SIGGRAPH ASIA 2021 TOKYO LIVE

Interactive Cutting and Tearing in Projective Dynamics with Progressive Cholesky Updates

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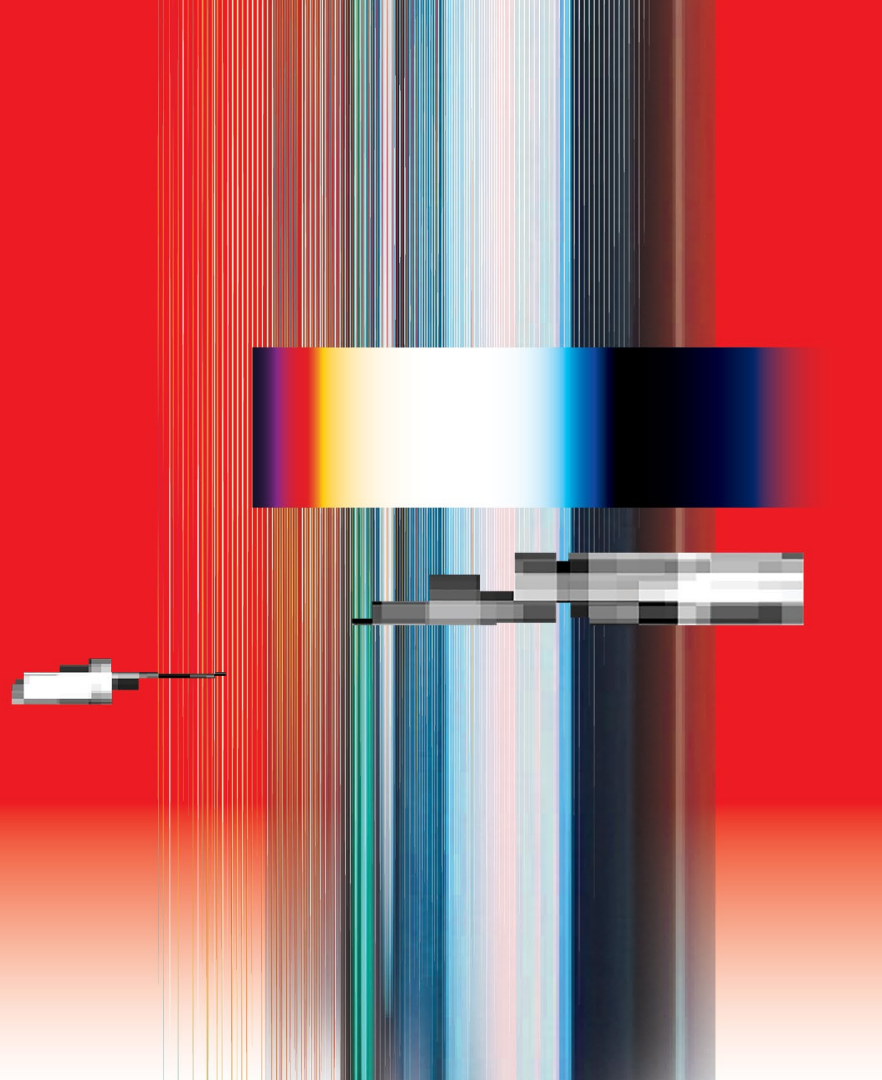
Interactive Applications

Background



Projective Dynamics

local step:

$$\mathbf{x} \rightarrow \mathbf{p}$$

global step:

$$\left(\frac{\mathbf{M}}{h^2} + \mathbf{H}_L\mathbf{C}\right)\mathbf{x}^* = \left(\mathbf{J}\mathbf{p} + \frac{\mathbf{M}\mathbf{y}}{h^2}\right)$$

$$\mathbf{C} = \mathbf{L}\mathbf{L}^\intercal$$

Sparse Cholesky Modification

Timothy A Davis and William W Hager. 2001. Multiple-rank modifications of a sparse Cholesky factorization. SIAM J. Matrix Anal. Appl. 22, 4 (2001), 997–1013.

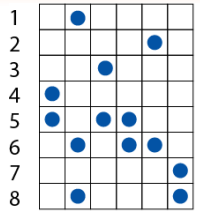
$$\mathbf{A}\mathbf{A}^\intercal + \mathbf{W}\mathbf{W}^\intercal = [\mathbf{A}|\mathbf{W}][\mathbf{A}|\mathbf{W}]^\intercal$$

symbolic update

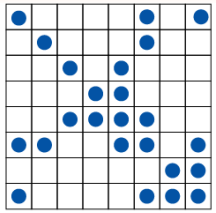
$$\mathcal{L}_j = \{j\} \cup \left(\bigcup_{\{c:j=\pi(c)\}} \mathcal{L}_c \backslash \{c\}\right) \cup \left(\bigcup_{\min \mathcal{A}_k = j} \mathcal{A}_k\right)$$

$$\overline{\mathcal{L}}_j = \{j\} \cup \left(\bigcup_{\{c:j=\overline{\pi}(c)\}} \mathcal{L}_c \backslash \{c\}\right) \cup \left(\bigcup_{\min \mathcal{A}_k = j} \mathcal{A}_k\right) \cup \left(\bigcup_{\min \mathcal{W}_i = j} \mathcal{W}_i\right)$$

