



近场动力学 (Peridynamics) 并行编程与实践

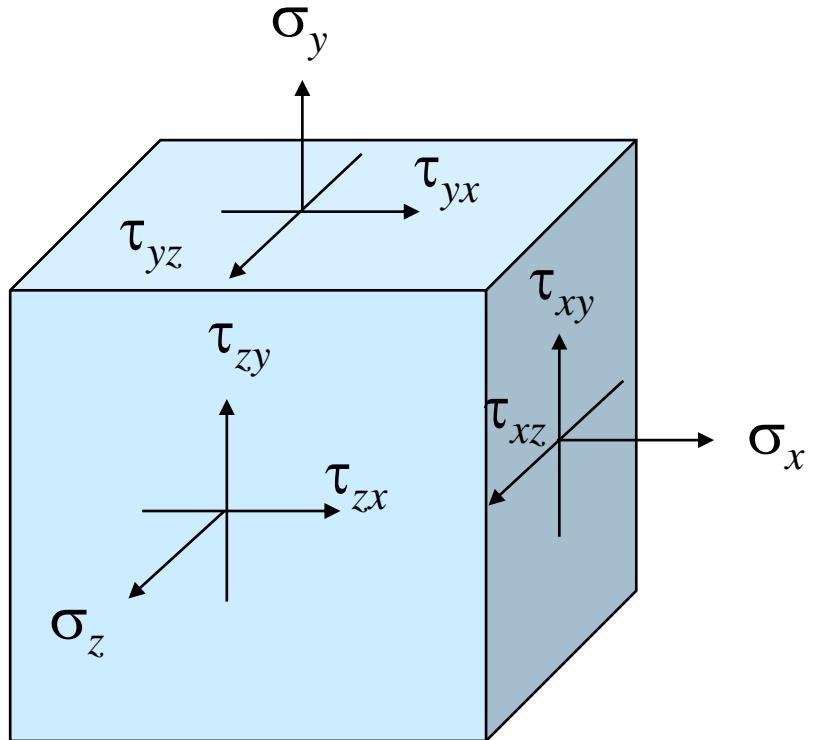
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2023.5.7

大纲

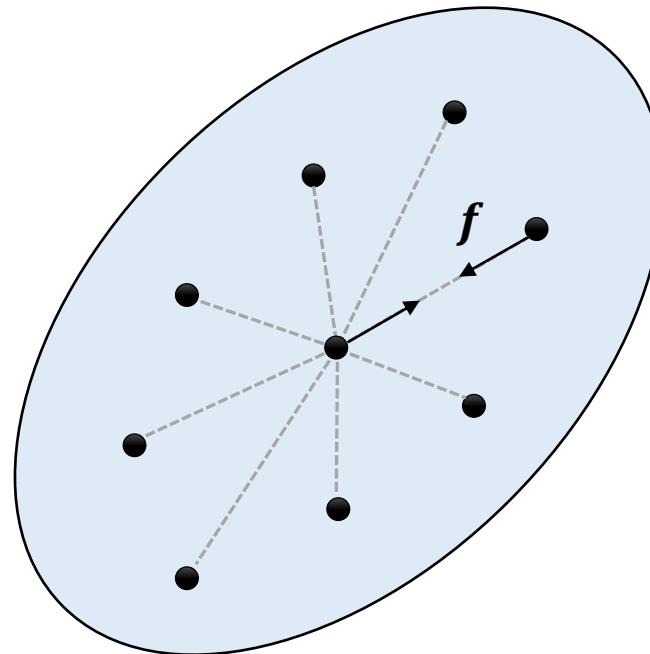
- 近场动力学 (Peridynamics) 基础
- 投影近场动力学 (Projective Peridynamics)
- 半隐式连续迭代法 (Semi-implicit Successive Substitution Method)
- 场景演示

