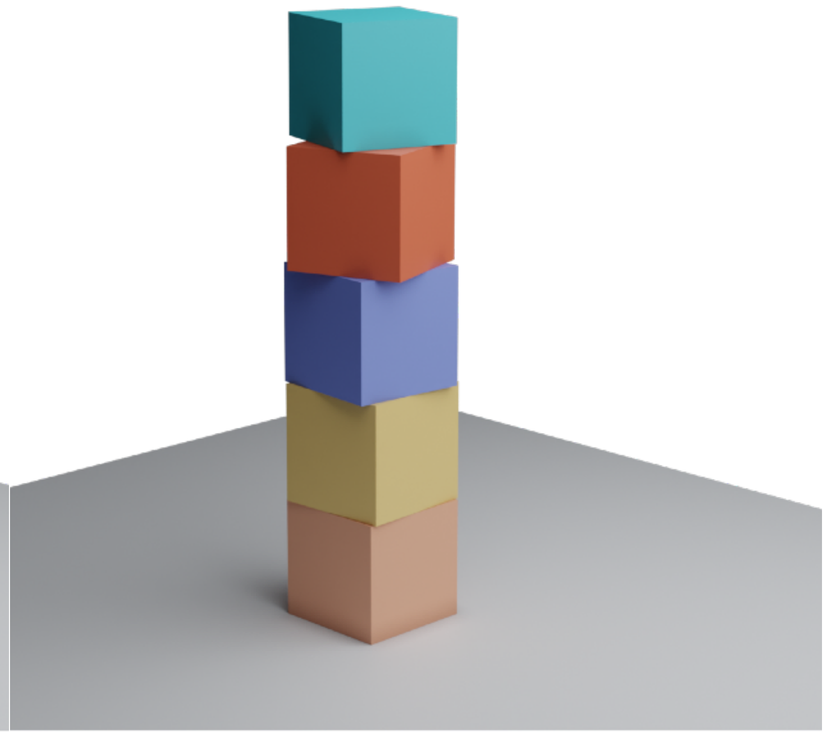
Mujoco



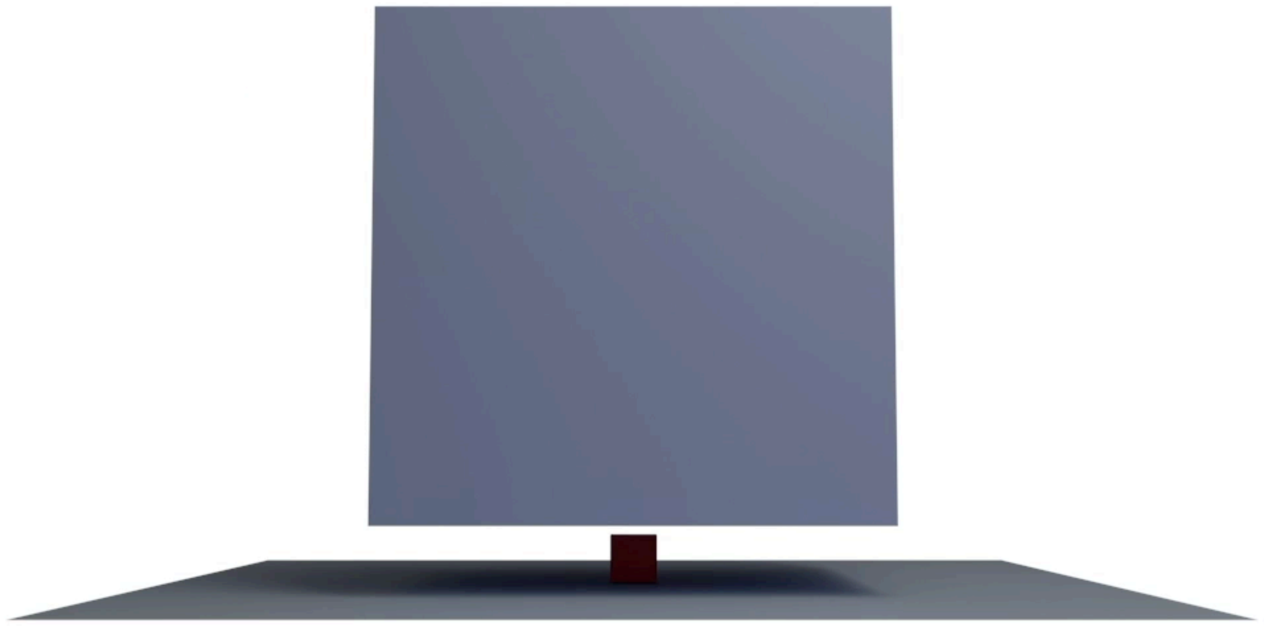
Solution



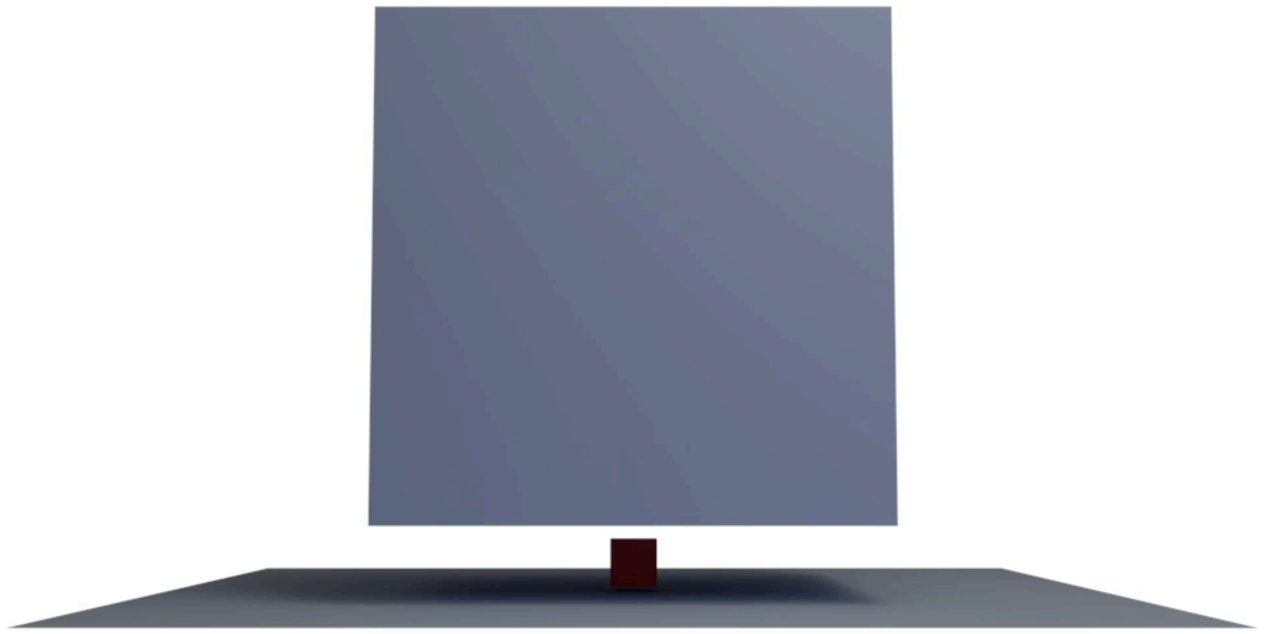
Chrono



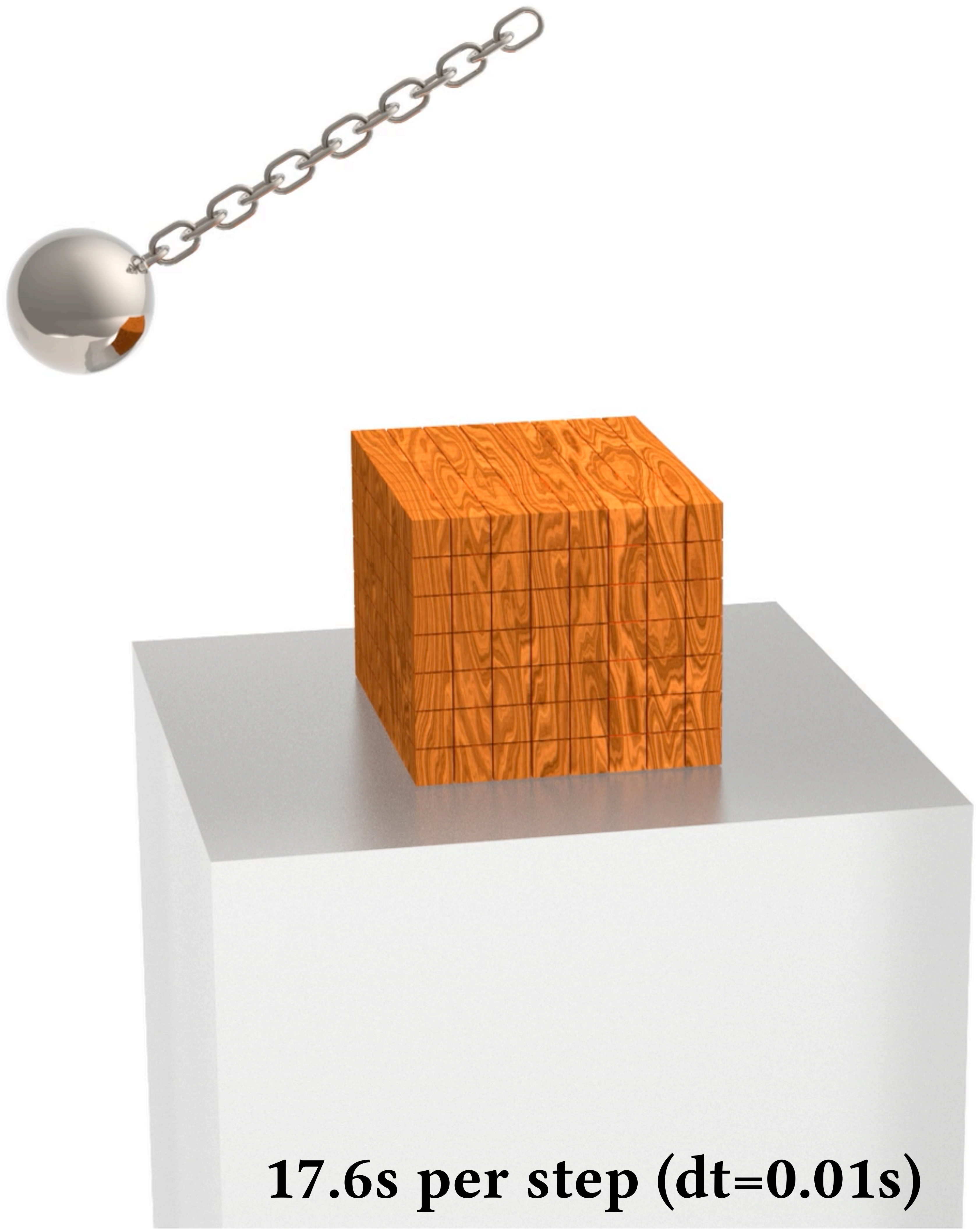
Solution

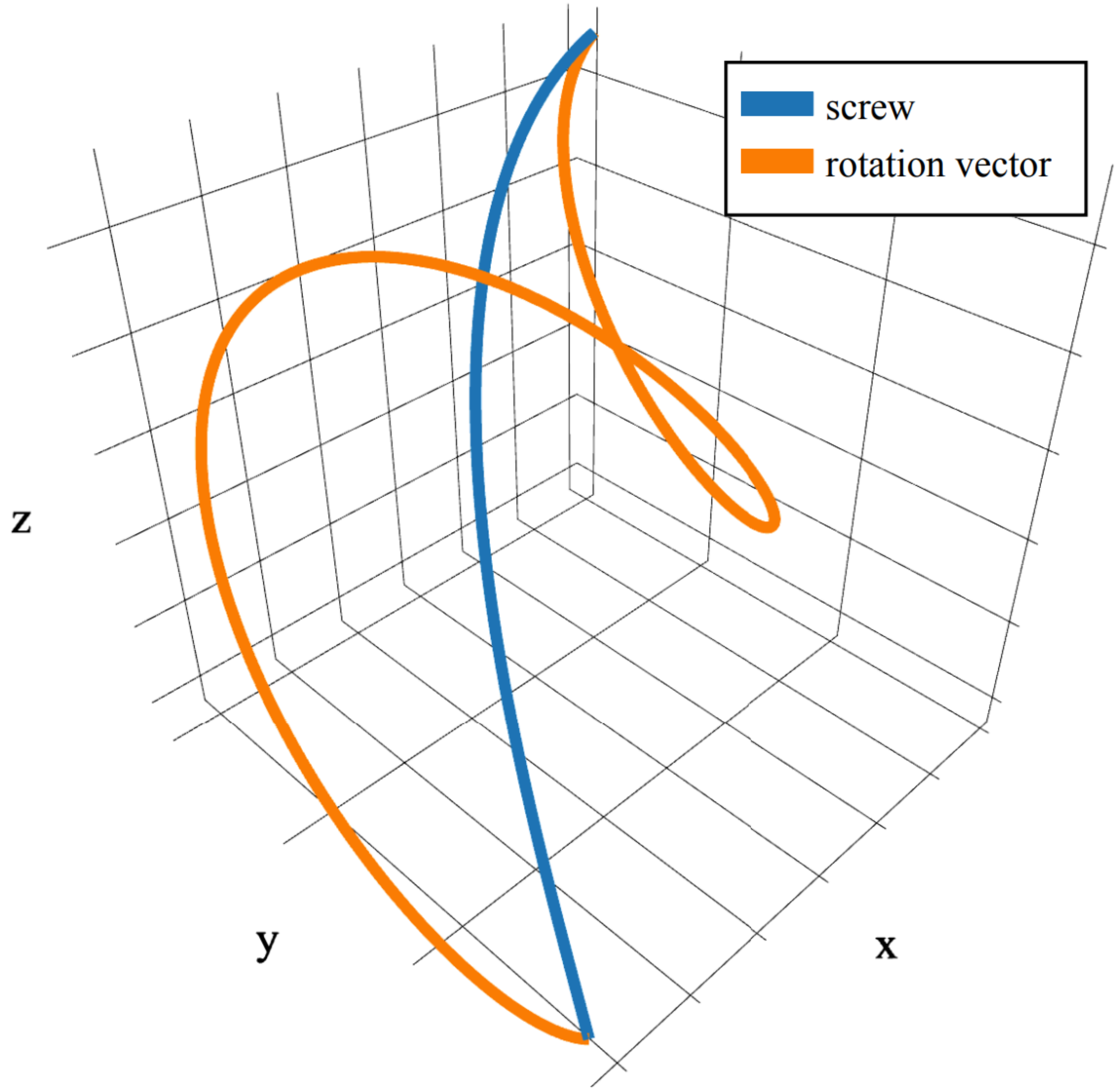


Bullet



Solution





Collision Detection on nonlinear trajectories: Expensive!

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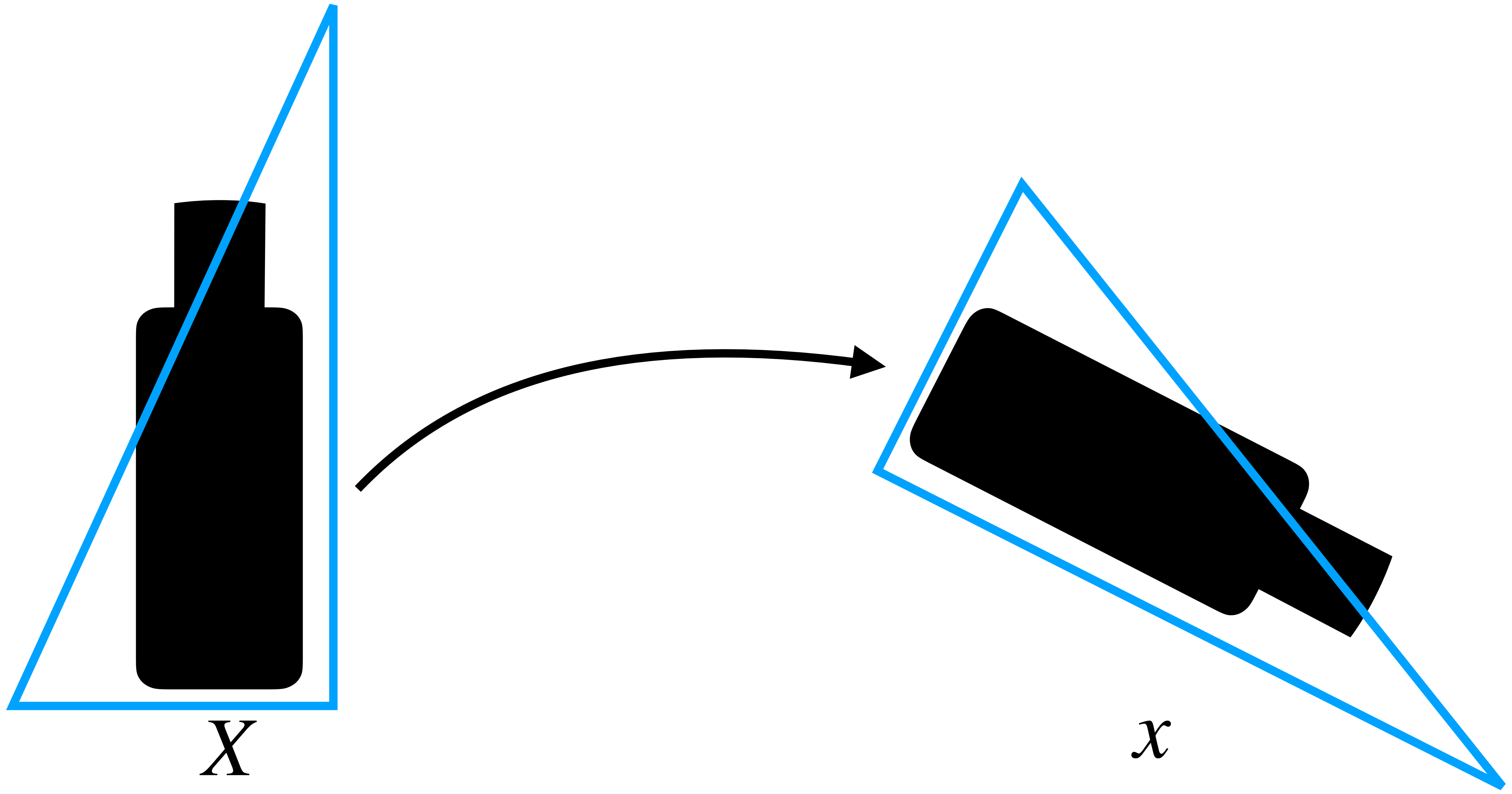
Lei Lan, Danny M. Kaufman, Minchen Li, Chenfanfu Jiang, Yin Yang. Affine Body Dynamics: Fast, Stable & Intersection-free Simulation of Stiff Materials. SIGGRAPH 2022.

ABD Formulation

Goal: Robust contact with IPC, as efficient as Bullet!