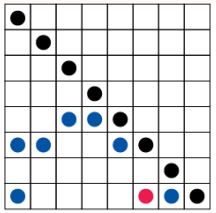
Sparse Cholesky Modification



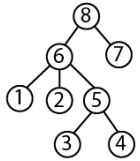
A



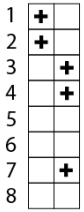
$C = AA^T$



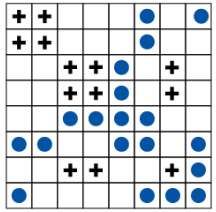
original factor L



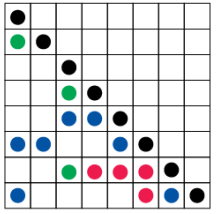
original elimination tree



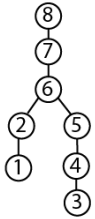
W



$\overline{C} = C + WW^T$



new factor $\overline{L}$

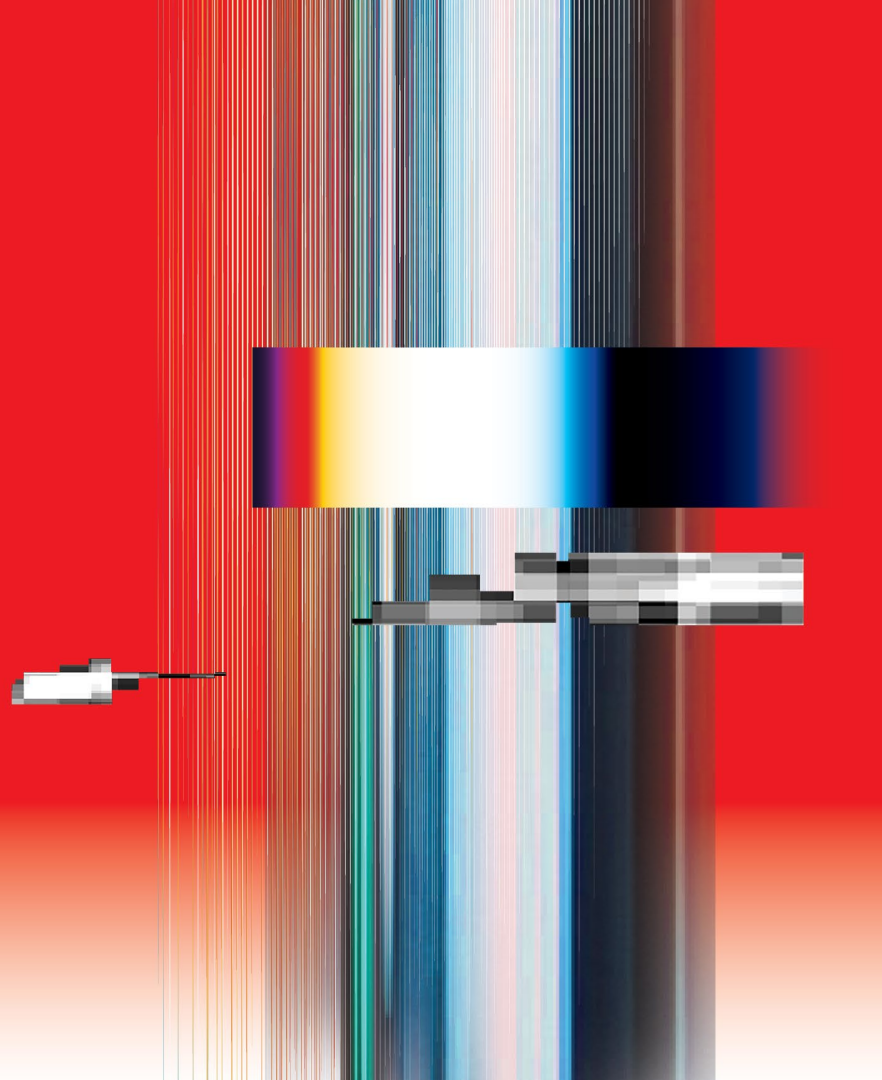


new elimination tree

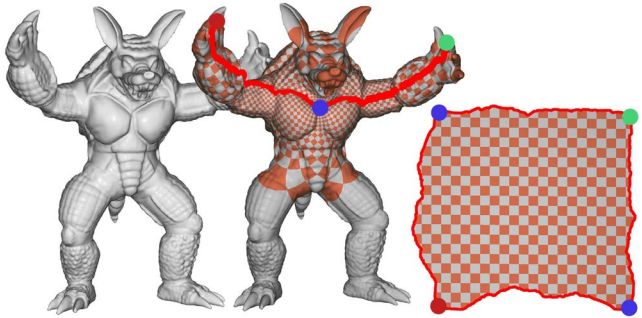
$$\mathcal{L}_j = \{j\} \cup \left(\bigcup_{\{c:j=\pi(c)\}} \mathcal{L}_c \backslash \{c\} \right) \cup \left(\bigcup_{\min \mathcal{A}_k = j} \mathcal{A}_k \right)$$

$$\overline{\mathcal{L}}_j = \{j\} \cup \left(\bigcup_{\{c:j=\overline{\pi}(c)\}} \mathcal{L}_c \backslash \{c\} \right) \cup \left(\bigcup_{\min \mathcal{A}_k = j} \mathcal{A}_k \right) \cup \left(\bigcup_{\min \mathcal{W}_i = j} \mathcal{W}_i \right)$$