近场动力学 (Peridynamics) 基础

- Local vs. Nonlocal



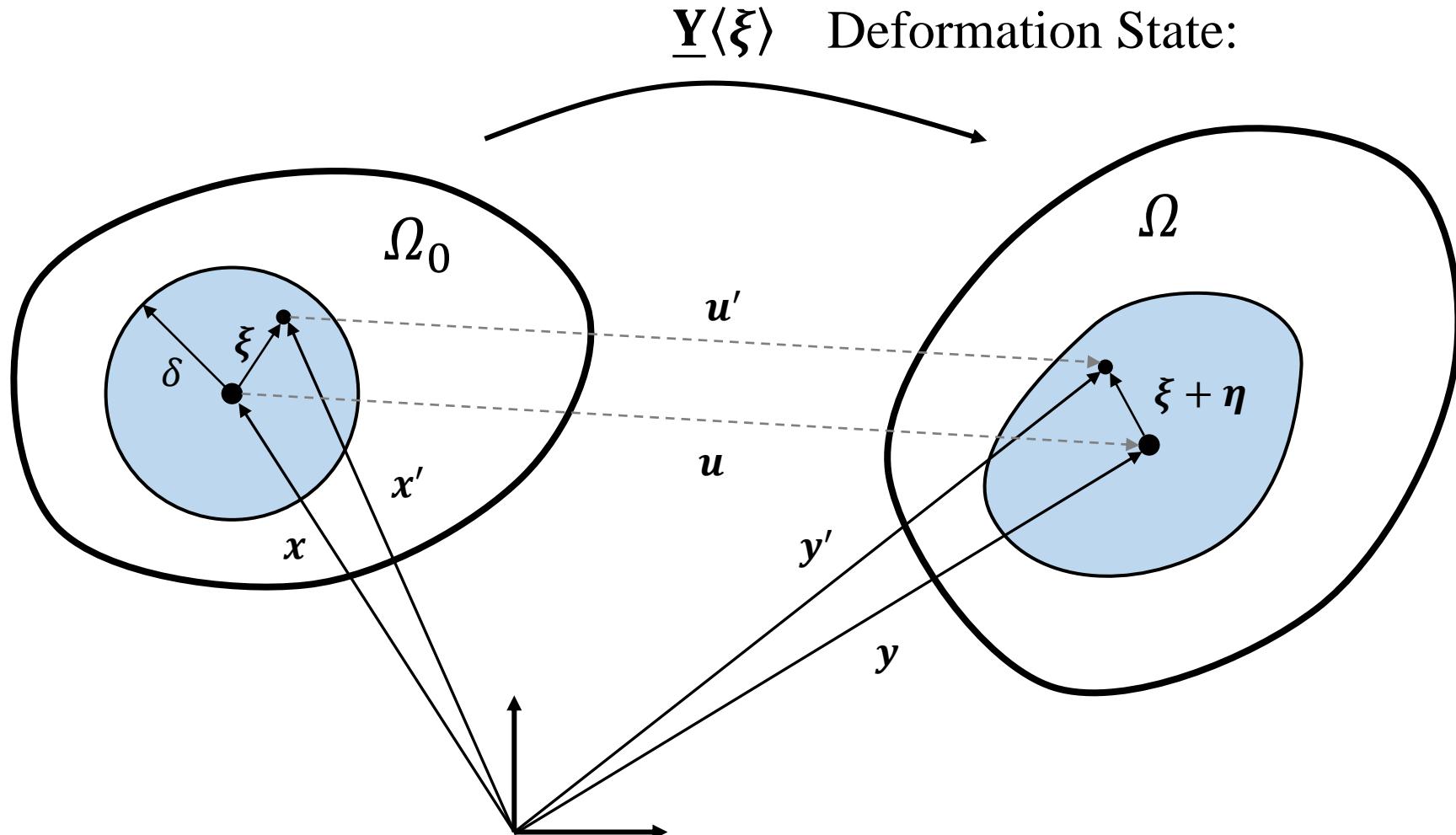
Continuum Mechanics (Local)



Peridynamics (Nonlocal)