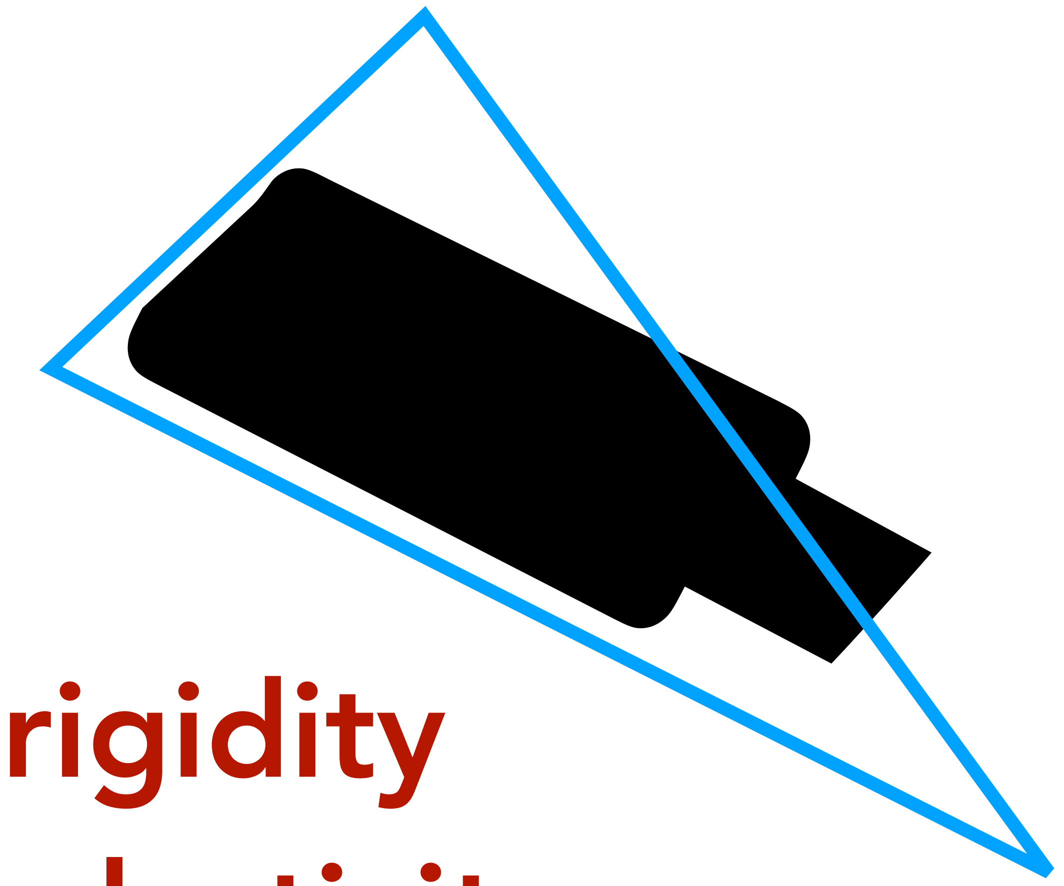


$$x_i - x_0 = A(X_i - X_0)$$

$$x_0 = X_0 + a$$

A: **Affine** matrix — **9 DOFs**

a: Translation vector — **3 DOFs**



Enforce rigidity through elasticity:

$$\Phi(x^{n+1}) = \frac{Y}{8}\|A^T A - I\|^2$$

$$\min_{x^{n+1}} \frac{1}{2}\|x^{n+1} - \tilde{x}^n\|_M^2 + h^2 \sum P(x^{n+1})$$

Rigid IPC v.s. ABD

Rigid IPC
- **Rigidity**: Enforced exactly

ABD
- **Rigidity**: Applied with stiff potential

Rigid IPC v.s. ABD

Rigid IPC

- **Rigidity** — Enforced exactly
- **Collision** — Handled with barrier potential

ABD

- **Rigidity** — Applied with stiff potential
- **Collision** — Handled with barrier potential

→

1000x Speedup over Rigid-IPC

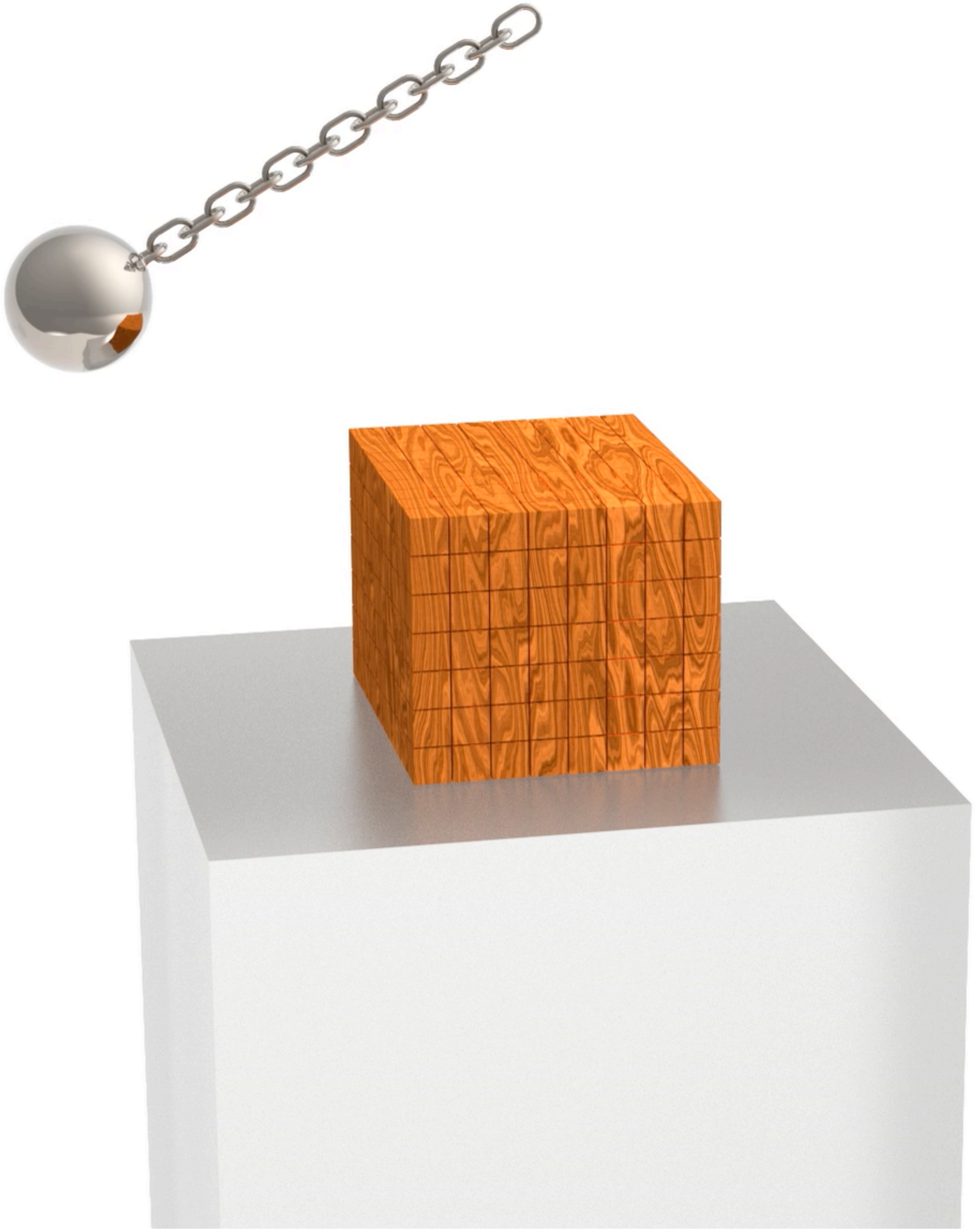
Several times faster than Bullet with guarantees of stability and nonintersection.

Comparison with Rigid-IPC

Wrecking Ball

Rigid-IPC

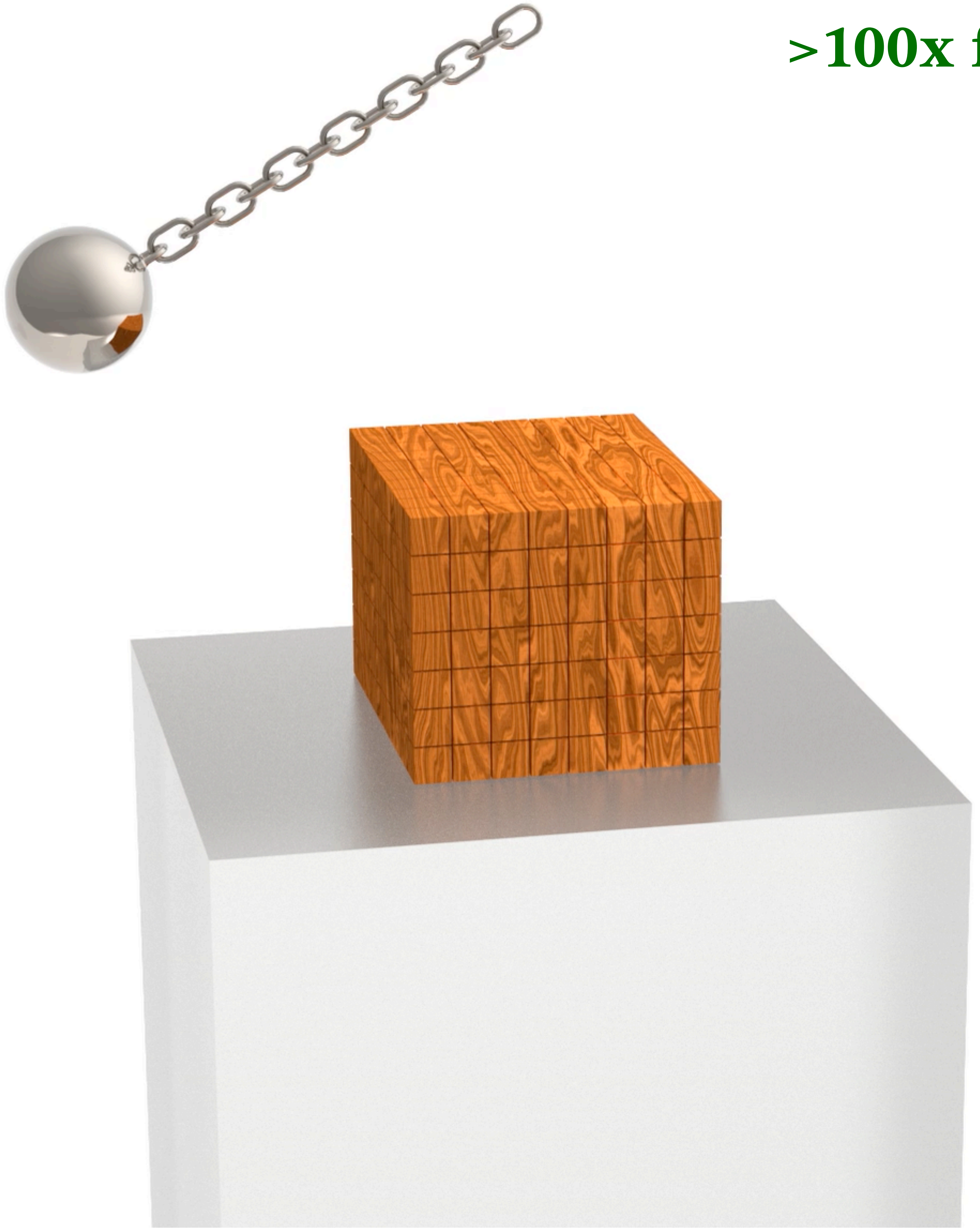
17.6s per step (dt=0.01s)



ABD

0.14s per step (dt=0.01s)

>100x faster



14K triangles

575 bodies

House of Cards

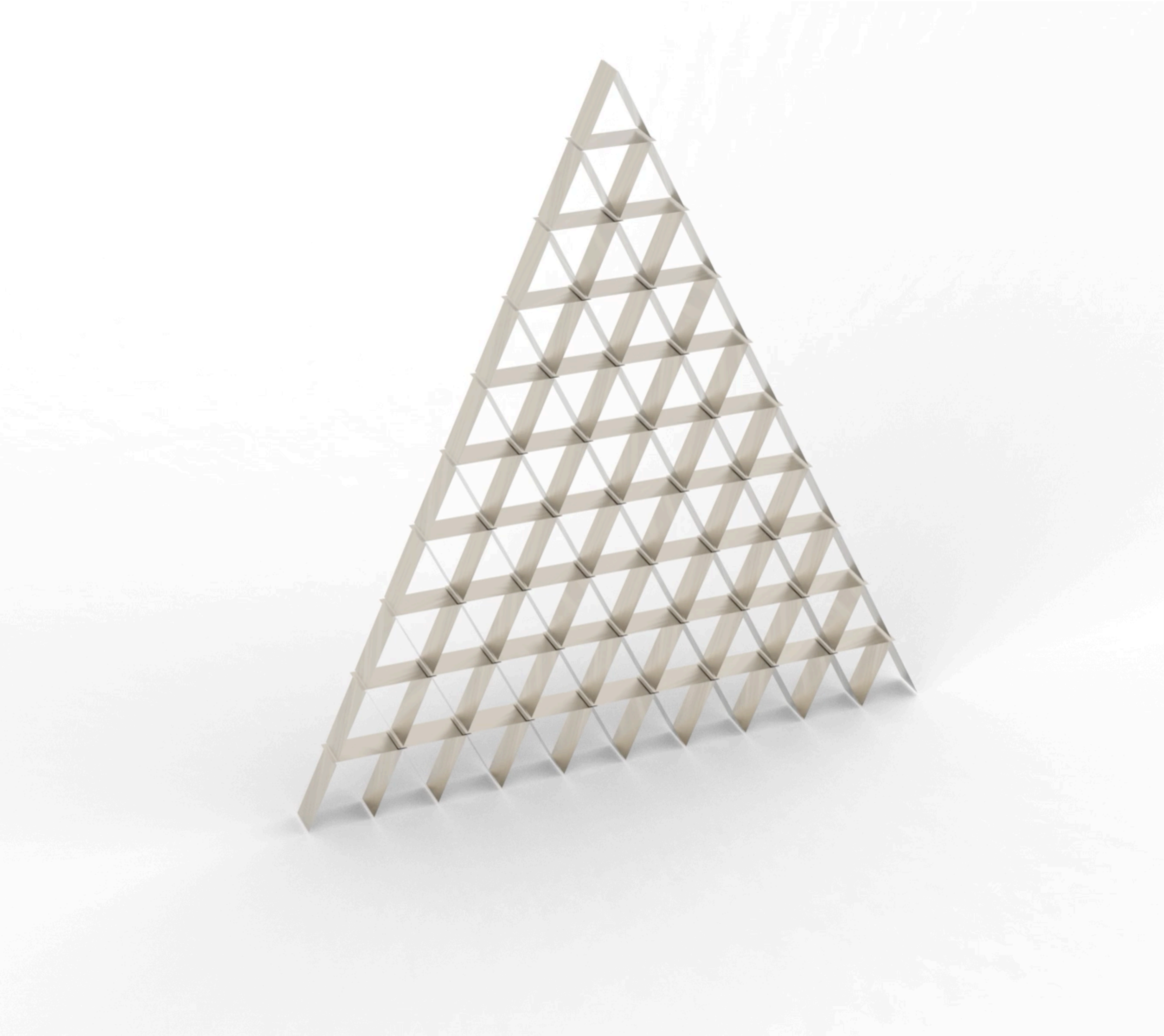
Rigid-IPC

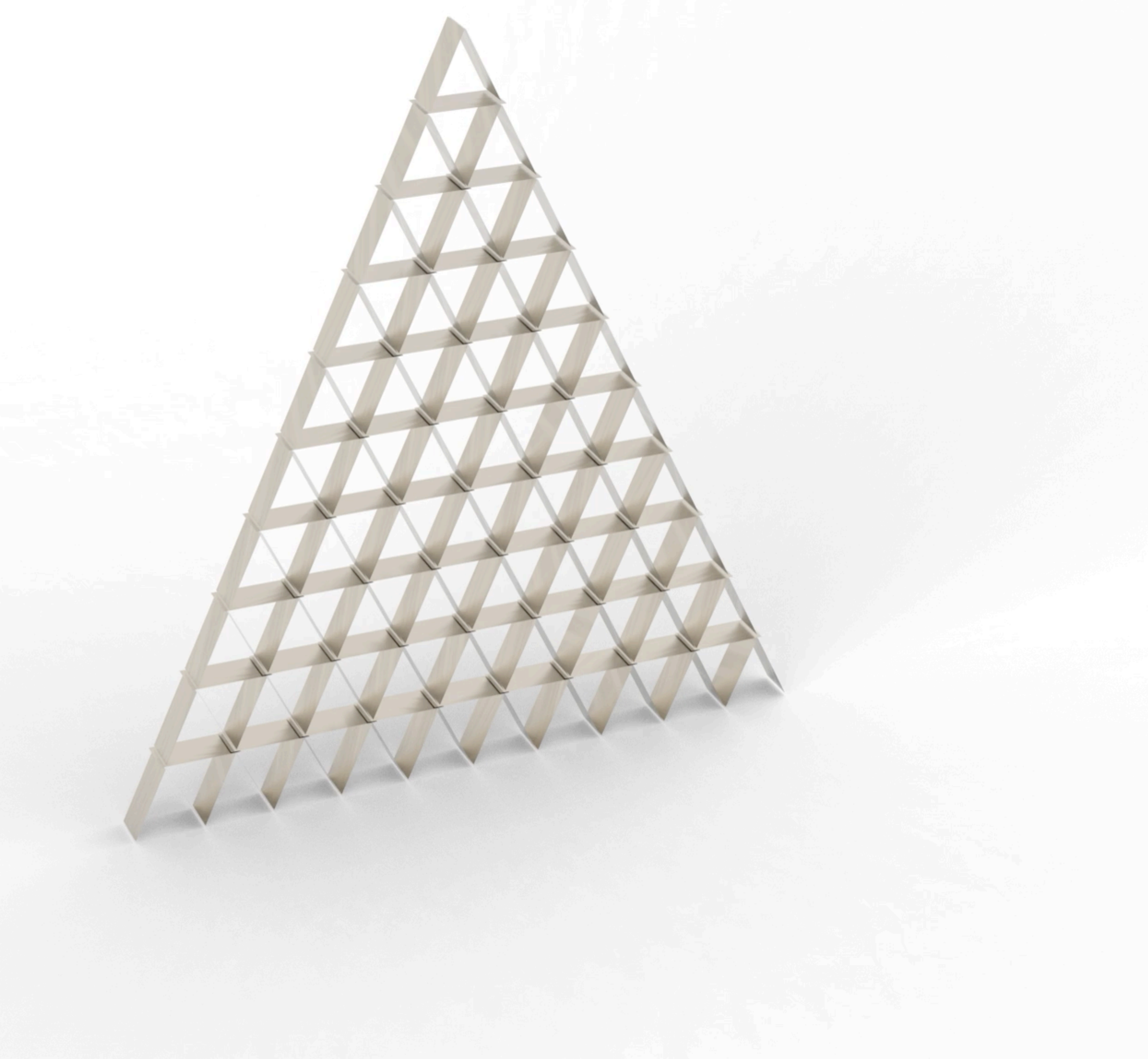
8.9s per step (dt=0.01s)

ABD

0.086s per step (dt=0.01s)