Our Method



Related Work



Philipp Herholz and Olga Sorkine-Hornung. 2020.
Sparse Cholesky updates for interactive mesh parameterization.
ACM Transactions on Graphics (TOG) 39, 6 (2020), 1–14.

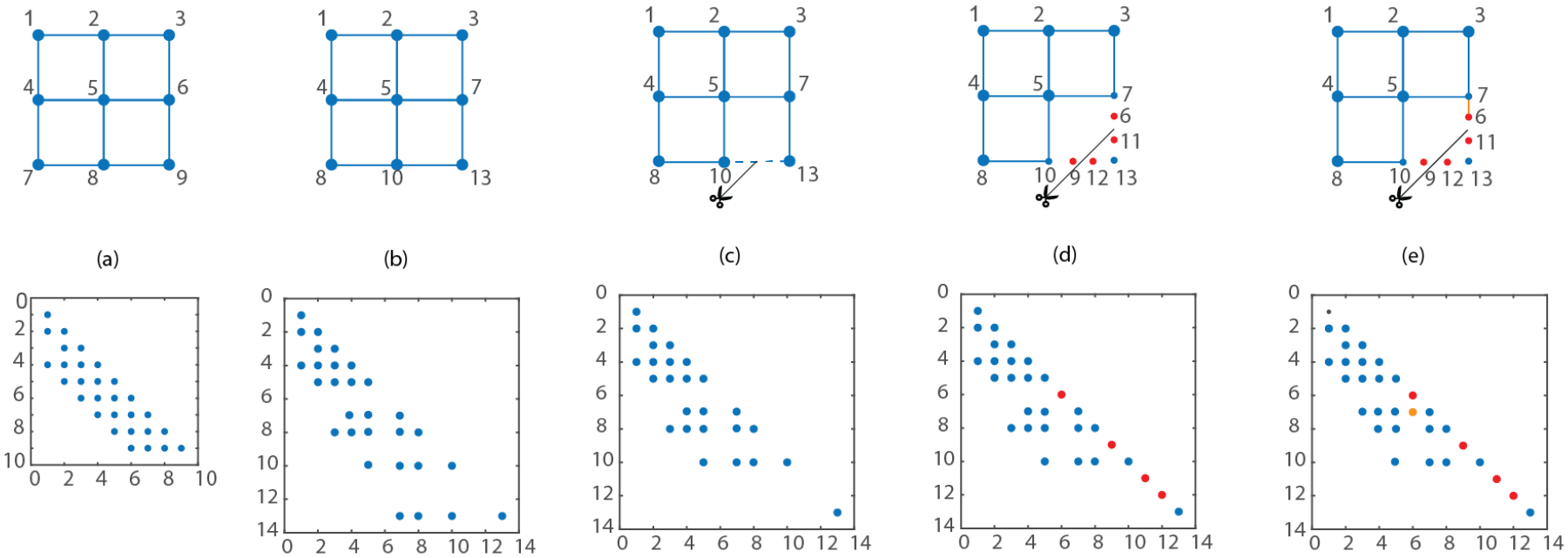
Yeung, Yu-Hong, Alex Pothen, Mahantesh Halappanavar, and Zhenyu Huang. "AMPS: An augmented matrix formulation for principal submatrix updates with application to power grids." SIAM Journal on Scientific Computing 39, no. 5 (2017): S809-S827.

Our Method

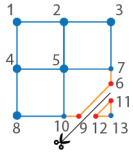
elasticity $\sigma \mathbf{W}_e \mathbf{W}_e^\intercal$ $\mathbf{w}_e = \sqrt{w_e}\mathbf{G}_e^\intercal$

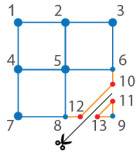
mass $\sigma \mathbf{W}_v \mathbf{W}_v^\intercal$ $\mathbf{w}_v = \sqrt{\frac{m_v}{h^2}}\delta_v$

Our Method

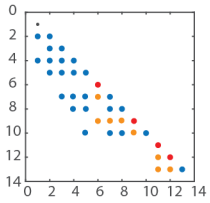

1 2 3
4 5 6
7 8 9
1 2 3
4 5 7
8 10 13
1 2 3
4 5 7
8 10 13
1 2 3
4 5 7
6
11
8 10 9 12 13
1 2 3
4 5 7
6
11
8 10 9 12 13
(a)
(b)
(c)
(d)
(e)