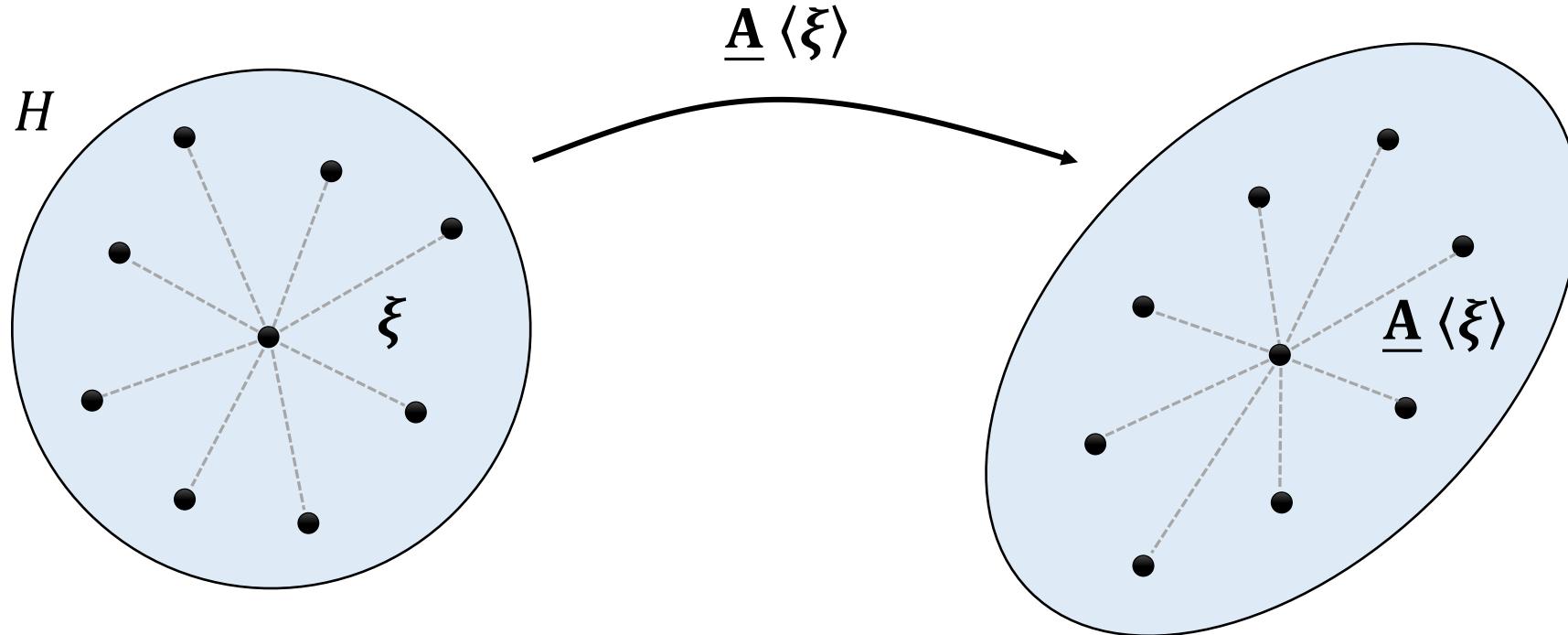
近场动力学 (Peridynamics) 基础

- 变形 (Deformation)



近场动力学 (Peridynamics) 基础

- 键 (Bond) 和态 (State)



Bond: $\xi = x' - x$

State: $\underline{A} \langle \xi \rangle: H \rightarrow L_m$, L_m : m 阶张量

近场动力学 (Peridynamics) 基础

- 常用State

- Scalar State

$\underline{a} \langle \xi \rangle : H \rightarrow \mathbb{R}$, \mathbb{R} : 实数空间

举例: 权重函数

$$\underline{a} \langle \xi \rangle \equiv W(\|\xi\|)$$

- Vector State

$\underline{V} \langle \xi \rangle : H \rightarrow \mathbb{R}^3$, \mathbb{R}^3 : 三维笛卡尔空间