>100x faster





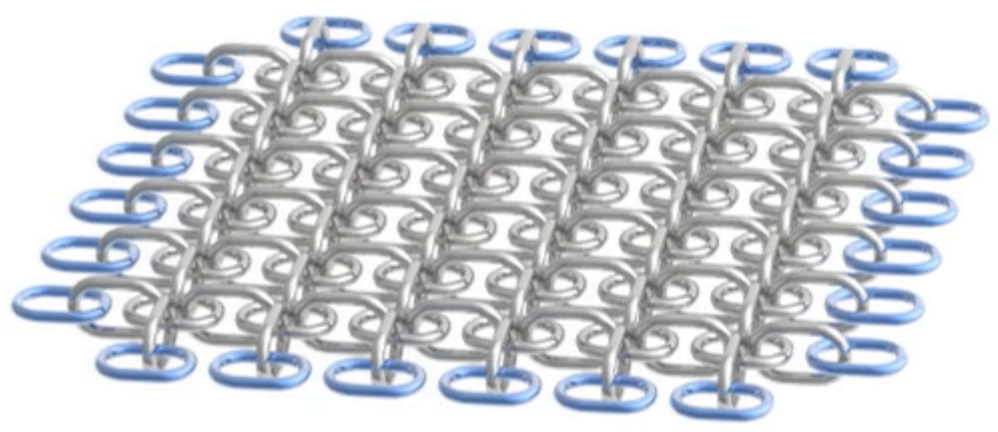
336 triangles

158 bodies

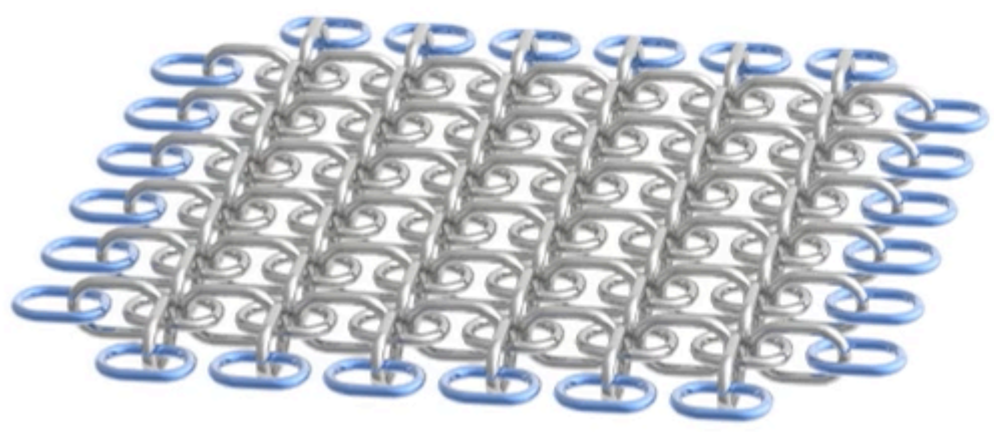
Scalability

8x8 Chain Net

Rigid-IPC
2.1s per step (dt=0.01s)



ABD
0.06s per step (dt=0.01s)
>30x faster



63K triangles
114 bodies

16x16 Chain Net

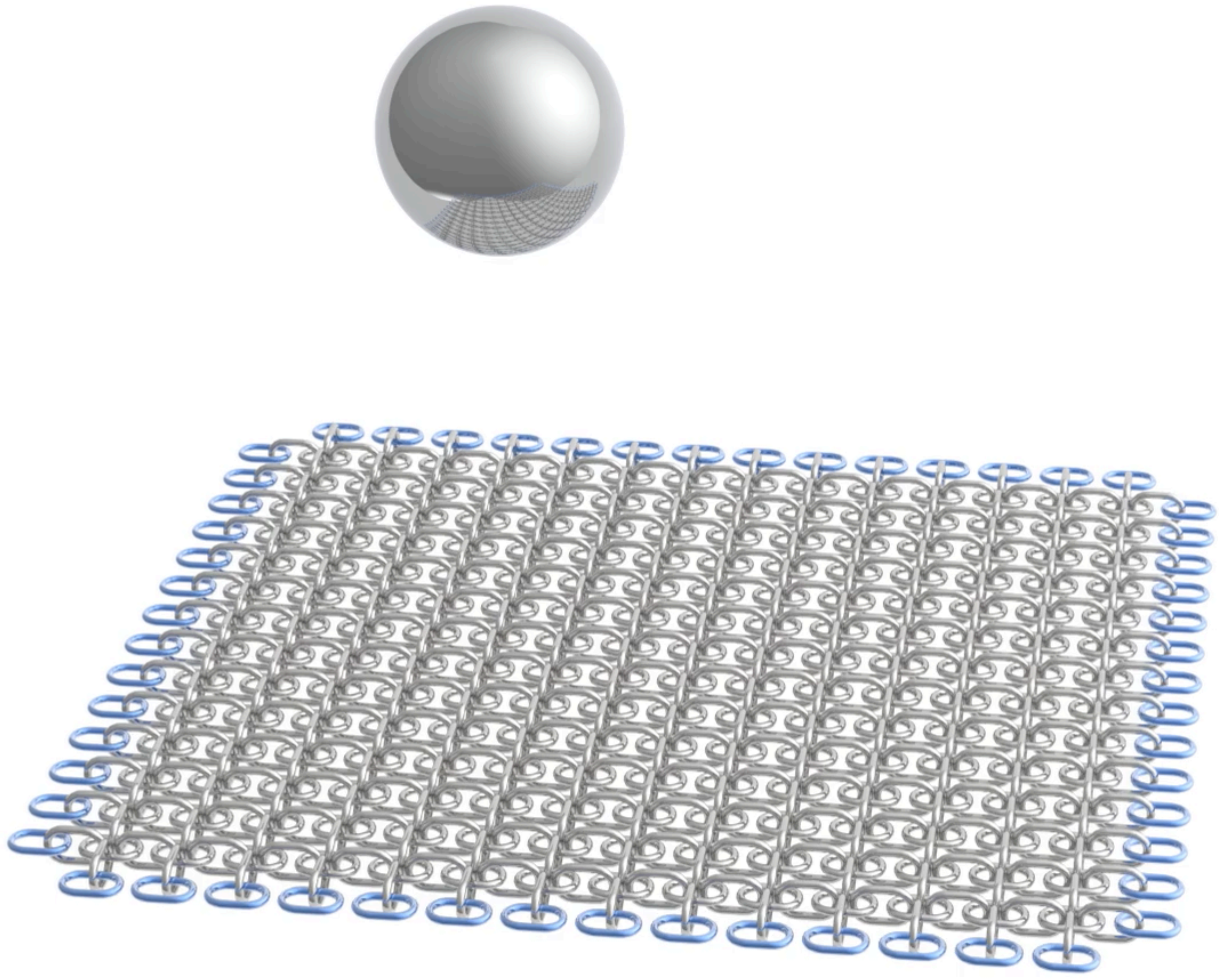
Rigid-IPC

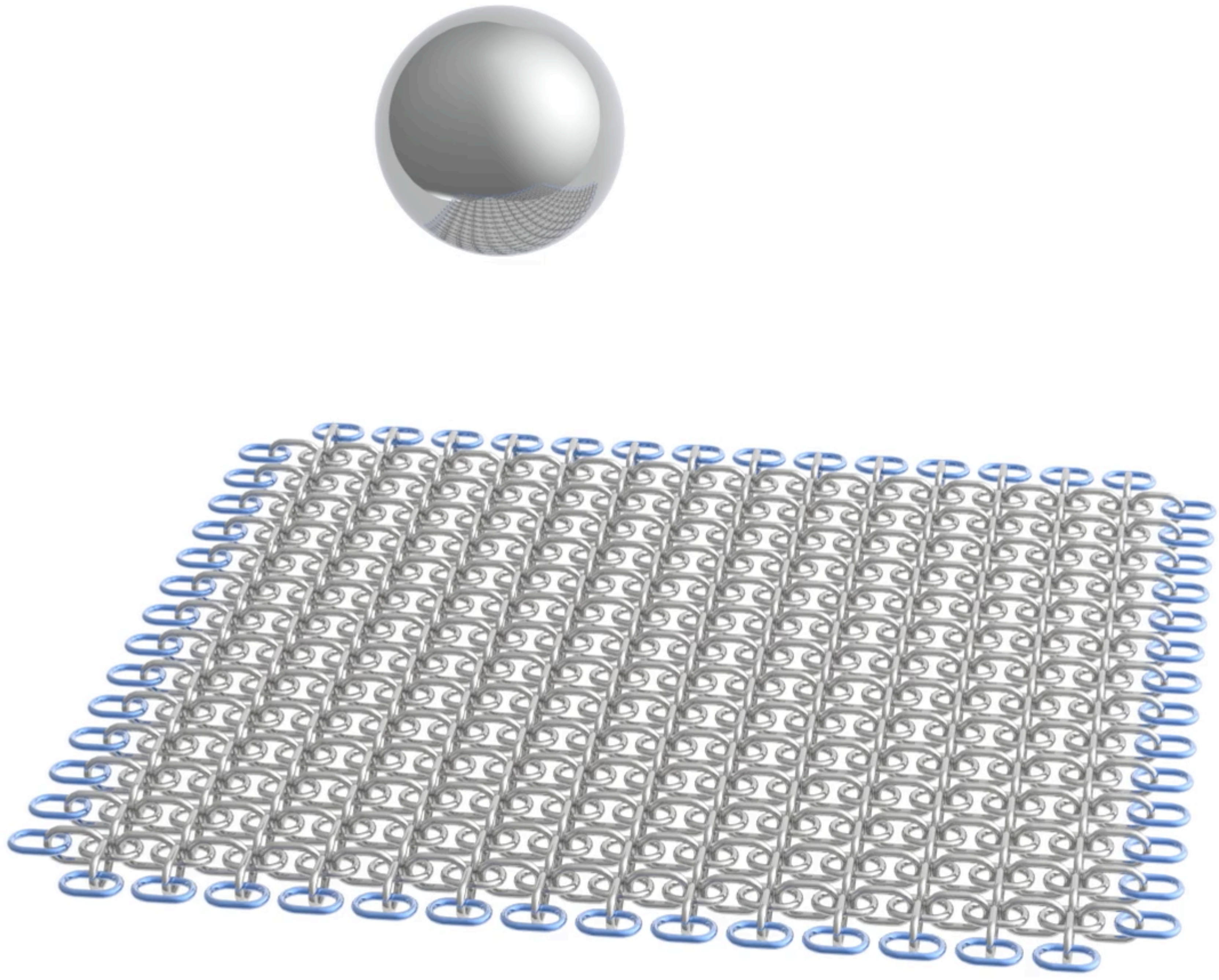
804s per step (dt=0.01s)

ABD

1.2s per step (dt=0.01s)

>1200x faster

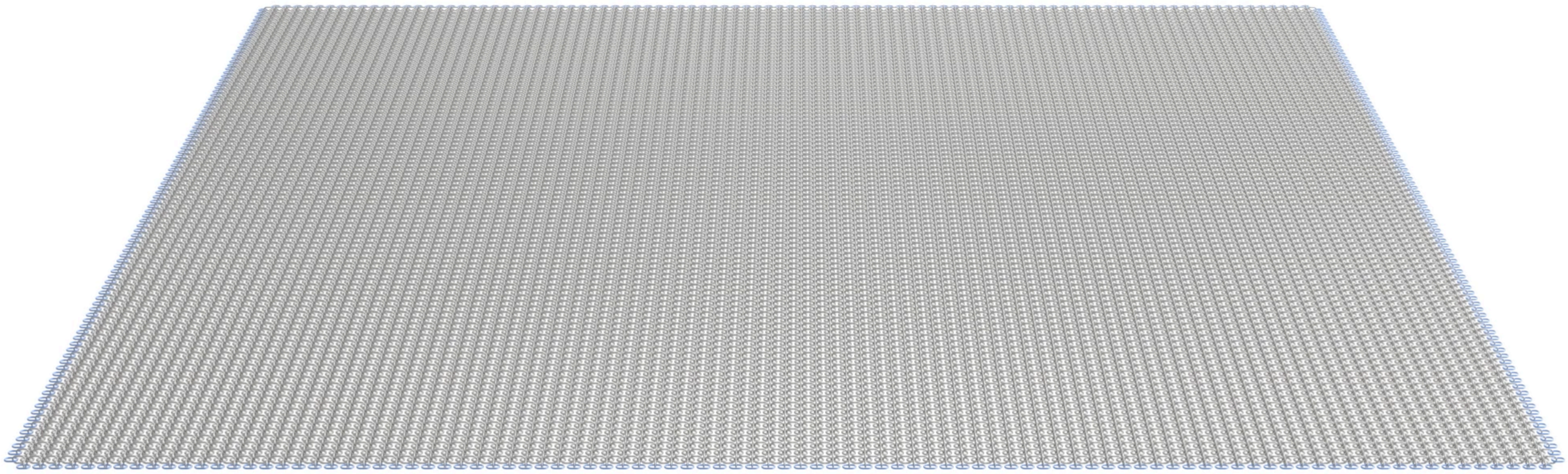




445K triangles

673 bodies

96x96 Chain Net



ABD Simulation

12M triangles

28K bodies

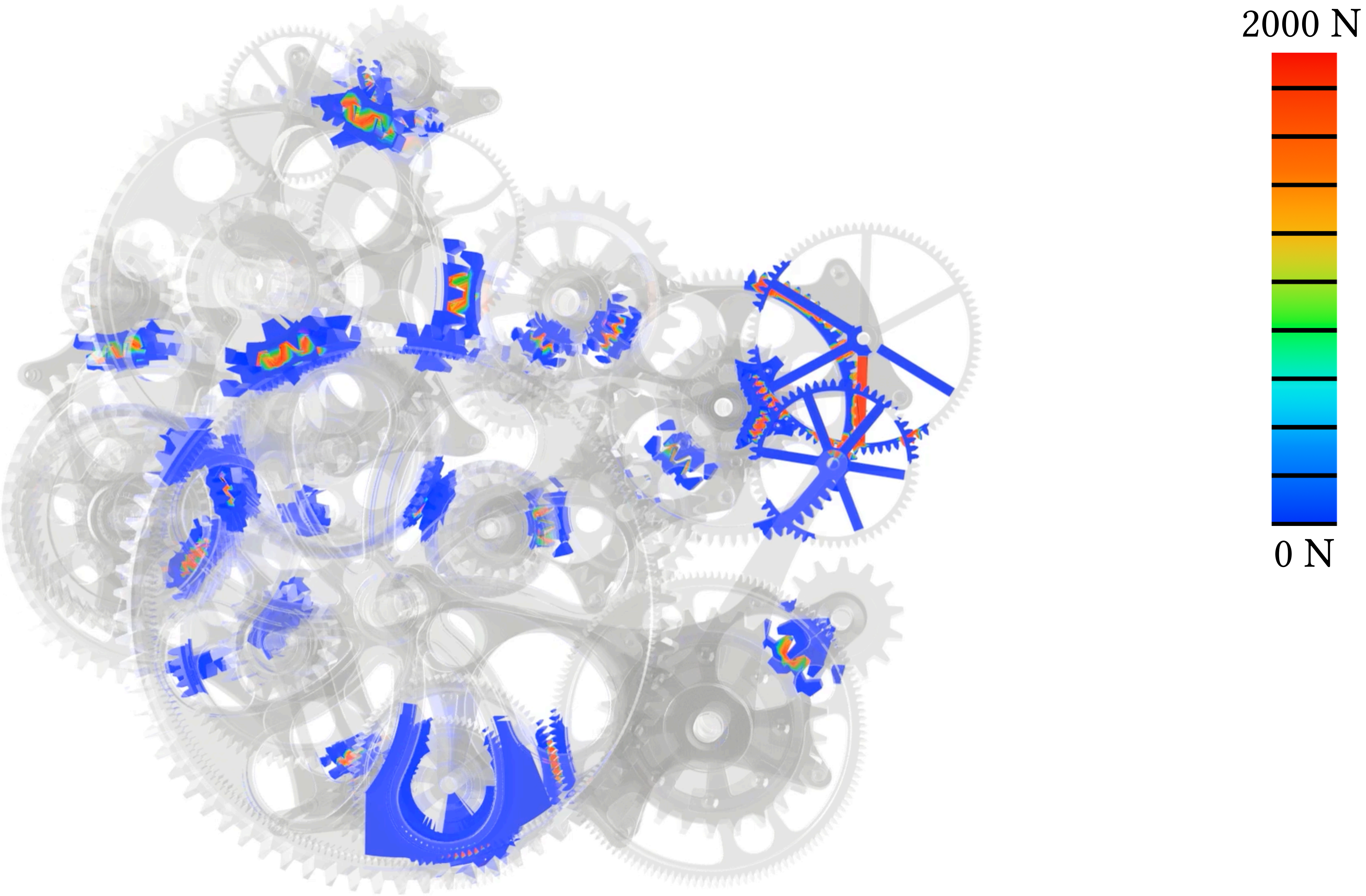
310s per step (dt=0.01s)

Gears

ABD Simulation
2.5M triangles
28 bodies
11.7s per step (dt=0.01s)

On GPU: 85x faster!