Our Method

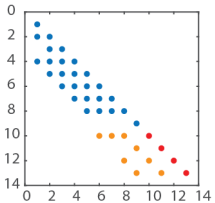


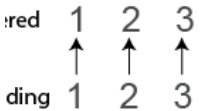


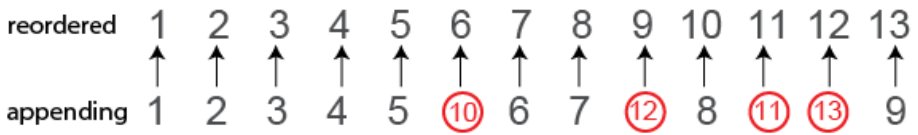
(e)



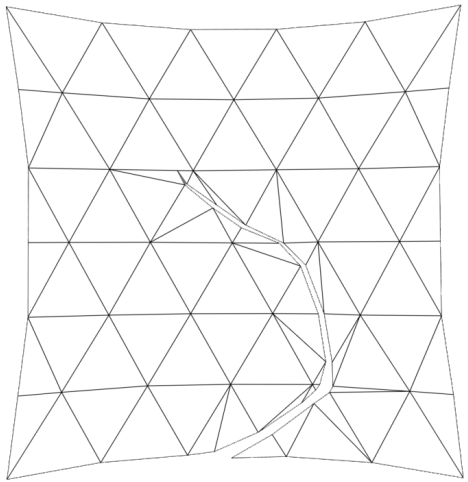
(f)