变形态

$$\underline{V} \langle \xi \rangle \equiv F\xi$$

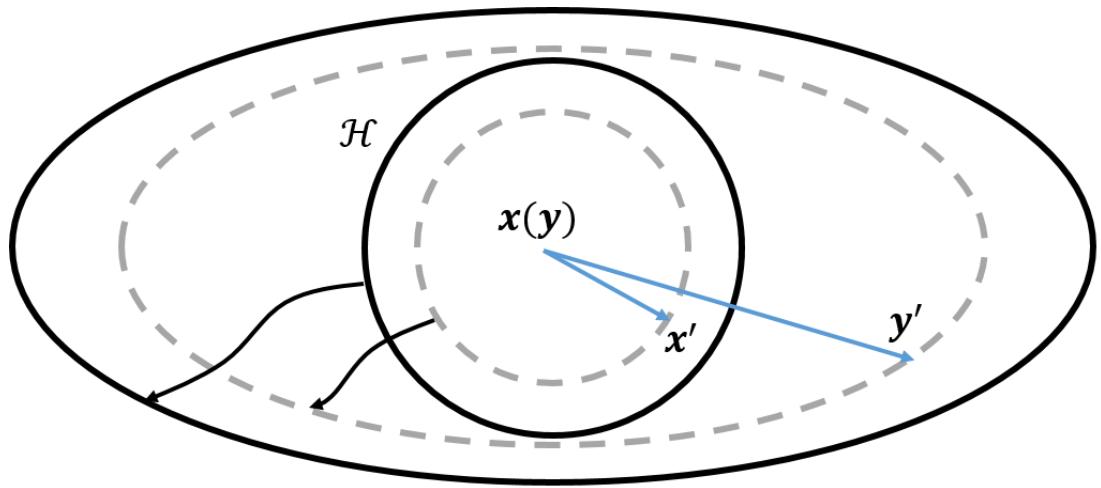
近场动力学 (Peridynamics) 基础

• 态的代数运算

设 $\mathbf{A}, \mathbf{B} \in \mathcal{A}_n$, \mathbf{A} 和 \mathbf{B} 的和 (Sum)	$(\mathbf{A} + \mathbf{B})\langle \xi \rangle = \mathbf{A}\langle \xi \rangle + \mathbf{B}\langle \xi \rangle$
设 $\mathbf{A} \in \mathcal{A}_n, \mathbf{B} \in \mathcal{A}_l$, \mathbf{A} 和 \mathbf{B} 的组合 (Composition)	$(\mathbf{A} \circ \mathbf{B})\langle \xi \rangle = \mathbf{A}\langle \mathbf{B}\langle \xi \rangle \rangle$
设 $\mathbf{A} \in \mathcal{A}_{m+n}, \mathbf{B} \in \mathcal{A}_n$, \mathbf{A} 和 \mathbf{B} 的点积 (Point product)	$(\mathbf{AB})_{i_1 \dots i_m} \langle \xi \rangle = A_{i_1 \dots i_m j_1 \dots j_n} \langle \xi \rangle B_{j_1 \dots j_n} \langle \xi \rangle$
设 $\mathbf{A} \in \mathcal{A}_{m+n}, \mathbf{B} \in \mathcal{A}_n$, \mathbf{A} 和 \mathbf{B} 的内积 (Dot product)	$\mathbf{A} \bullet \mathbf{B} = \int_{\mathcal{H}} (\mathbf{AB})\langle \xi \rangle d\xi$
设 $\mathbf{A} \in \mathcal{A}_n$, \mathbf{A} 的长度 (Magnitude)	$ \mathbf{A} \langle \xi \rangle = \sqrt{(\mathbf{AA})\langle \xi \rangle}$
设 $\mathbf{A} \in \mathcal{A}_n$, \mathbf{A} 的模 (Norm)	$\ \mathbf{A}\ = \sqrt{\mathbf{A} \bullet \mathbf{A}}$
设 $\mathbf{A} \in \mathcal{A}_n$, \mathbf{A} 的方向 (Direction) 定义为	$(\text{Dir}\mathbf{A})\langle \xi \rangle = \begin{cases} 0 & , \text{ 如果 } \mathbf{A} \langle \xi \rangle = 0 \\ \mathbf{A}\langle \xi \rangle / \mathbf{A} \langle \xi \rangle & , \text{ 其他} \end{cases}$
设 $\mathbf{A}, \mathbf{B} \in \mathcal{A}_l$, 则 \mathbf{A} 与 \mathbf{B} 的二阶张量	$\mathbf{A} * \mathbf{B} = \int_{\mathcal{H}} \omega \langle \xi \rangle \mathbf{A}\langle \xi \rangle \otimes \mathbf{B}\langle \xi \rangle d\xi$

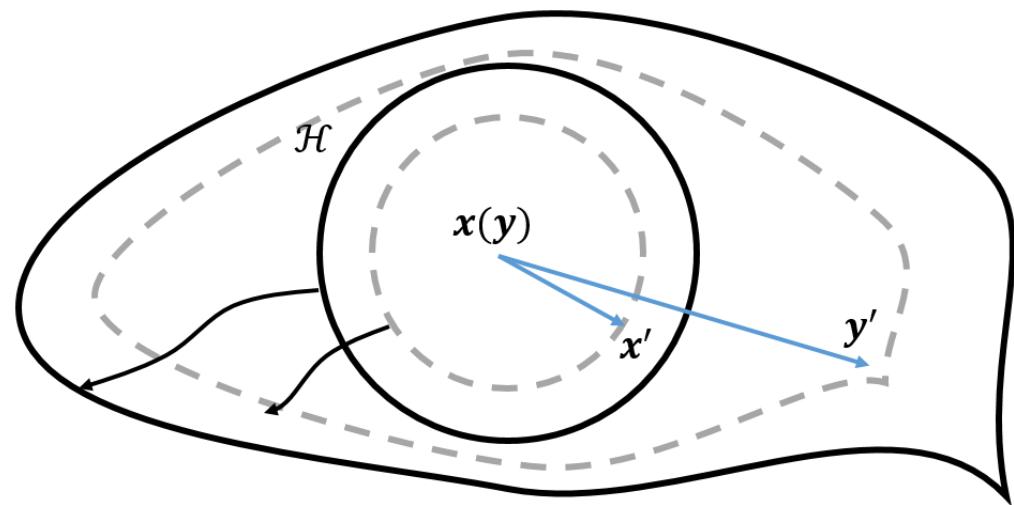
近场动力学 (Peridynamics) 基础

- 应变向量态 (Deformation Vector State)



$$[\boldsymbol{\varepsilon}] = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix}$$