Gears - Contact Force Visualization

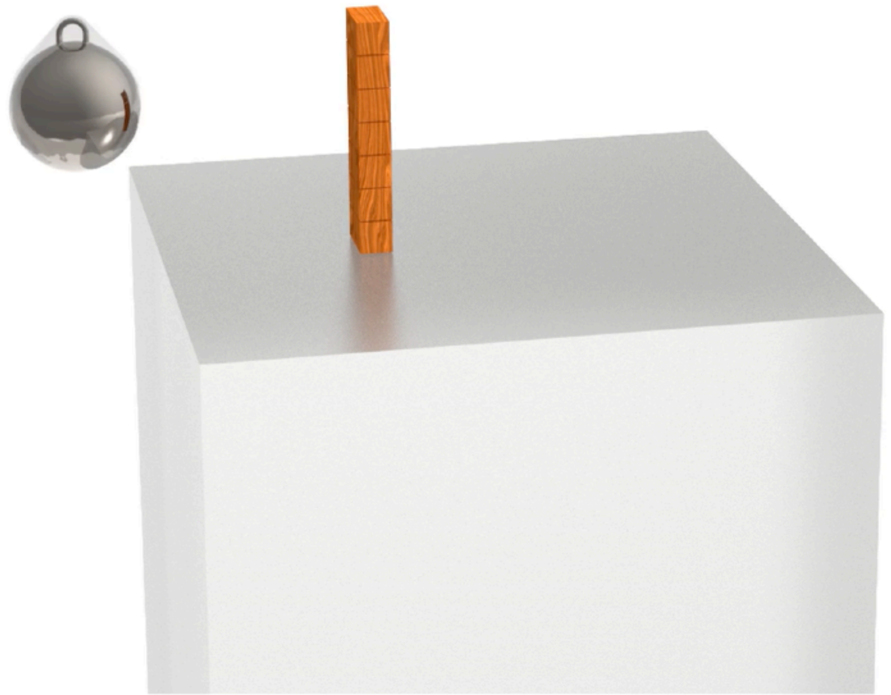


ABD Simulation
2.5M triangles
28 bodies
11.7s per step (dt=0.01s)

Comparison with Bullet

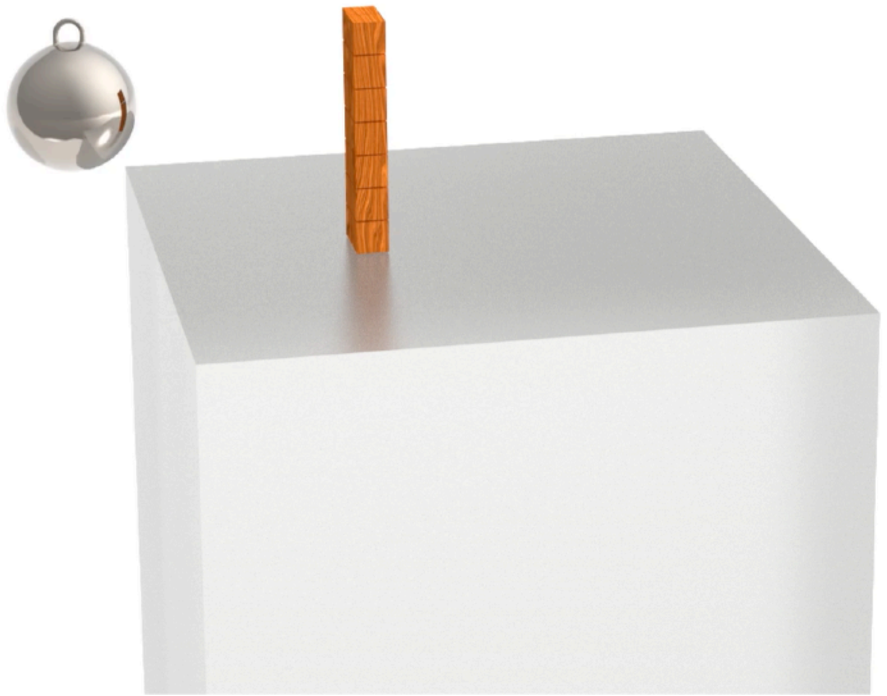
Comparison with Bullet

1.2K triangles
16 bodies



Bullet

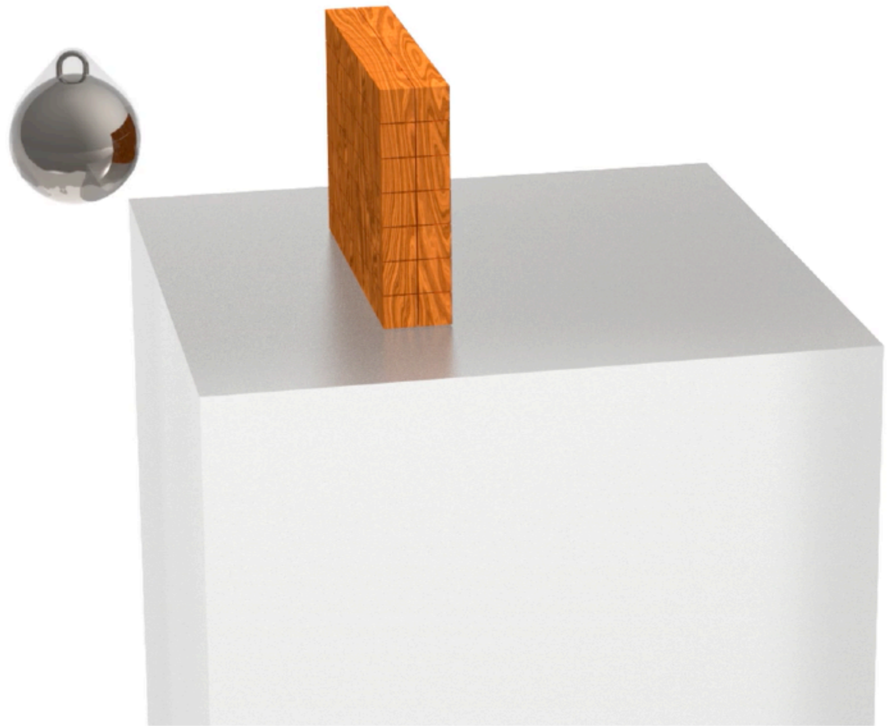
1.5ms per 1/240s step
3ms per 1ms step



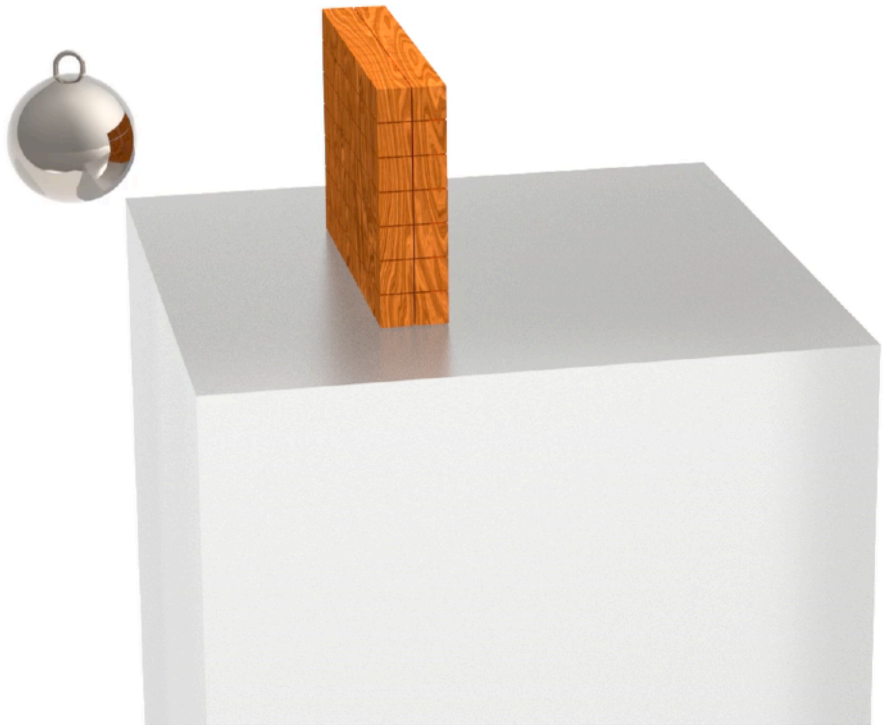
ABD

2.2ms per 1/240s step
2ms per 1ms step
Comparable

3.5K triangles
142 bodies

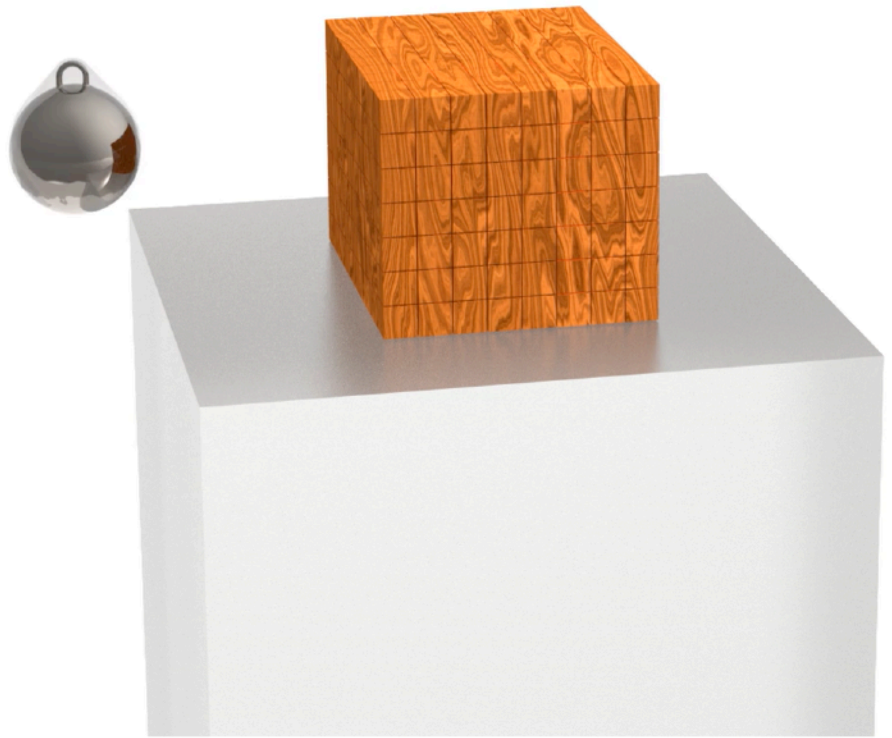


58ms per 1/240s step
82ms per 1ms step

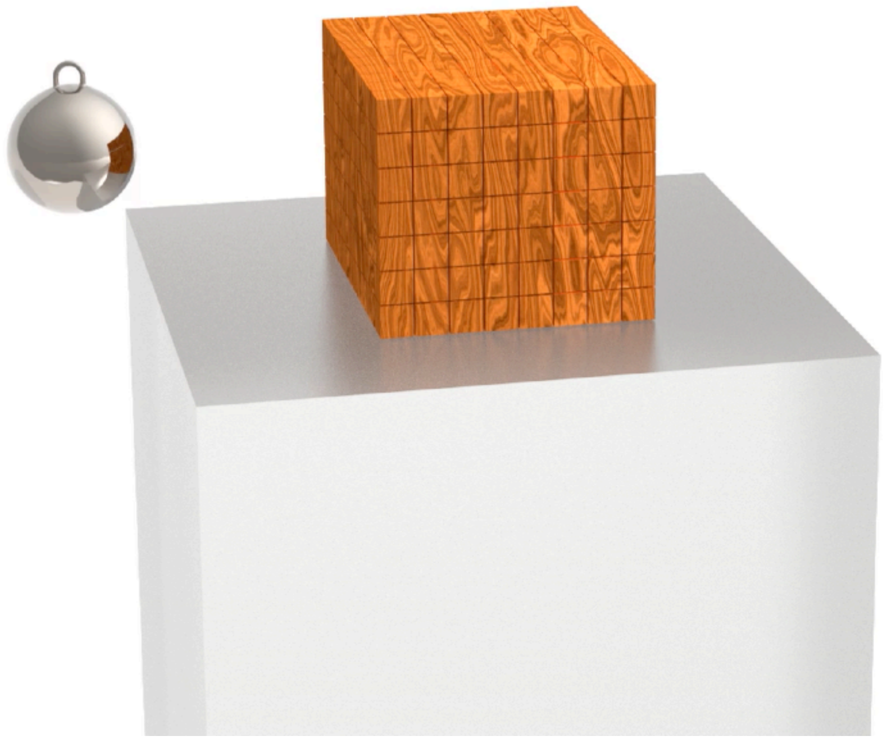


41ms per 1/240s step
19ms per 1ms step
>4x faster

11K triangles
562 bodies



809ms per 1/240s step
804ms per 1ms step



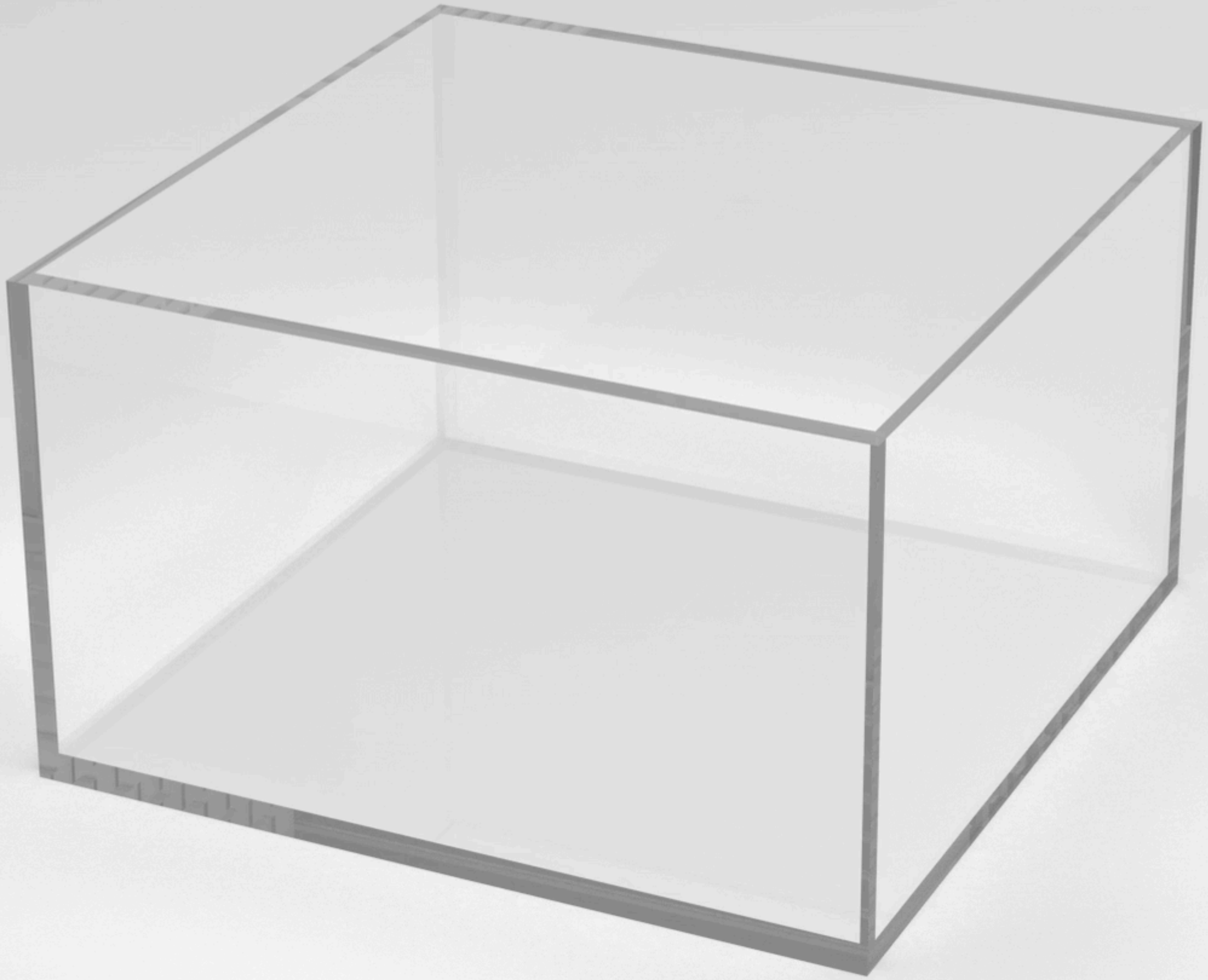
328ms per 1/240s step
102ms per 1ms step
>8x faster

Bullet

1.2M triangles
29 bodies

ABD
2.8s per step (dt=0.01s)

ABD Coupling with FEM Deformable Bodies



1.1M triangles
27 bodies
16.4s per step (dt=0.01s)

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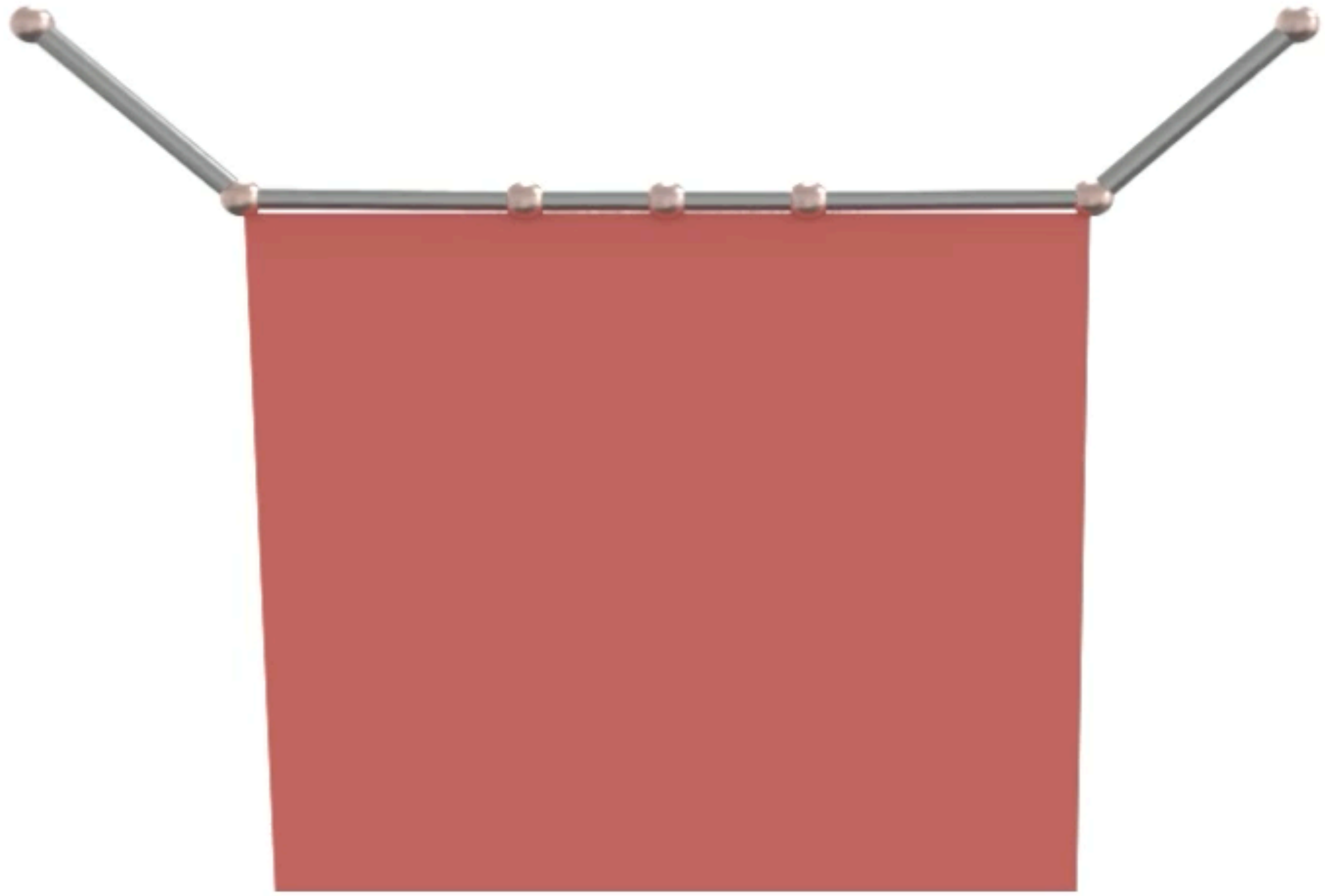
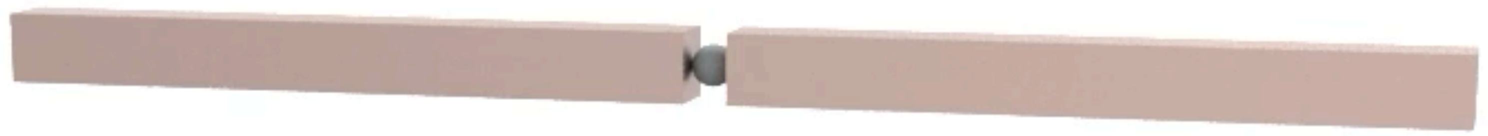
Yunuo Chen*, Minchen Li*, Lei Lan, Hao Su, Yin Yang, Chenfanfu Jiang. A Unified Newton Barrier Method for Multibody Dynamics. SIGGRAPH 2022.