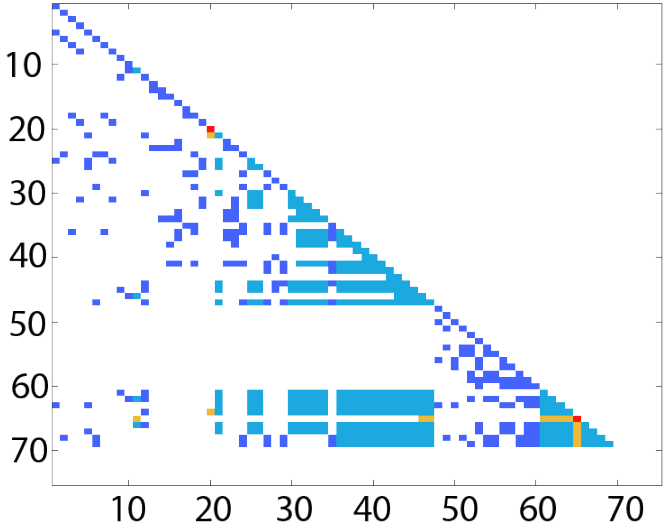






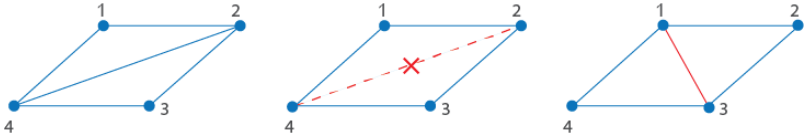
Our Method



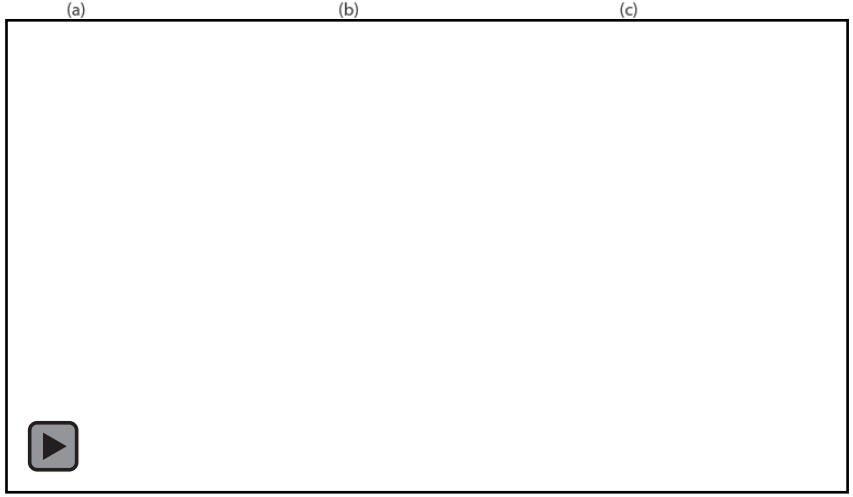


Our Method

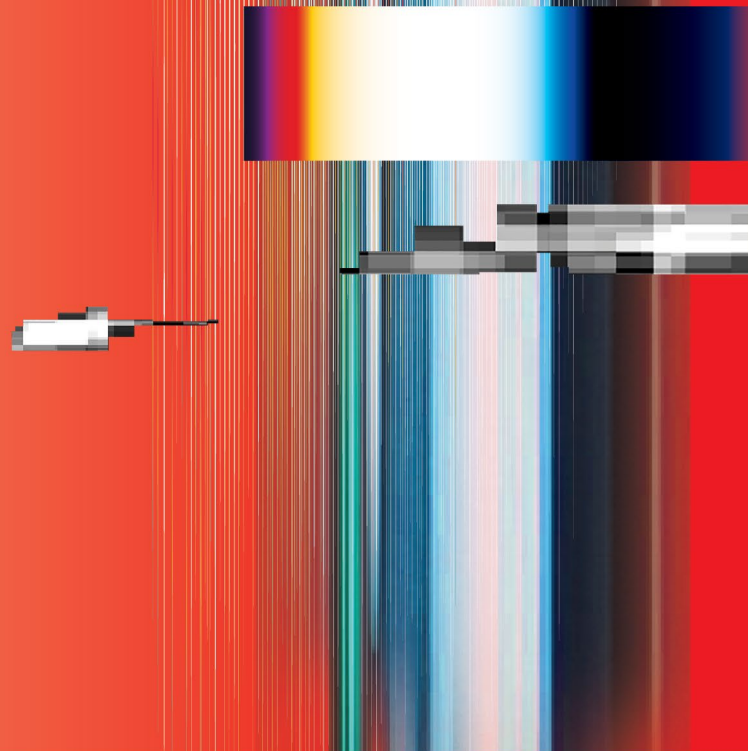
remesh



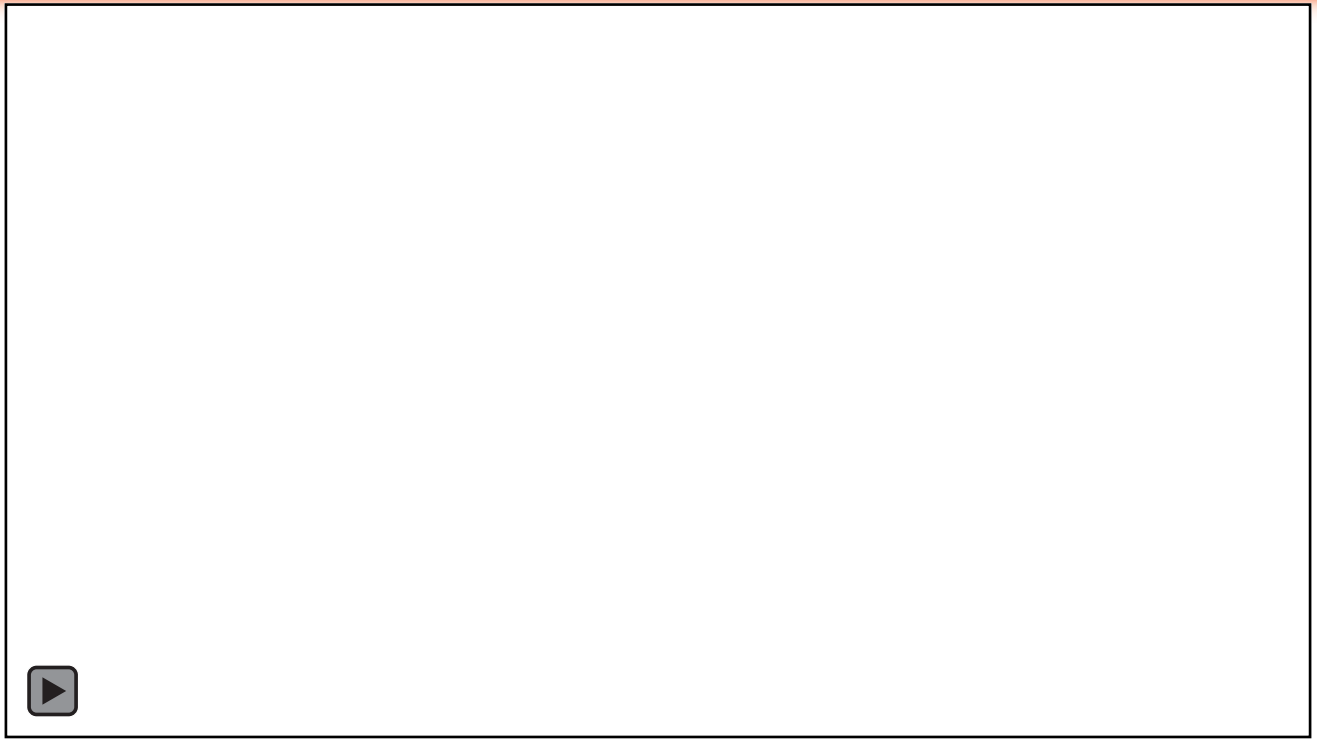
collision



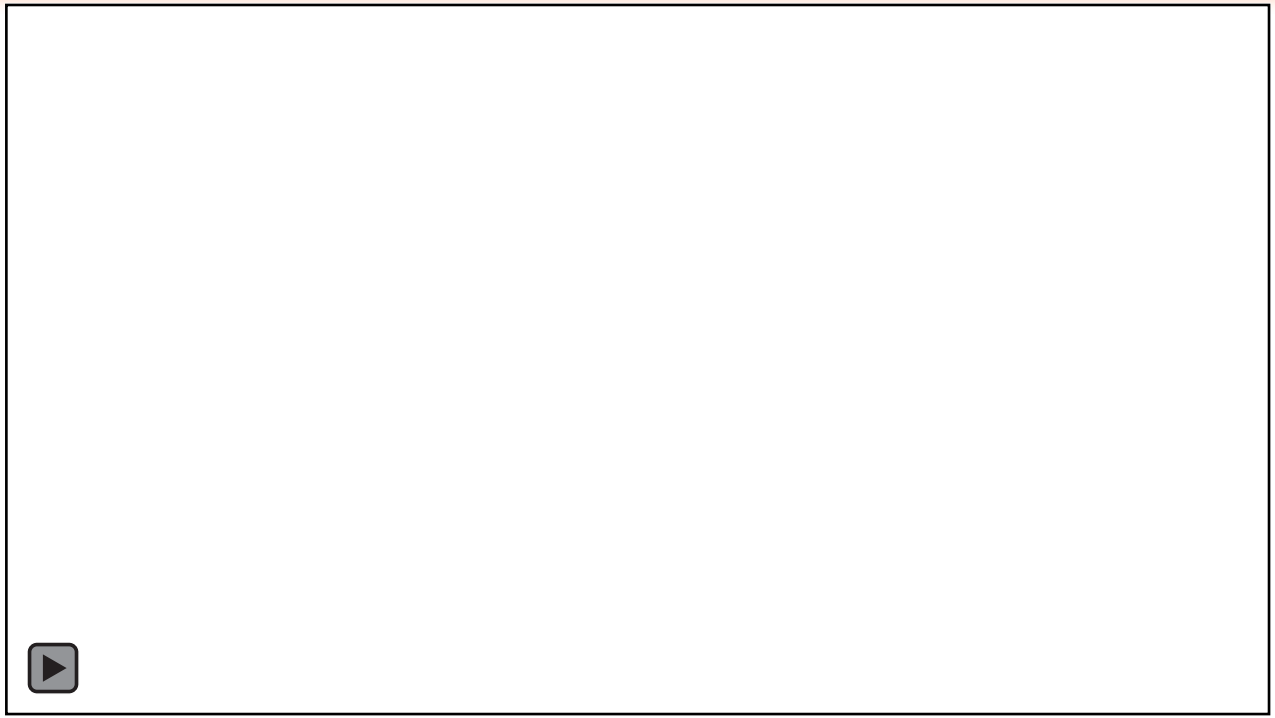
Results