Point Connection Constraints

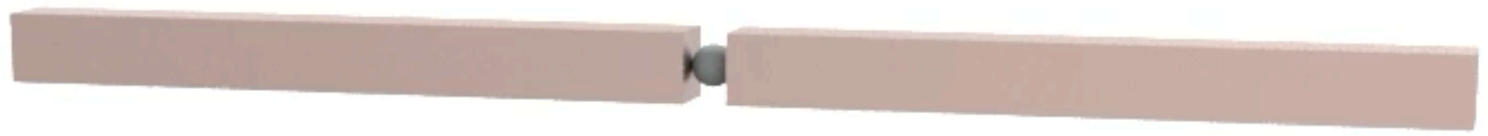


Sliding & Hinge Constraints

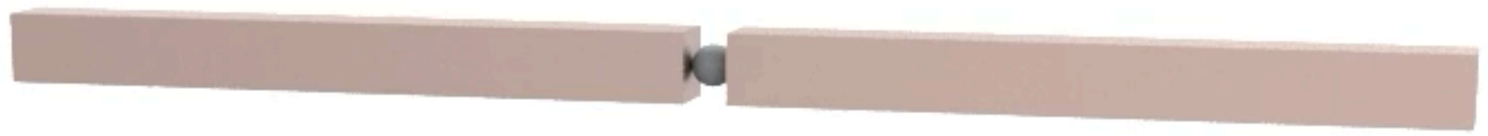
Cone Twist Constraints



Bending 1



Bending 2



Twisting

Articulation Constraints & Formulation

Linear Equality	Nonlinear Equality	Inequality
Point Connection	Distance	Bounded Distance
Hinge	Relative Sliding	Sliding Range
Cone Twist		Rotation Range

Sequential Quadratic Programming (SQP): no global convergence

Line search on constrained minimization?

Articulation Constraints & Formulation

Linear Equality	Nonlinear Equality	Inequality
Point Connection	Distance	Bounded Distance
Hinge	Relative Sliding	Sliding Range
Cone Twist		Rotation Range

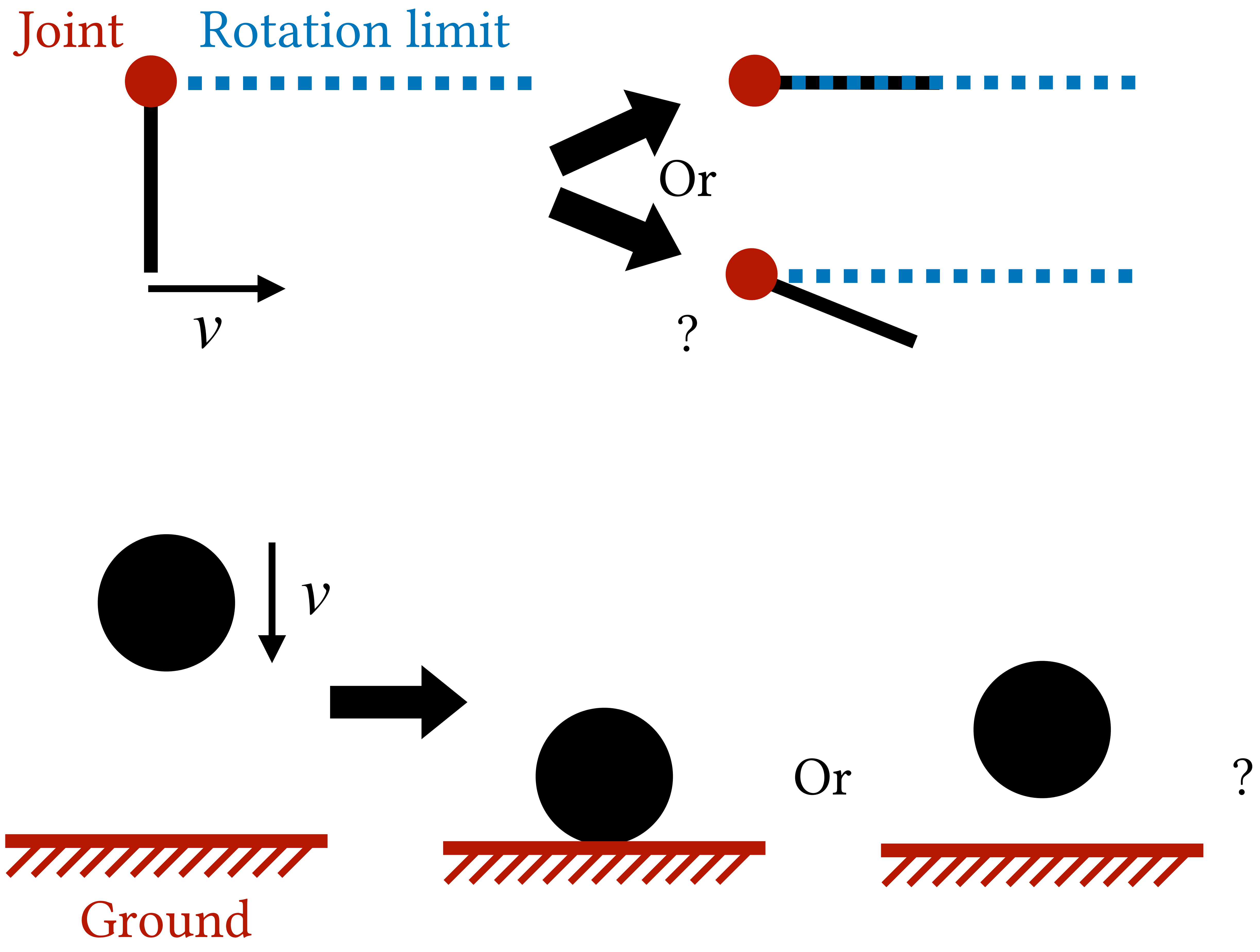
Change-of-variables

Stiff Penalty

Barrier Potential

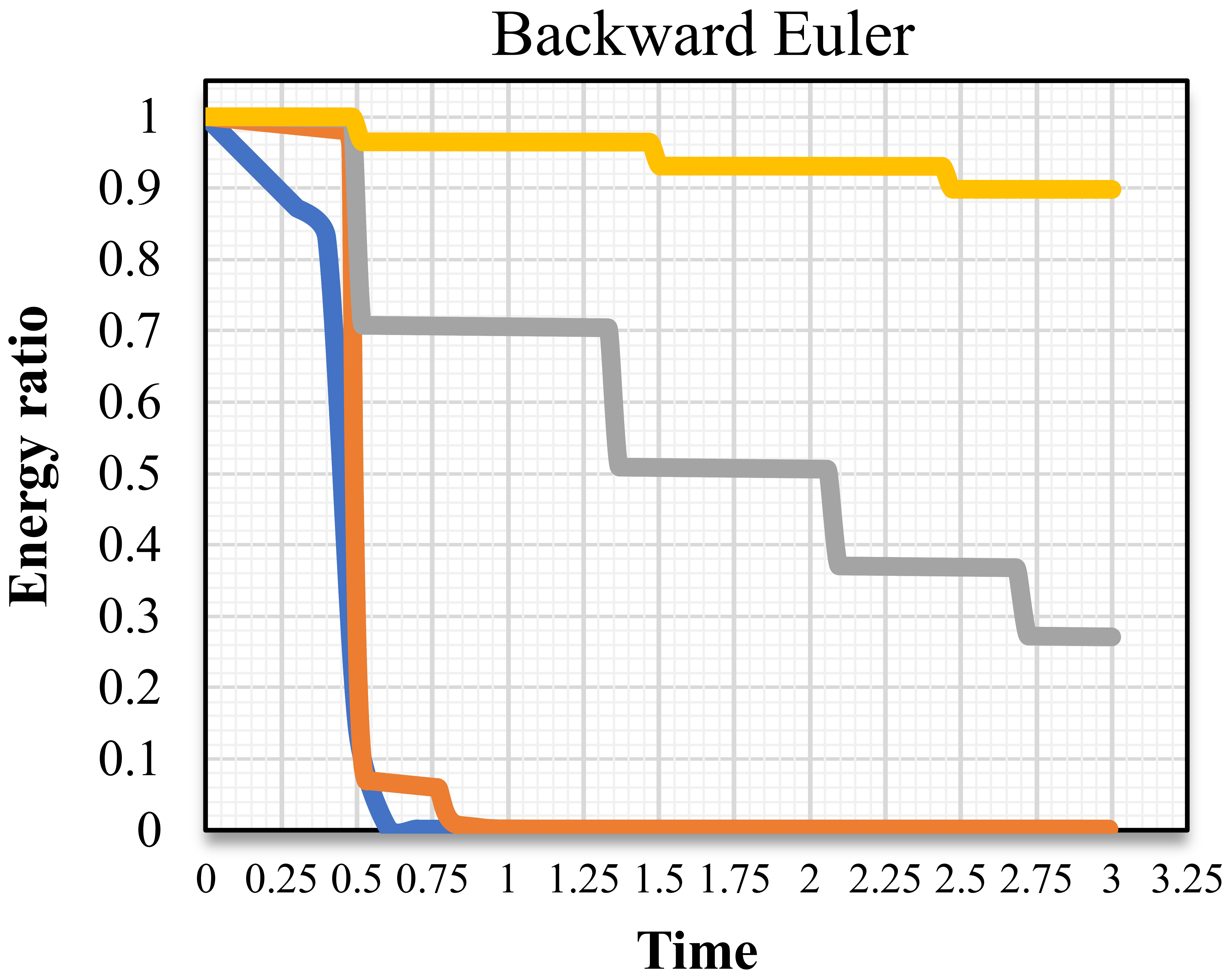
$$\min_{x^{n+1}} \frac{1}{2}\|x^{n+1} - \tilde{x}^n\|_M^2 + h^2 \sum P(x^{n+1})$$

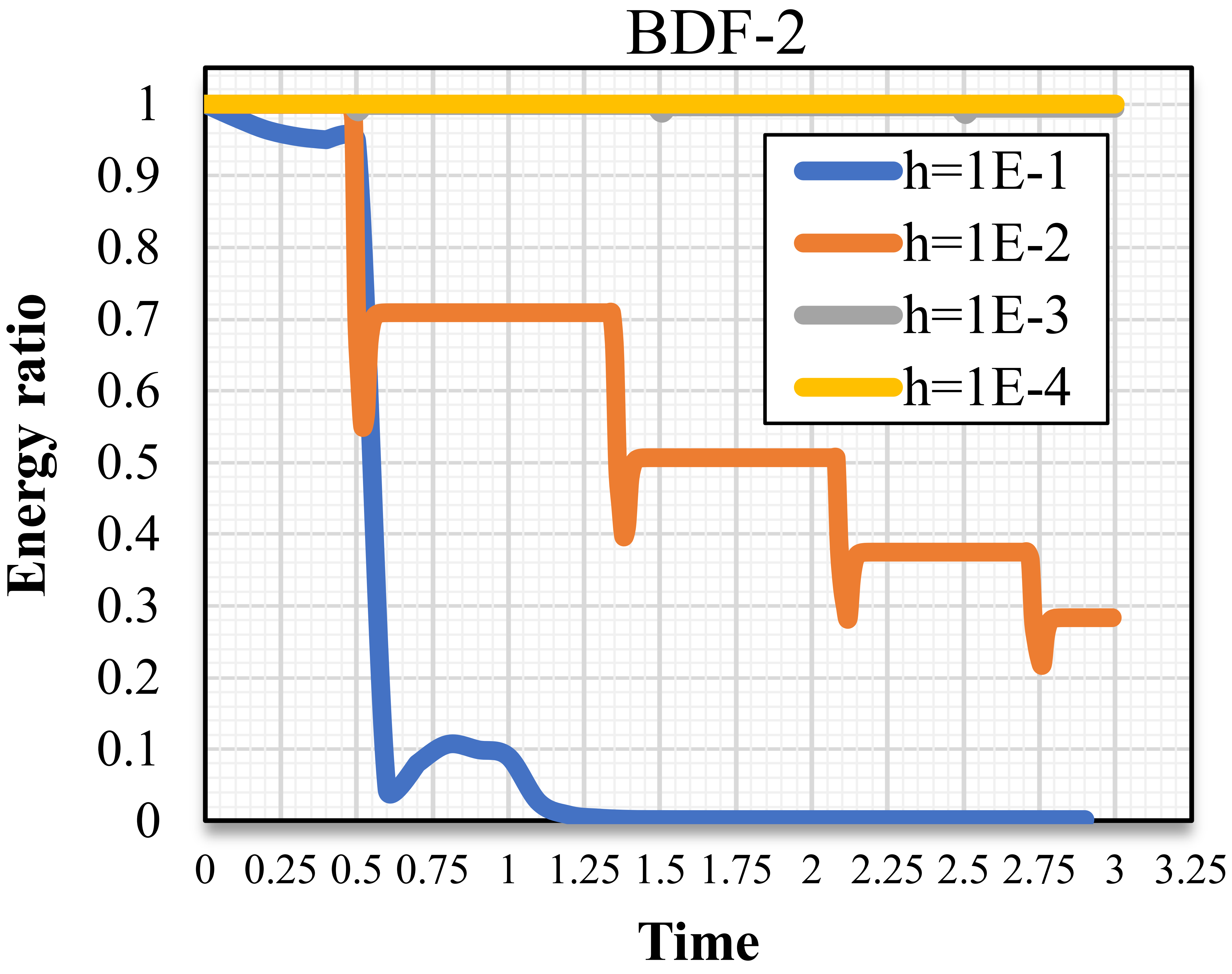
Restitution



Restitution ⇔ Energy Conservation of Time Stepping

Use high-order rules, e.g. BDF-2: $\min_{x^{n+1}} \frac{1}{2}\|x^{n+1} - \tilde{x}^n_{BDF2}\|^2_M + \frac{4}{9}h^2 \sum P(x^{n+1})$



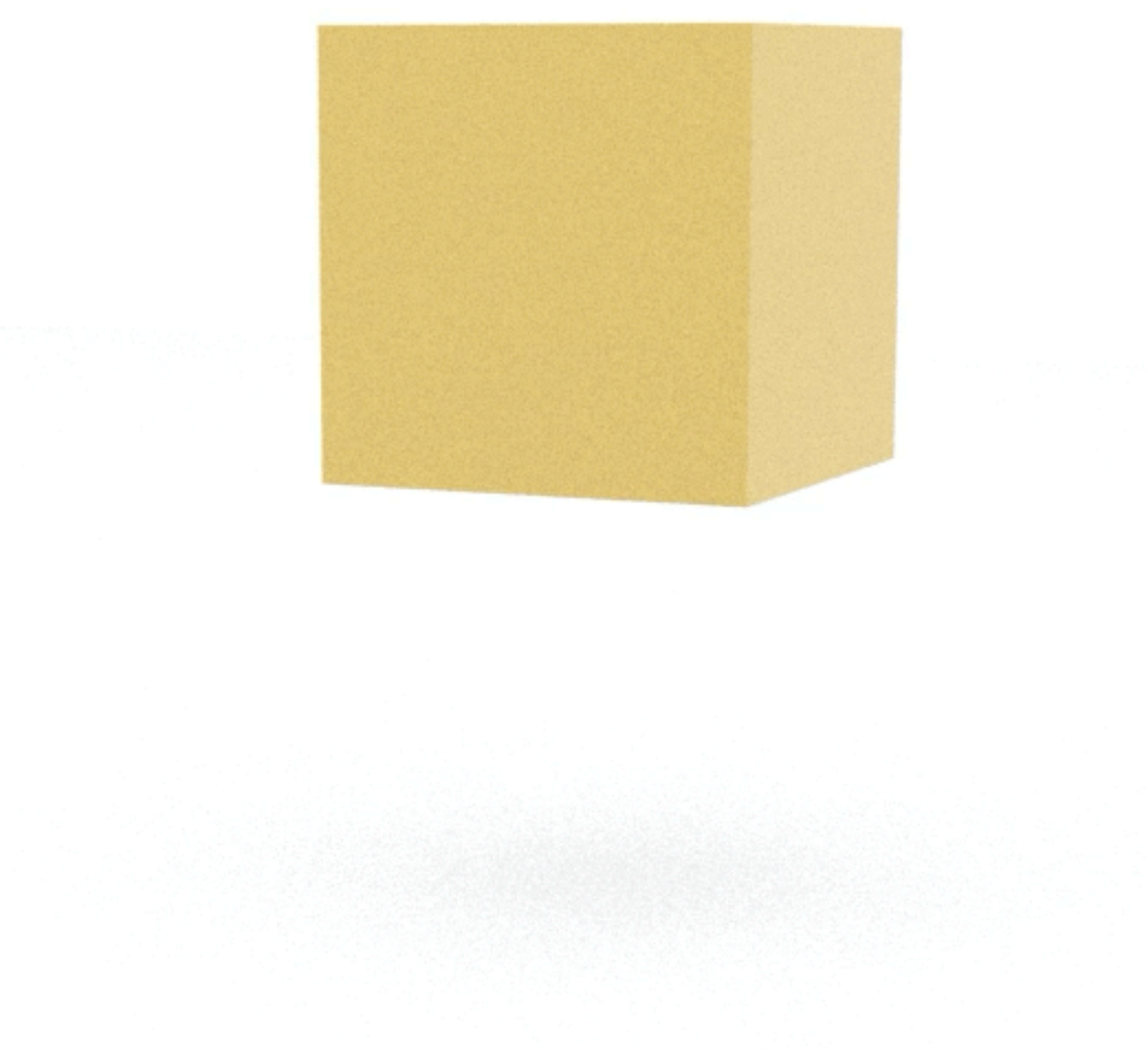


Perfectly Elastic Restitution

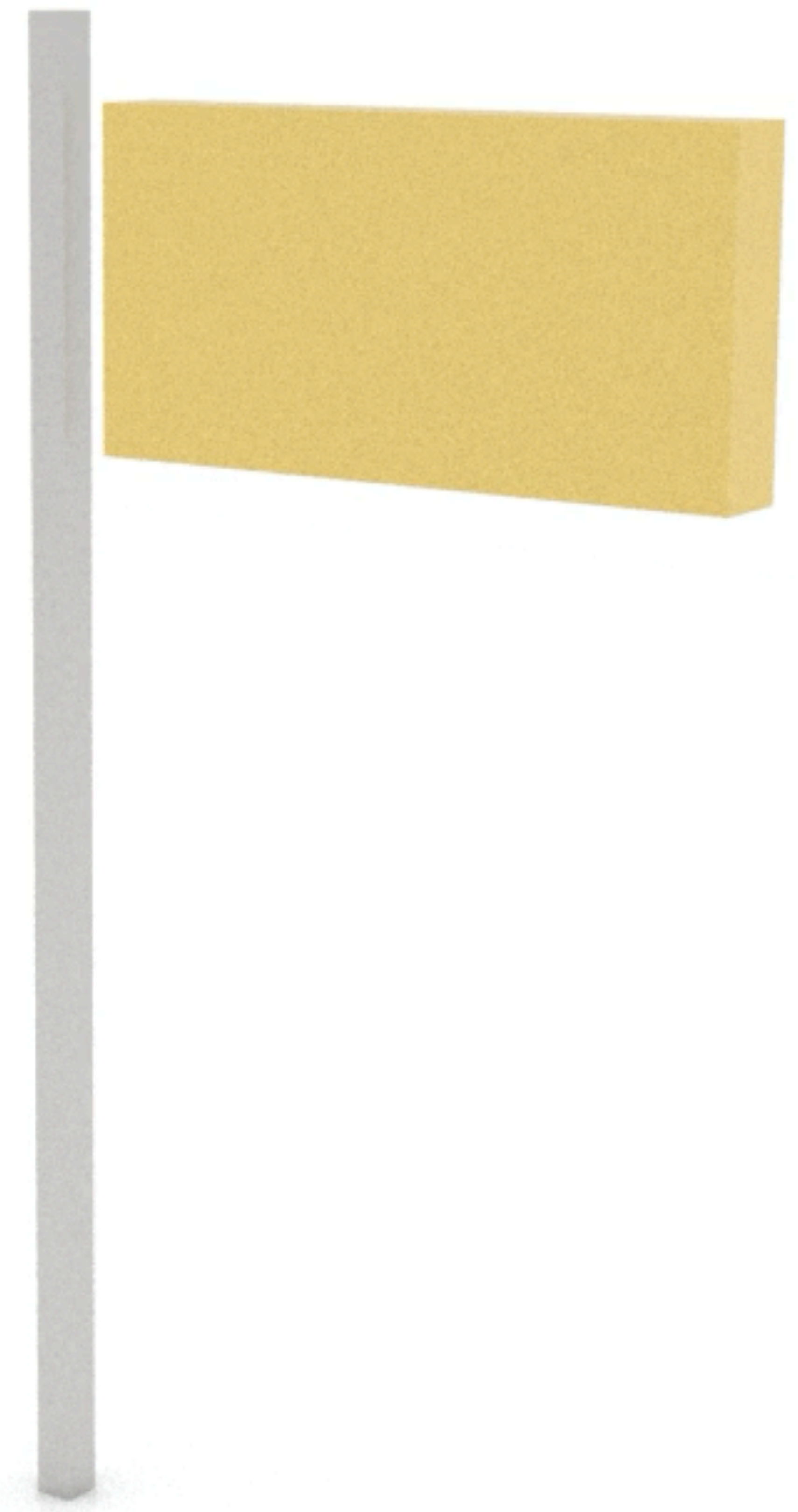
Restitution ⇔ Energy Conservation of Time Stepping

Use high-order rules, e.g. BDF-2:

$$\min_{x^{n+1}} \frac{1}{2}\|x^{n+1} - \tilde{x}^n_{BDF2}\|^2_M + \frac{4}{9}h^2 \sum P(x^{n+1})$$



Contact



Hinge Constraint

Use lagged Rayleigh Damping:

$$\frac{k_d}{2} v^T \frac{\partial^2 B_{contact}}{\partial x^2}(x^n) v$$

$$\min_{x^{n+1}} \frac{1}{2} \| x^{n+1} - \tilde{x}^n_{BDF2} \|^2_M + \frac{4}{9} h^2 \sum P(x^{n+1})$$

$k_d=0$

$k_d=0.005$

$k_d=0.1$

Controllable Restitution: Hinge

Use lagged Rayleigh Damping:

$$\frac{k_d}{2} v^T \frac{\partial^2 B_{hinge}}{\partial x^2}(x^n) v$$

$$\min_{x^{n+1}} \frac{1}{2} \| x^{n+1} - \tilde{x}^n_{BDF2} \|^2_M + \frac{4}{9} h^2 \sum P(x^{n+1})$$

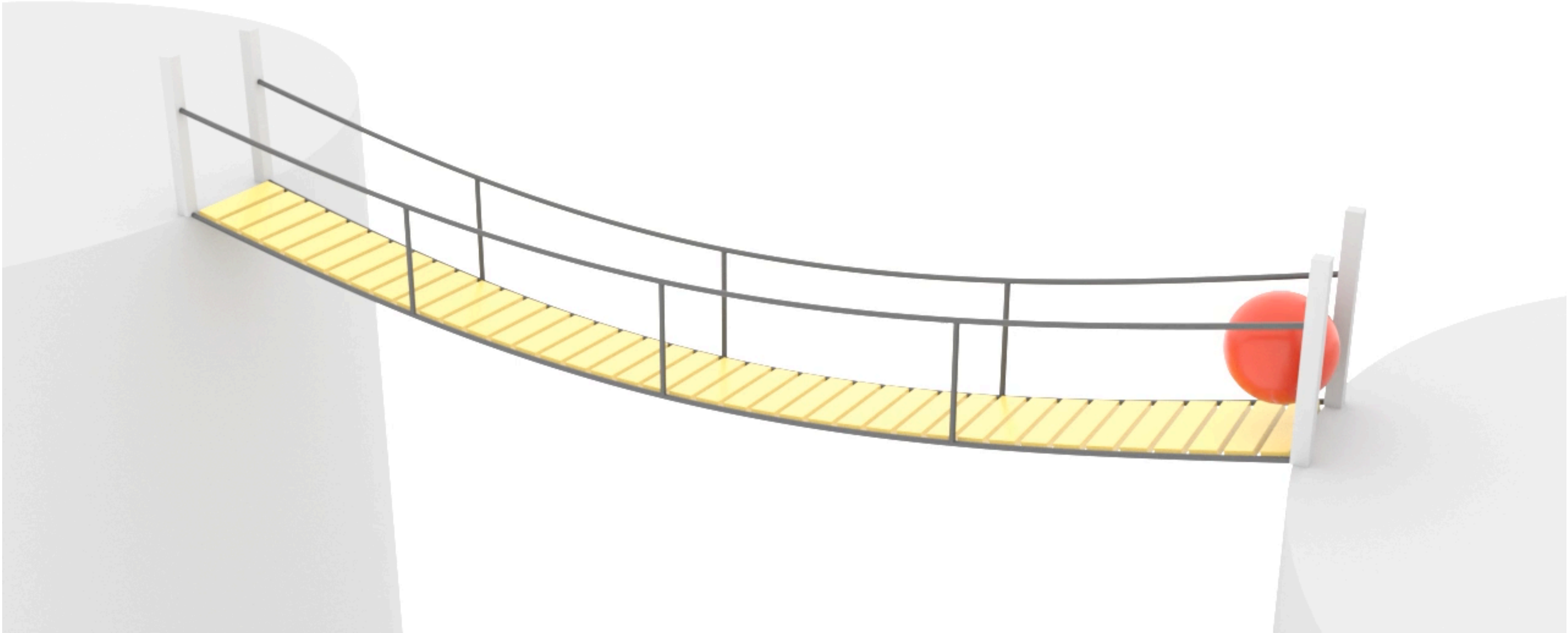
k_d=0

k_d=0.01

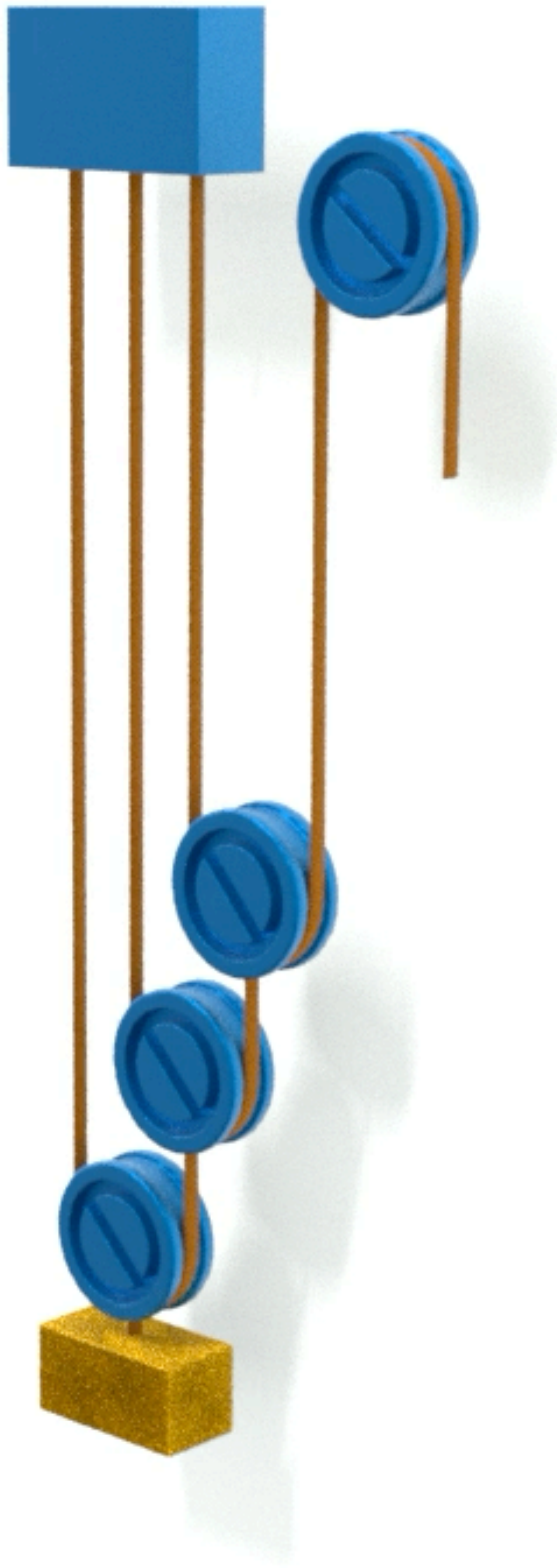
k_d=0.1

Complex Real-World Scenes

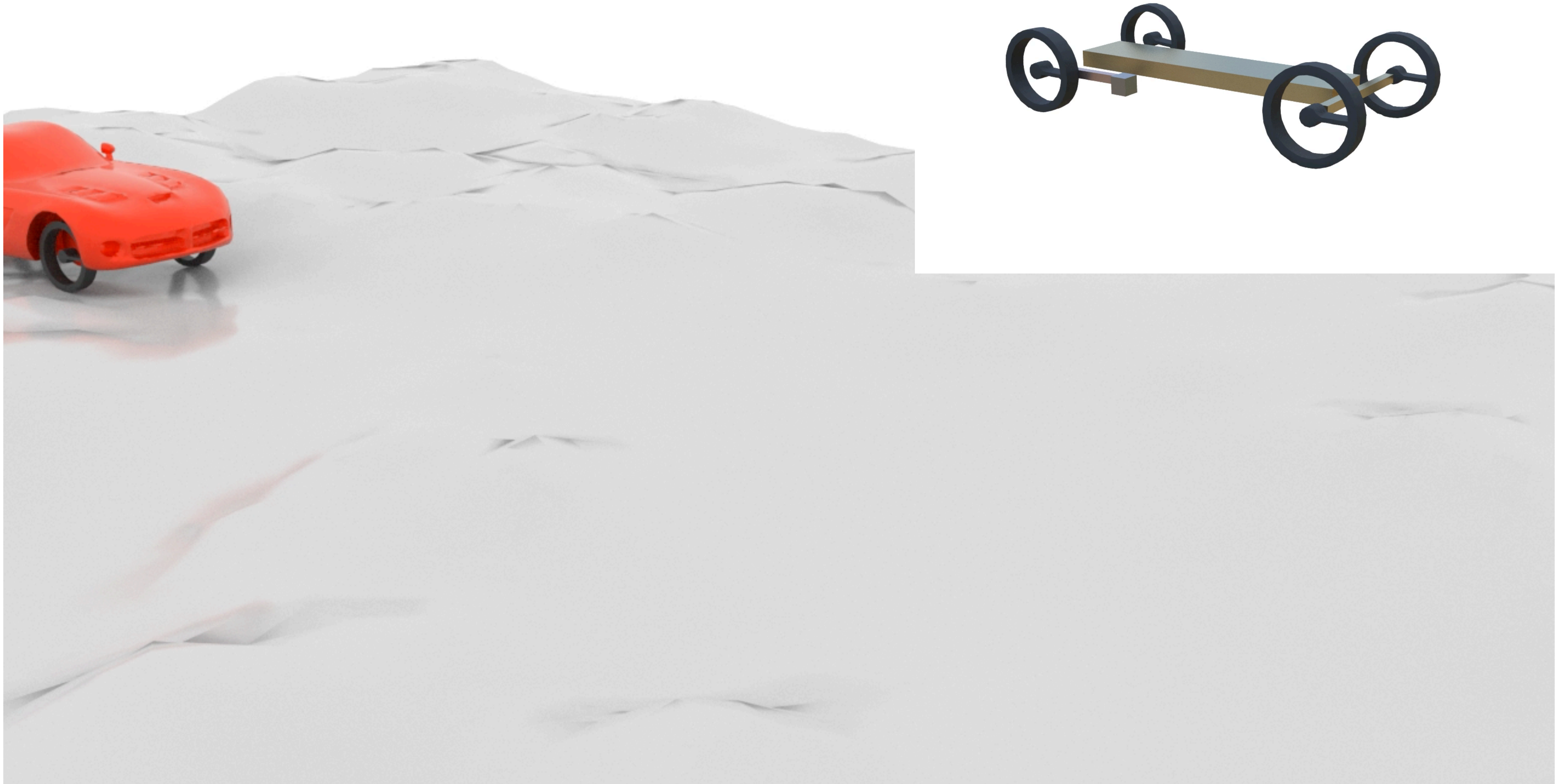
Hanging Bridge



Pulley System



Umbrella



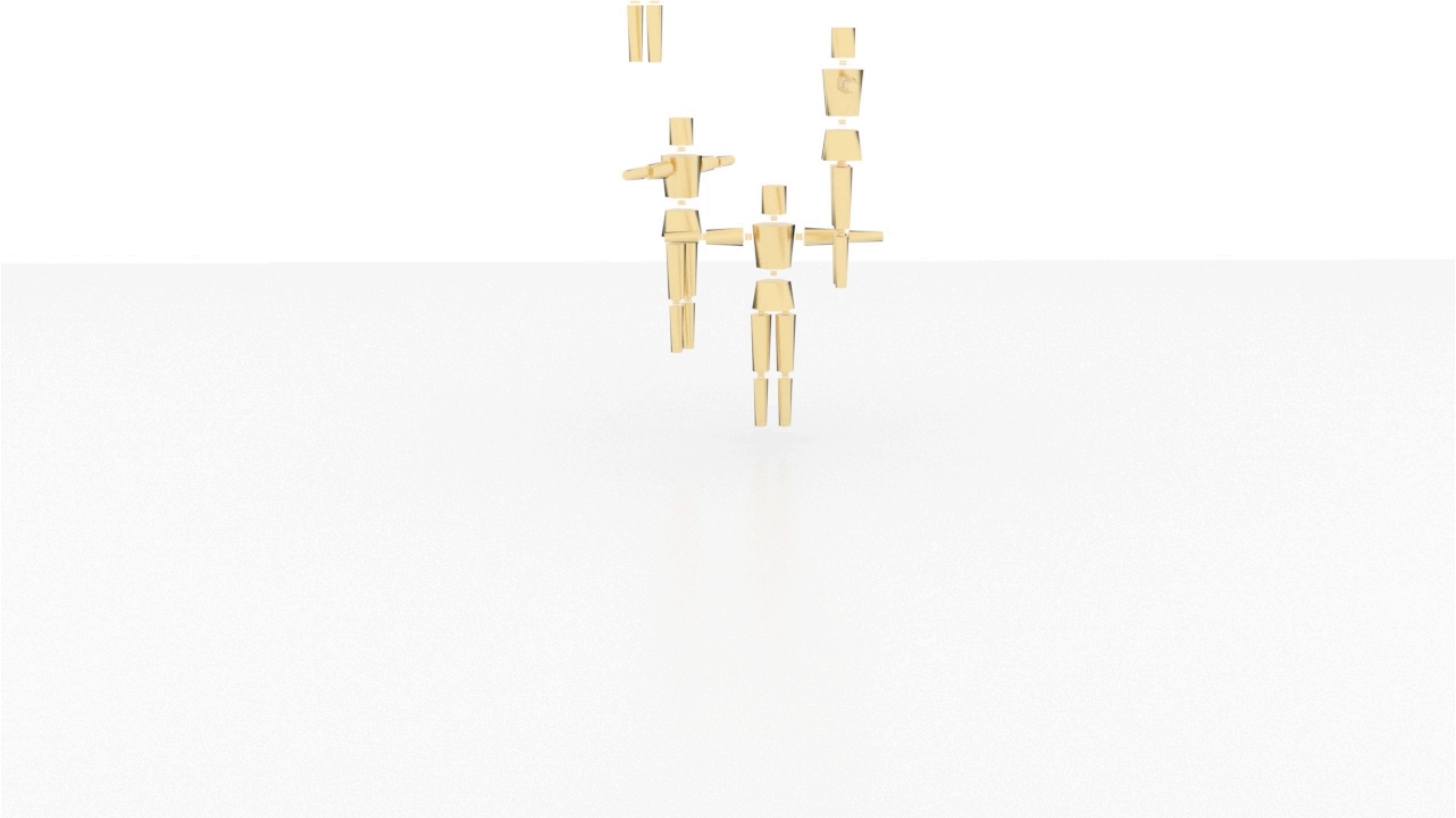
Precession

Slow

Medium

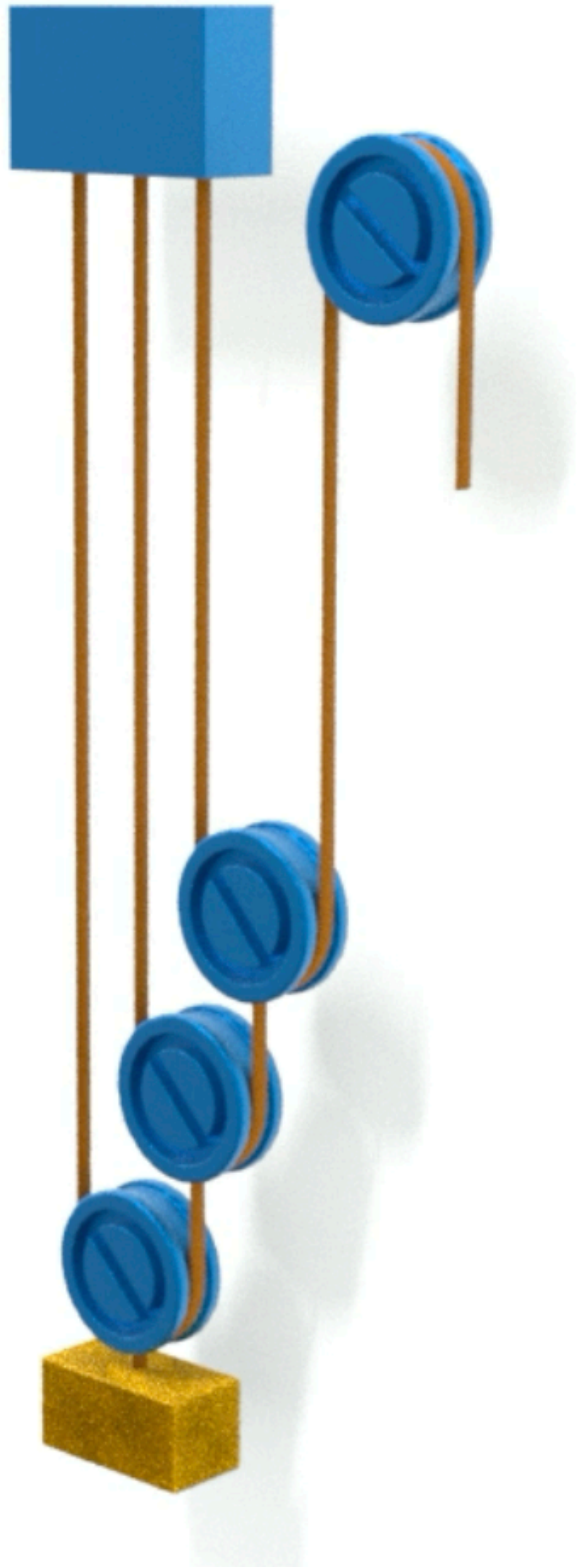
Fast

Ragdolls



"Lying Flat"