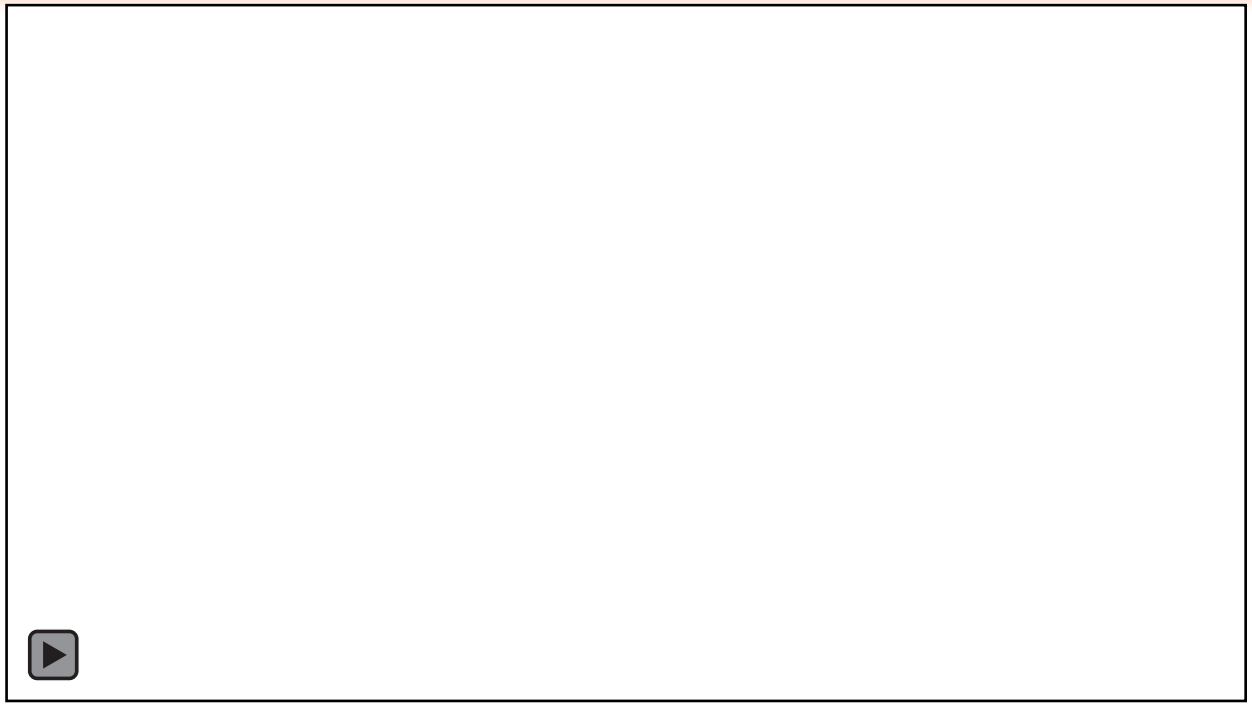
Results



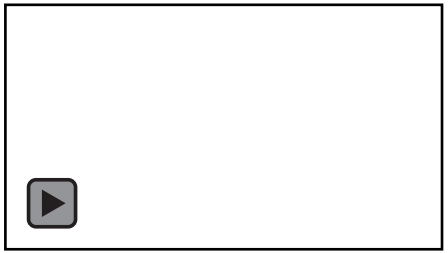
Results

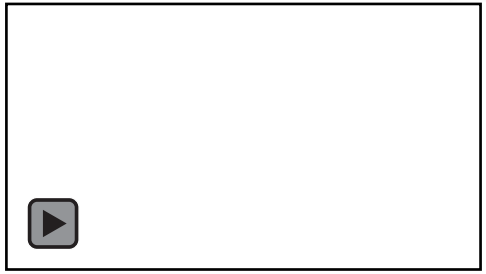


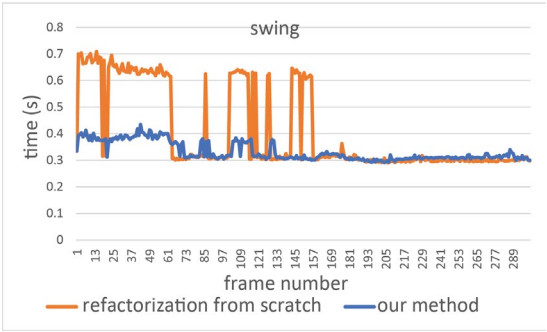
Results

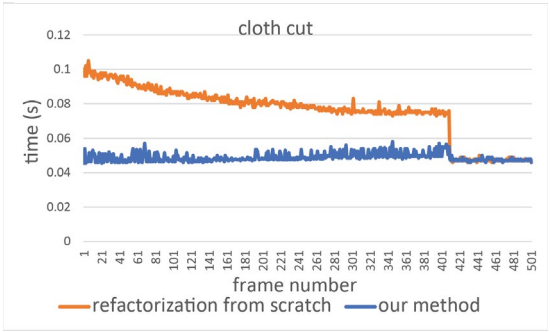


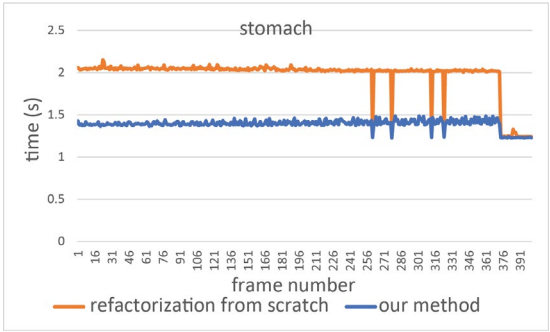
Results






swing
time (s)
frame number
refactorization from scratch
our method


cloth cut
time (s)
frame number
refactorization from scratch
our method


stomach
time (s)
frame number
refactorization from scratch
our method

Results

Table 1. Results on our example models. †: the bending springs are not considered in the cloth cut example.