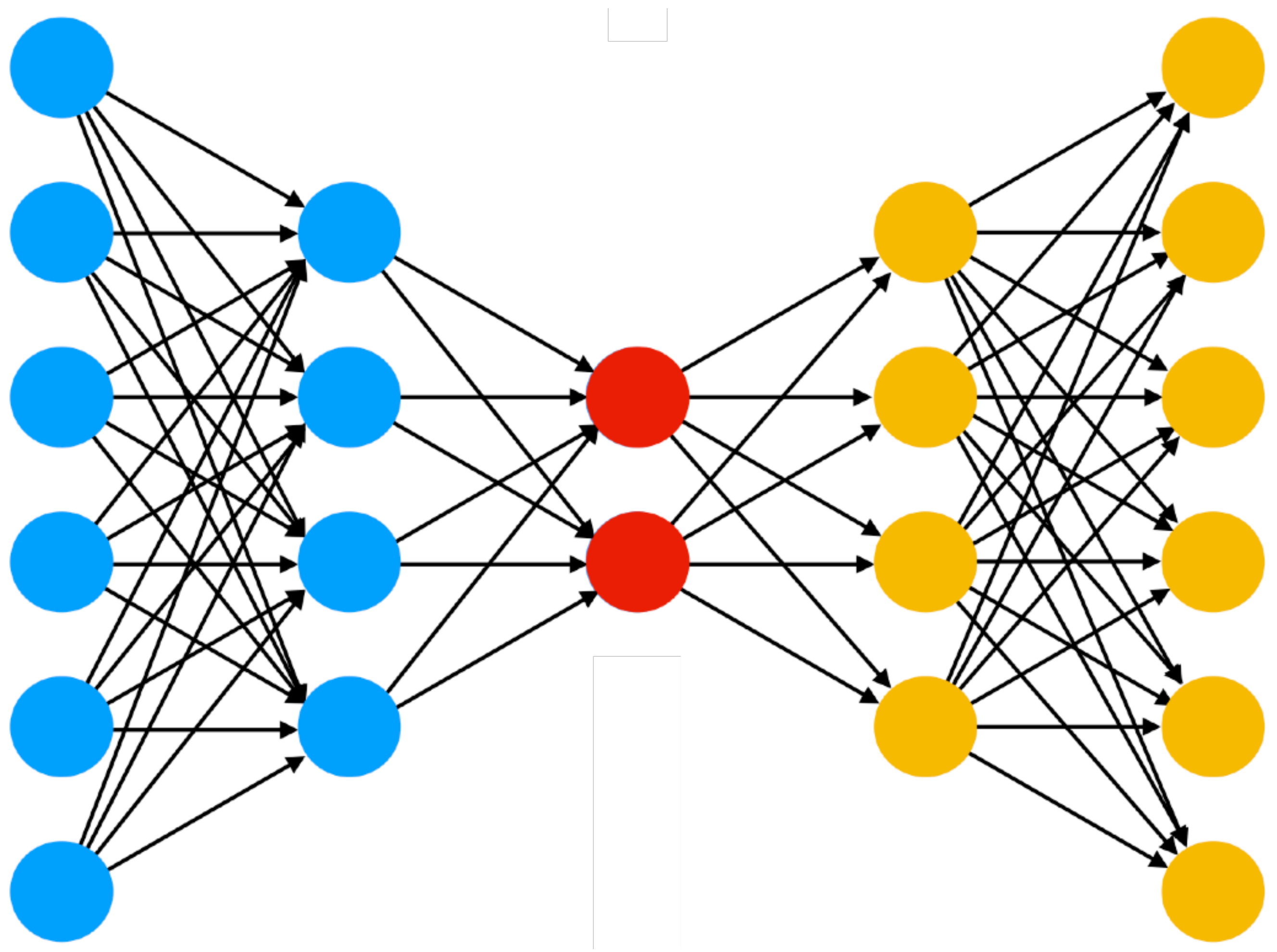
Summary



We presented **a robust multibody simulation framework** with

- fully implicit nonlinear elastodynamics for
  - Rigid and deformable solids in
  - Arbitrary codimensional geometries
- guaranteed non-interpenetration
- guaranteed algorithm convergence
- a wide range of articulation constraints
- a general restitution model for both contact and articulation

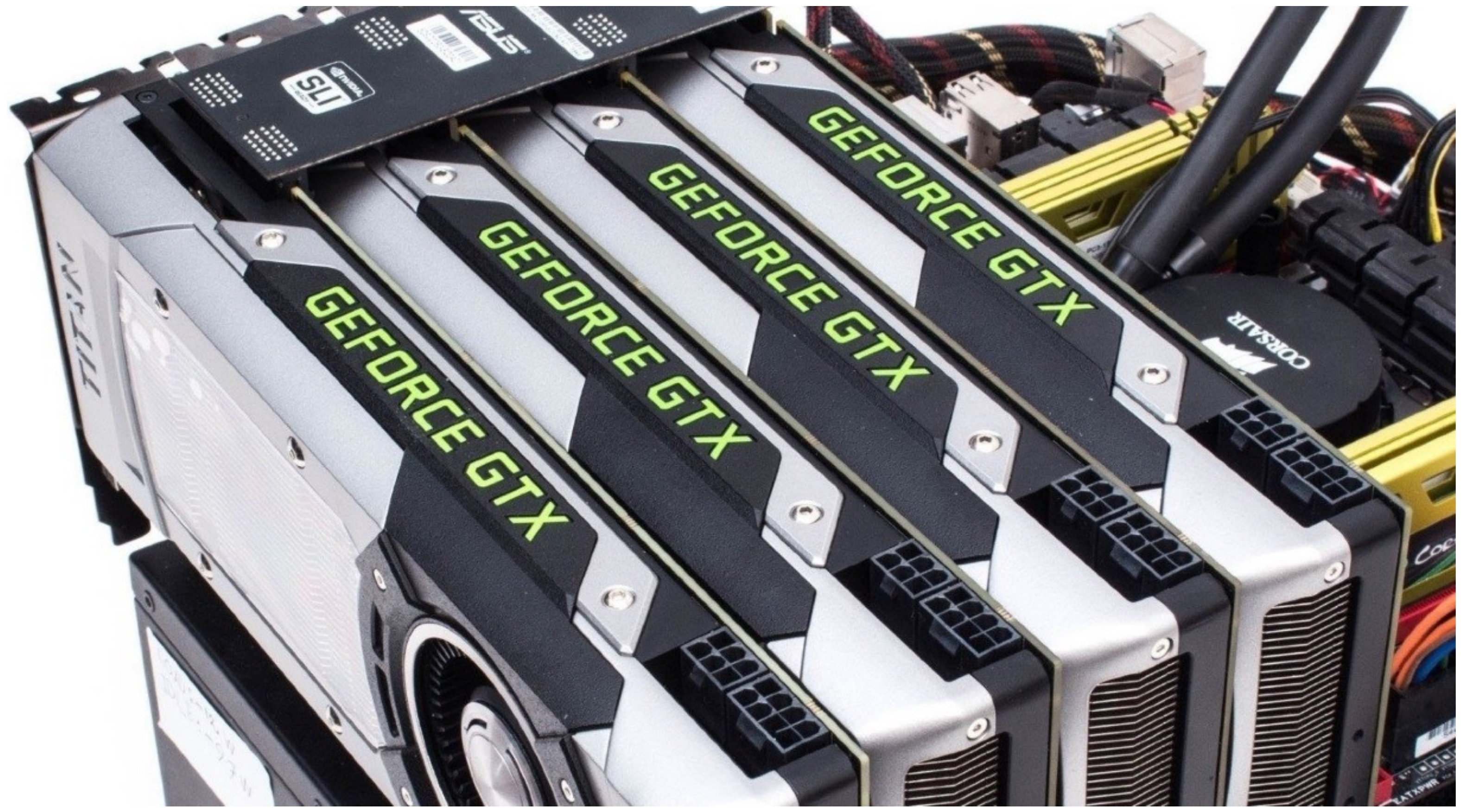
Higher Performance

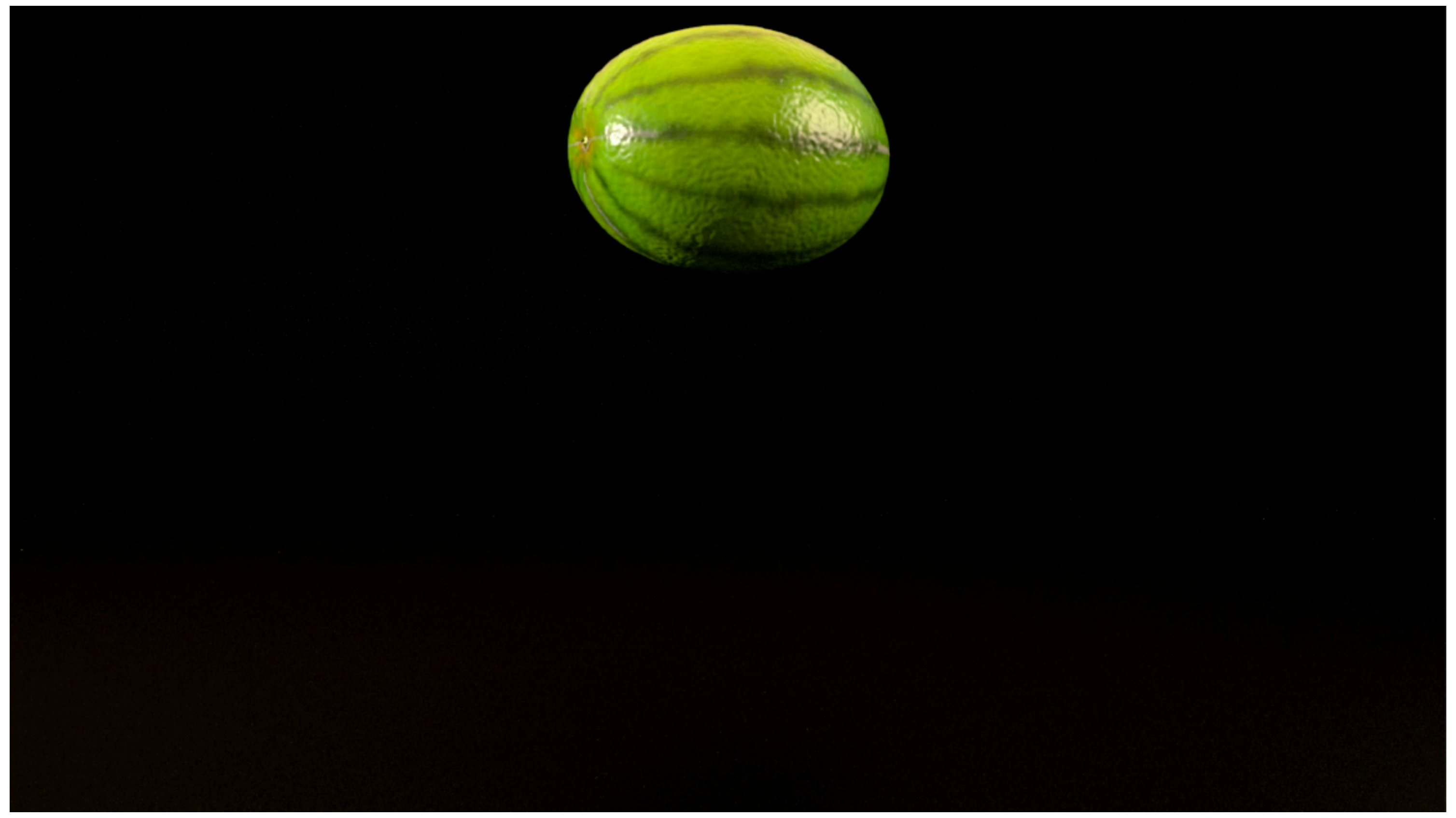


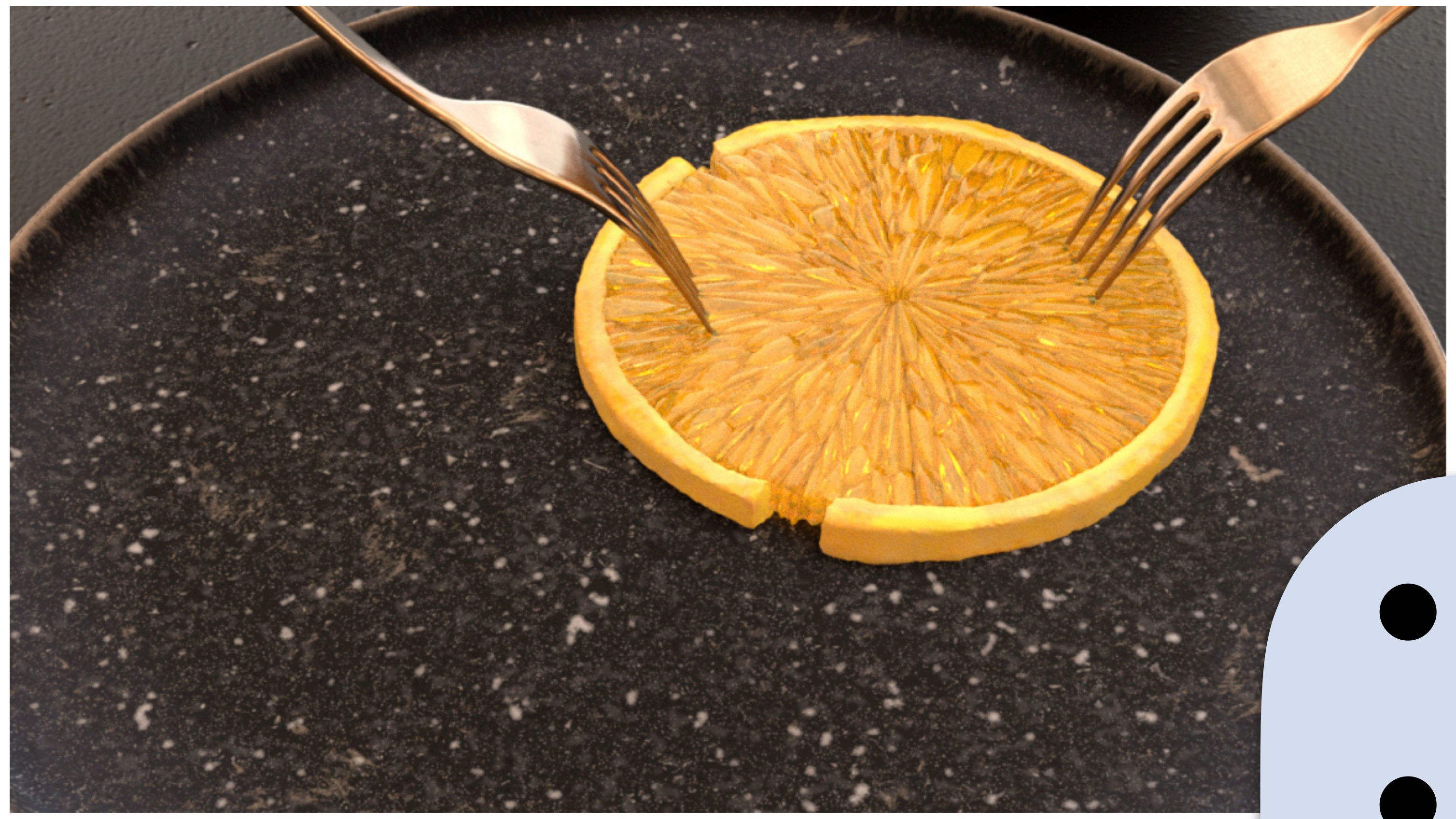
- Reduced models

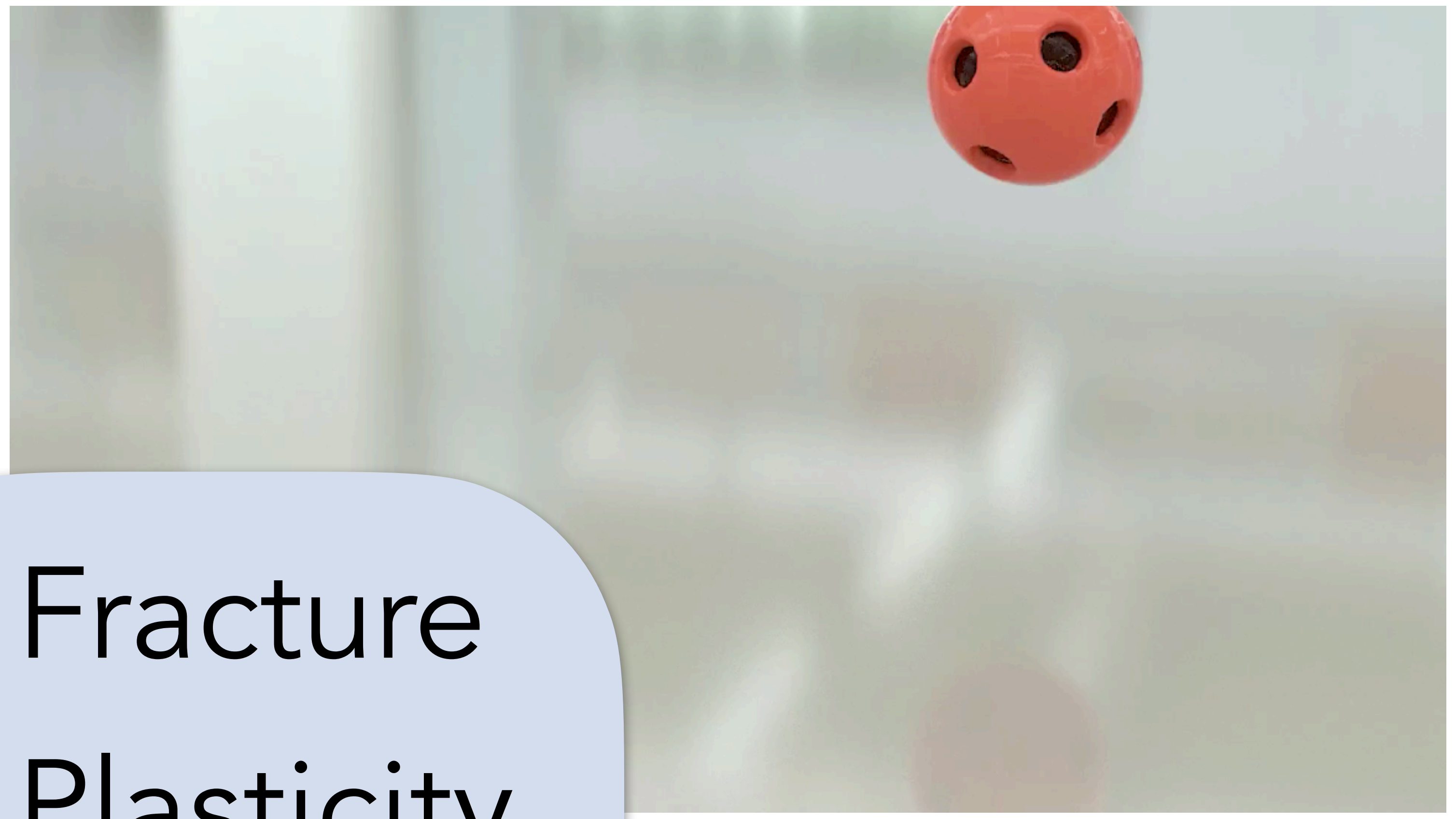
$$x^{n+1} = \textbf{argmin}_x \frac{1}{2}\|x - \tilde{x}^n\|_M^2 + \beta h^2 \sum_i P_i(x)$$

- GPU acceleration
- Cloud computing
- Advanced (non)linear solvers



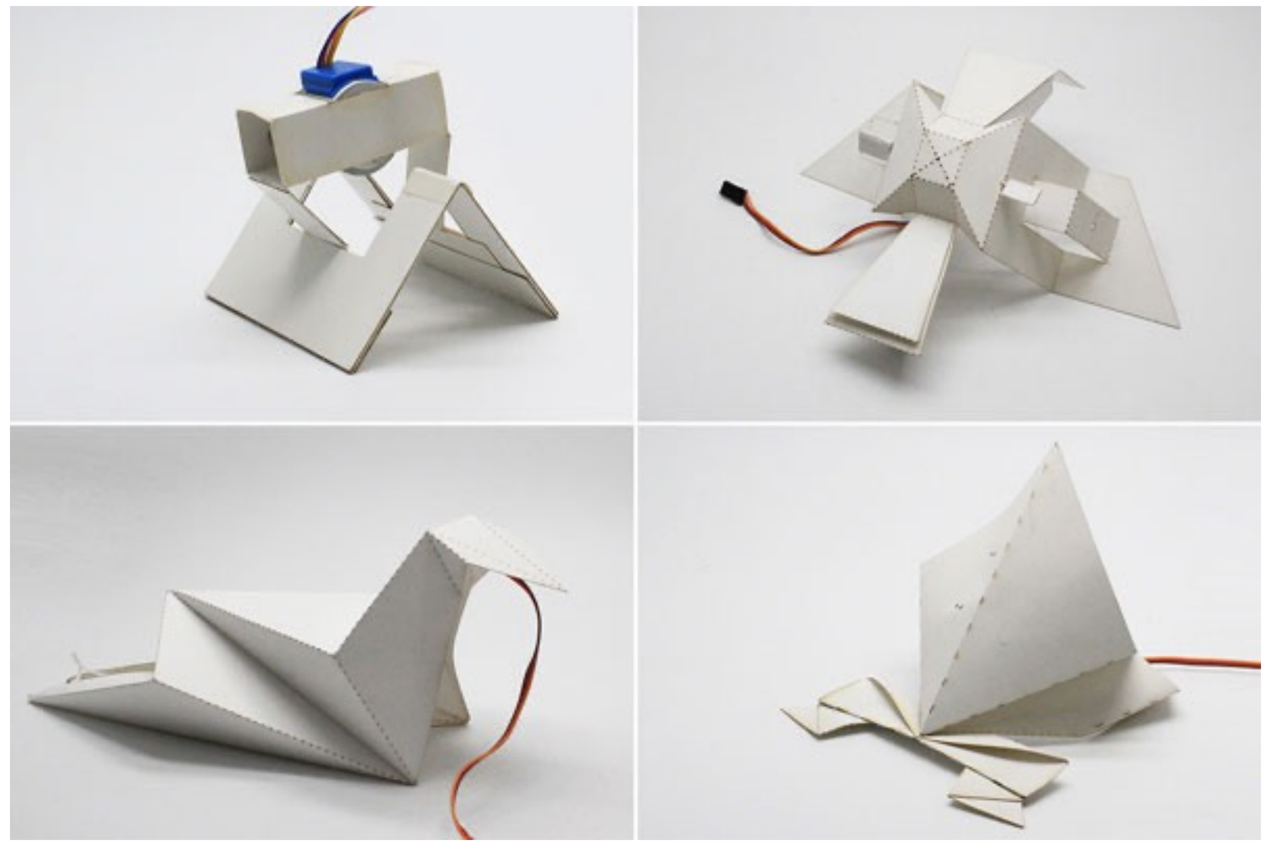




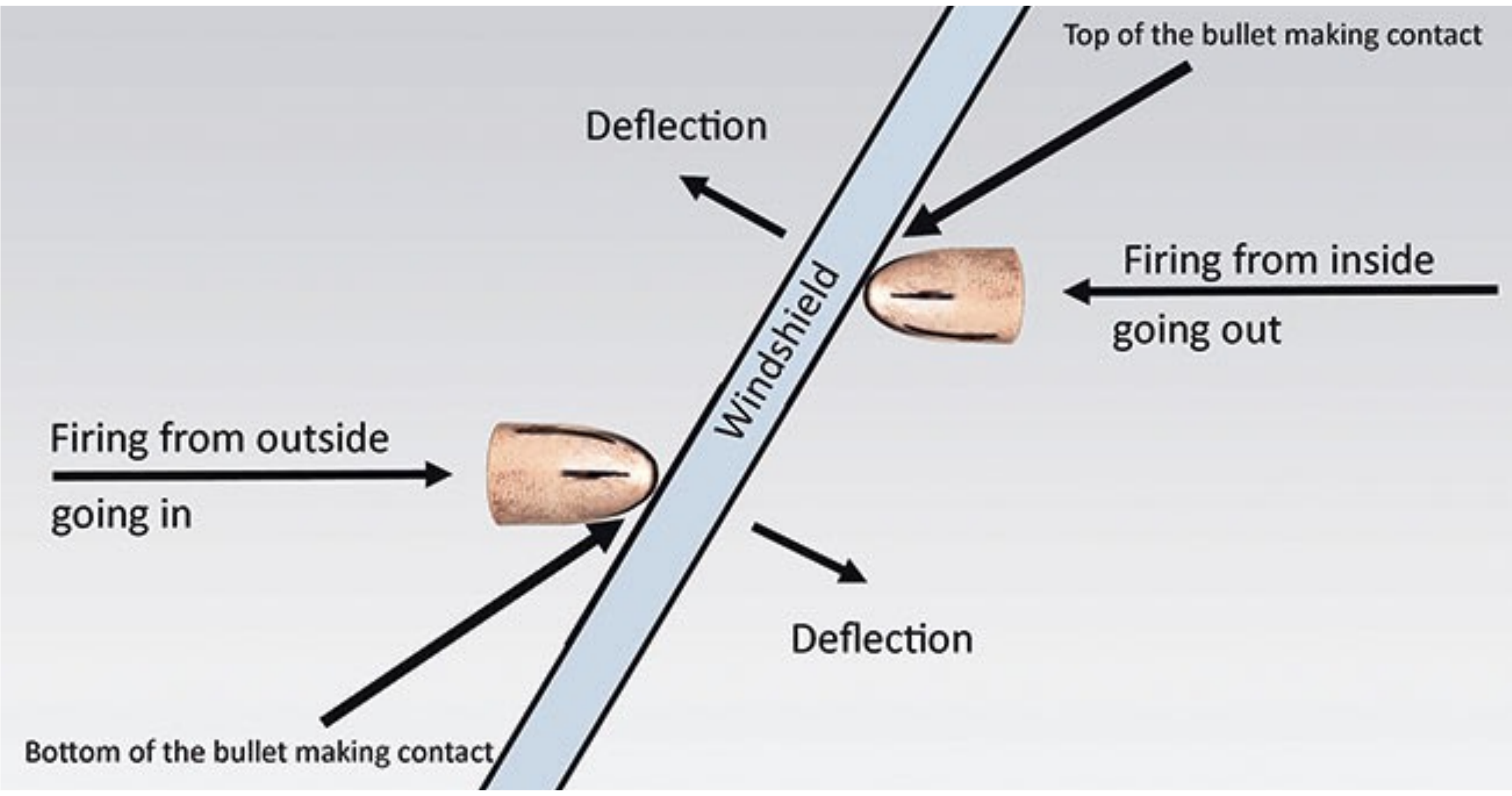


- Fracture
- Plasticity
- Fluids

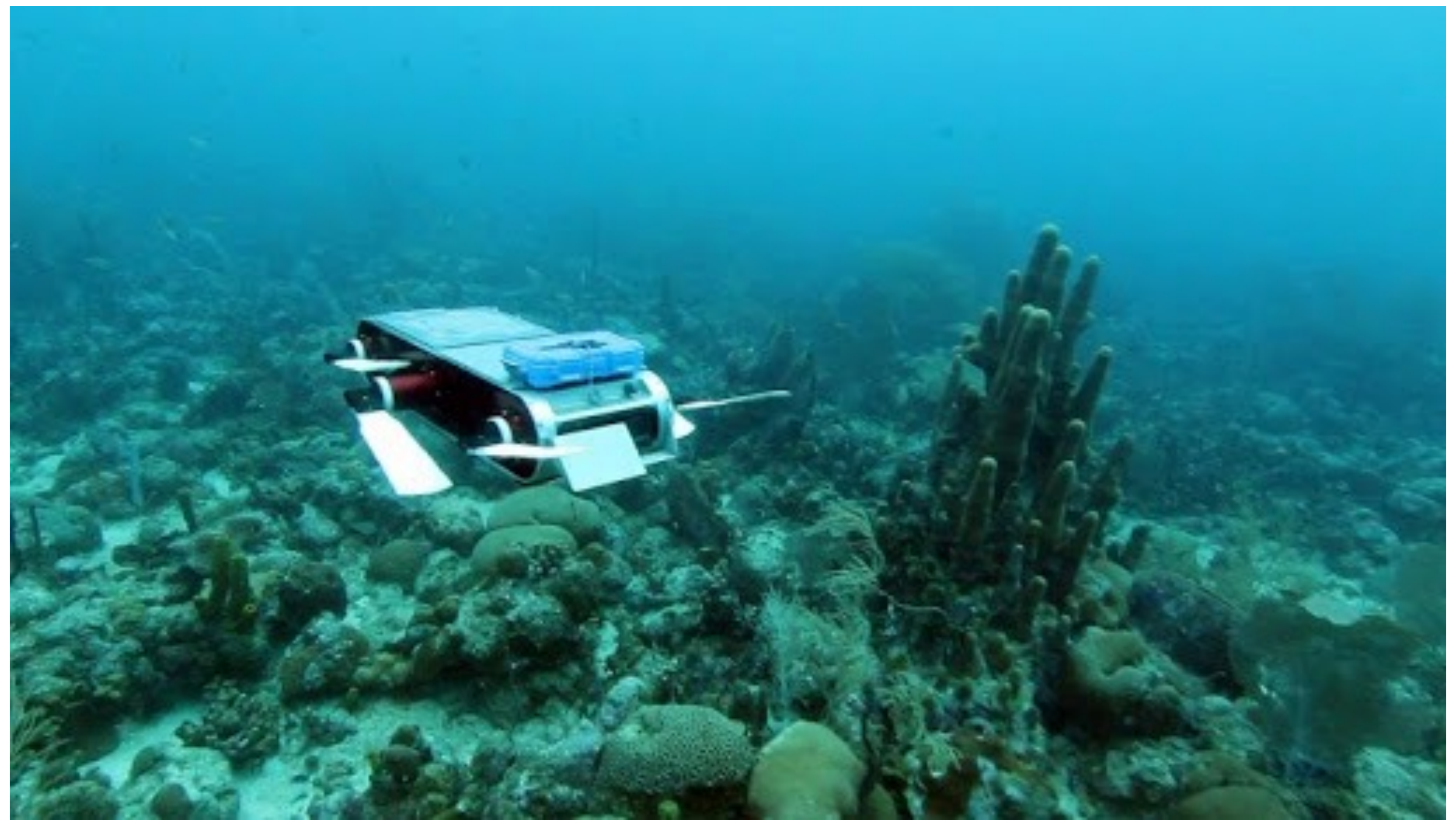
$$x^{n+1} = \textbf{argmin}_x \frac{1}{2}\|x - \tilde{x}^n\|_M^2 + \beta h^2 \sum_i P_i(x)$$



Origami robots



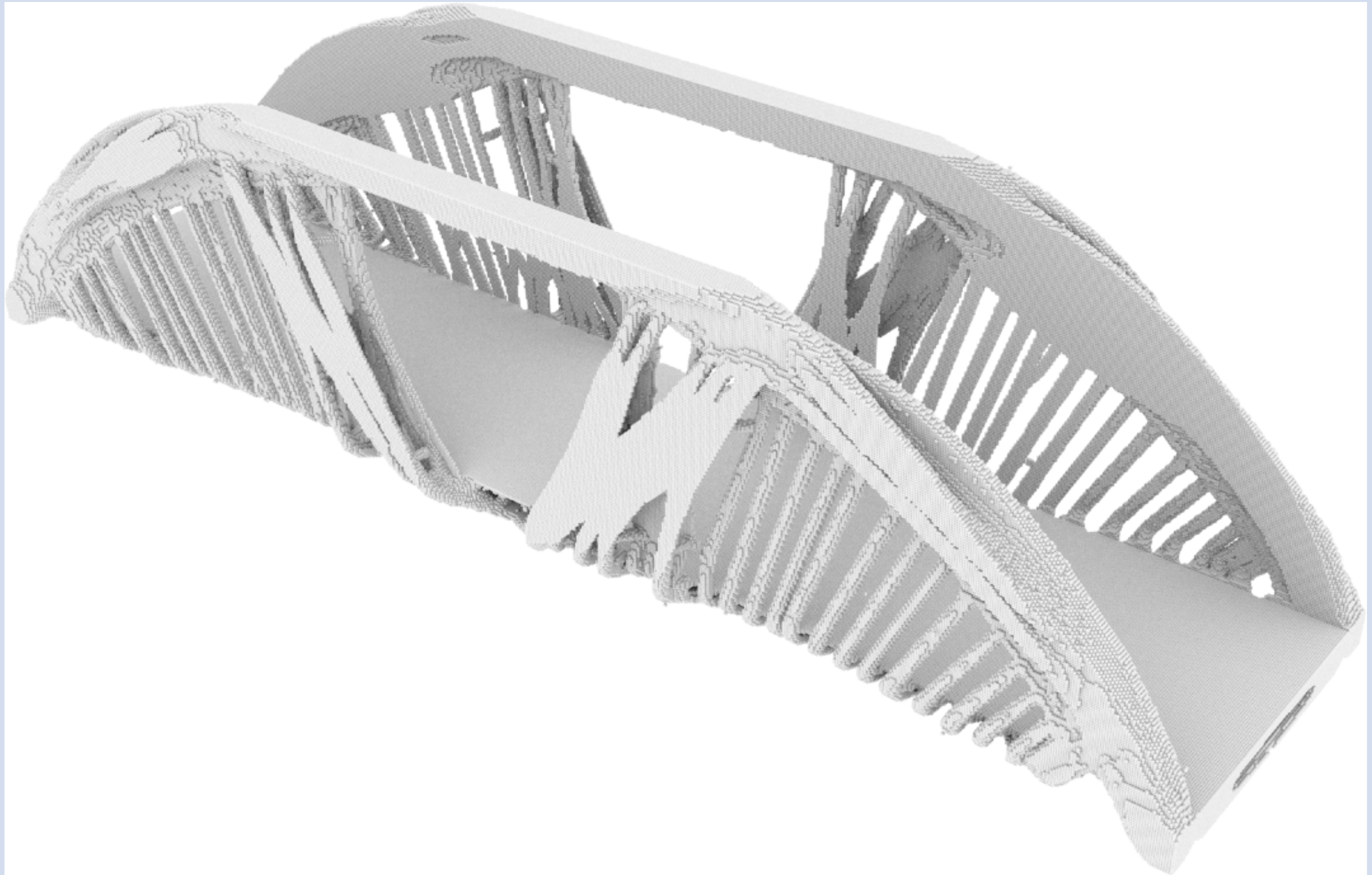
Bullet shot window



Swimming robots

Idea

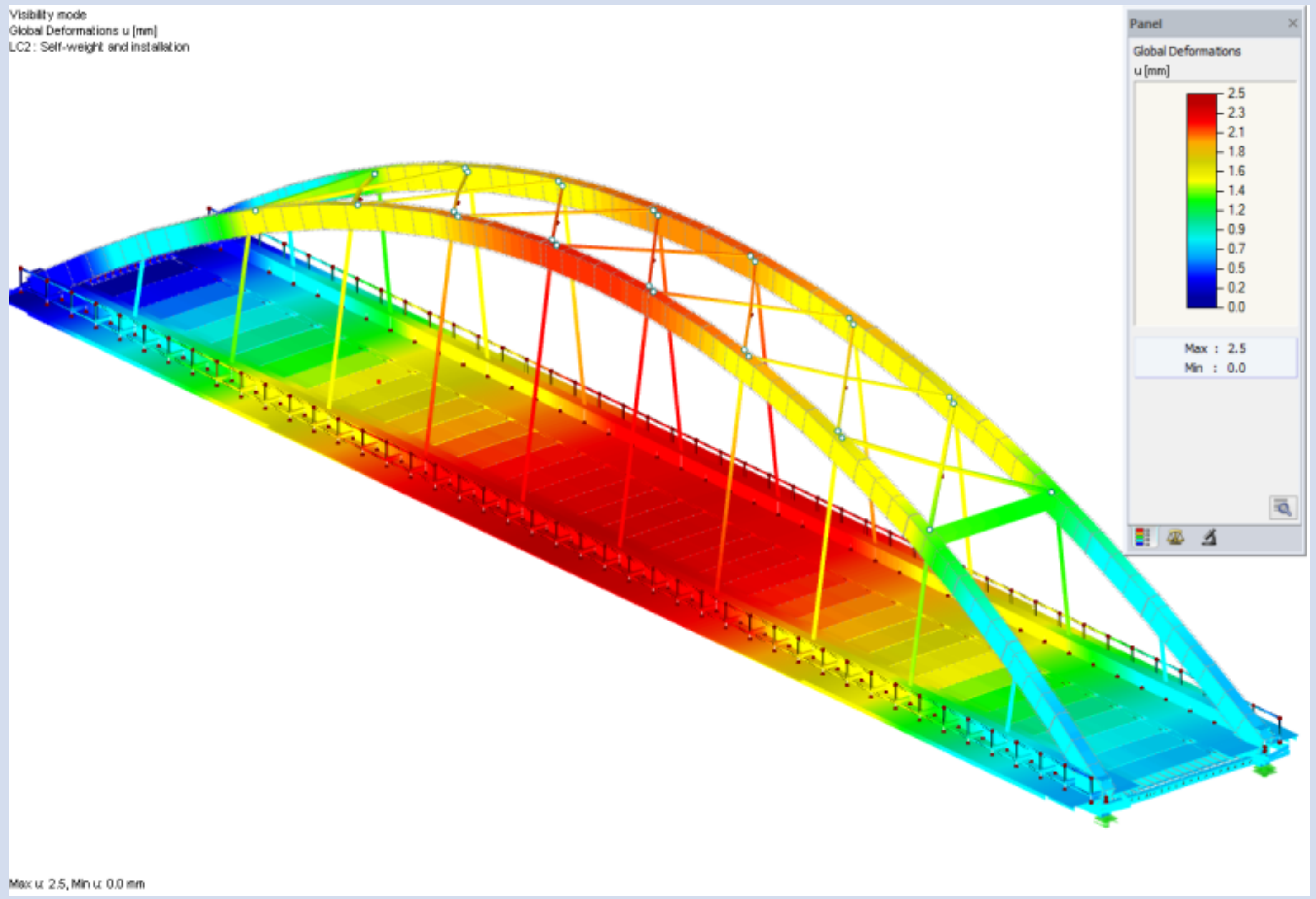
Design



Prototype

Differentiable Simulator

$$x^{n+1} = \textbf{argmin}_x \frac{1}{2}\|x - \tilde{x}^n\|_M^2 + \beta h^2 \sum_i P_i(x)$$



Feedback

Real-world experiments

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Collaborators

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