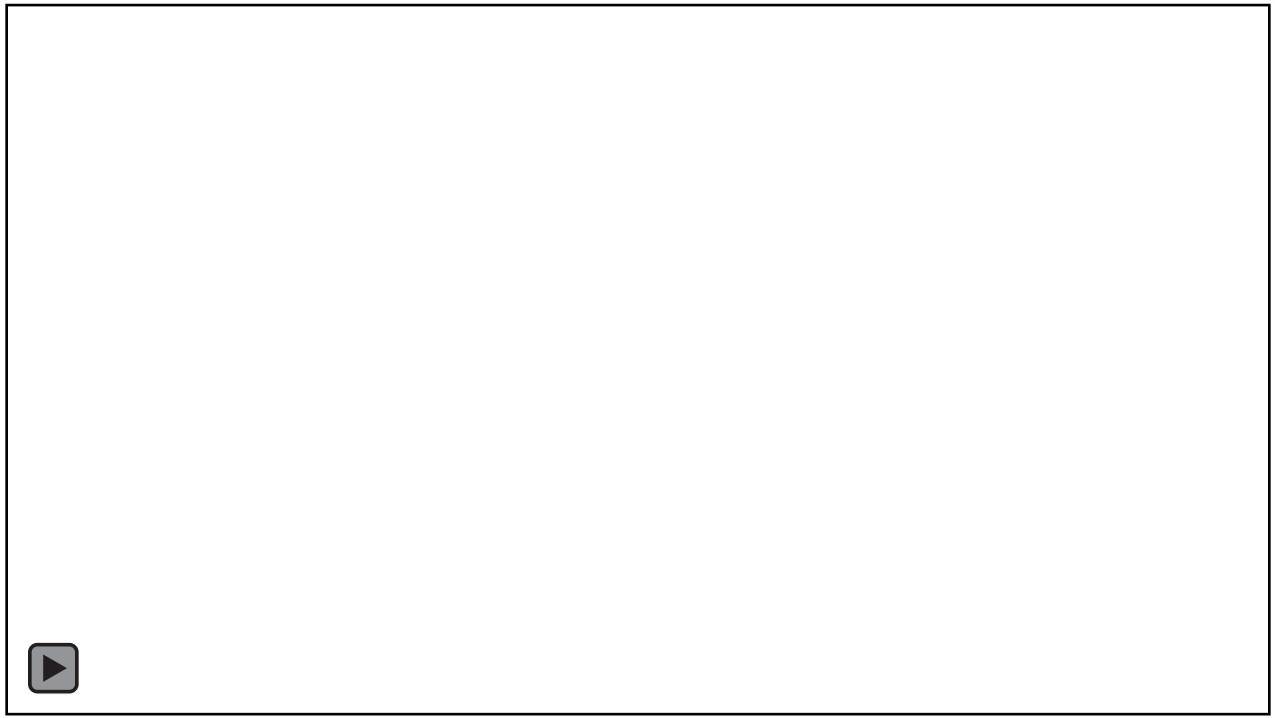
Example	model	#Verts	#Elems	local/global solve	factorization update		total time	
					refactorization	our method	PD + refactorization	PD + our method
cloth cut	cloth†	14,829	29,172	47ms	36ms	2ms	83ms	49ms
spinning cloth	cloth	14,829	29,172	52ms	77ms	15ms	129ms	67ms
swing	cloth	24,168	47,880	125ms	602ms	4ms	727ms	129ms
stomach	volumetric	36,055	13,7796	617ms	786ms	15ms	1403ms	632ms

Results

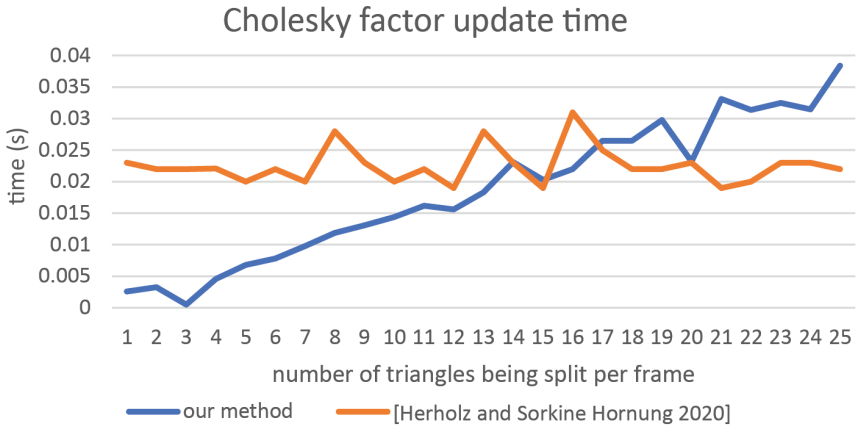
Table 2. Scalability test on the cloth cut example of different resolutions.

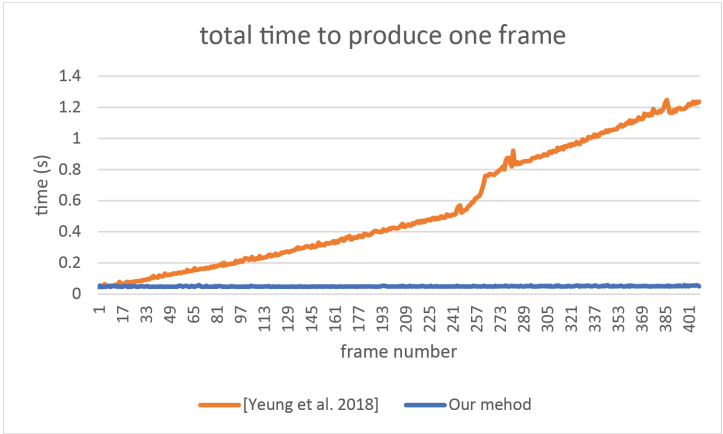
Example	#Verts	#Elems	cut per frame						recomputation
			1	4	8	16	32	64	
cloth cut	3,785	7,326	1ms	1ms	2ms	5ms	9ms	20ms	22ms
cloth cut	14,829	29,172	2ms	5ms	8ms	19ms	26ms	37ms	36ms
cloth cut	58,697	116,424	5ms	13ms	17ms	32ms	56ms	251ms	416ms

Comparisons

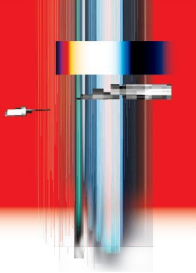


Comparisons





Conclusion



- Our method can efficiently update the Cholesky factor even when topological changes are produced with arbitrarily added DOFs.
- Our method requires that the update be low-rank, thus suitable for interactive applications where the update per frame is usually low-rank.
- Our method scales badly with rank of the update, i.e. the number of columns of W.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