Continuum Mechanics (Local)

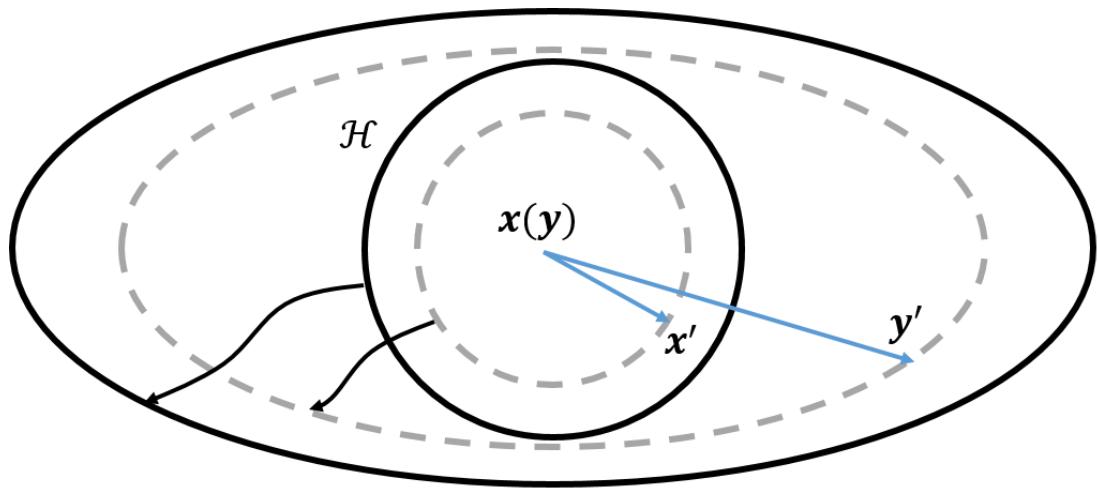


$$\mathbf{Y}[\mathbf{x}, t]\langle \xi \rangle = \mathbf{y}'(\mathbf{x} + \xi, t) - \mathbf{y}(\mathbf{x}, t)$$

Peridynamics (Nonlocal)

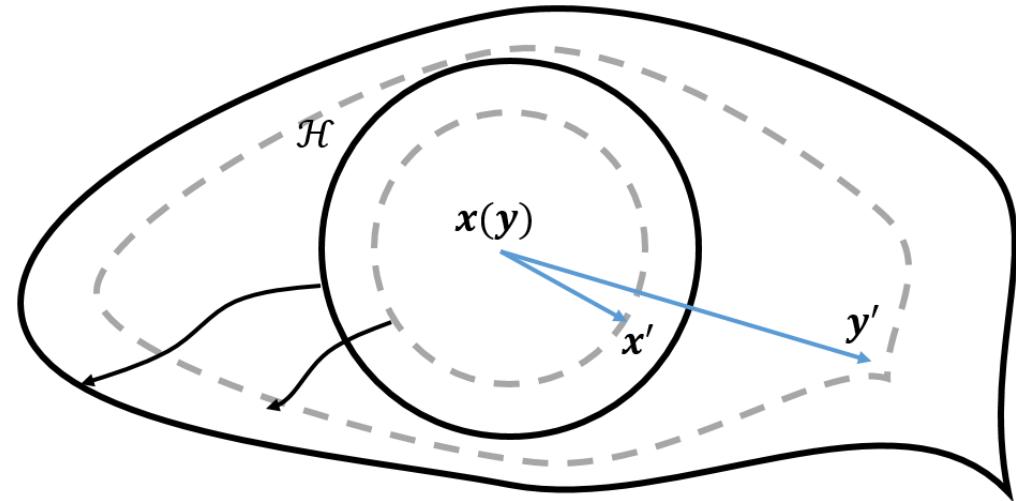
近场动力学 (Peridynamics) 基础

- 应力向量态 (Force Vector State)



$$\boldsymbol{\sigma} = C \boldsymbol{\varepsilon} \quad [\boldsymbol{\sigma}] = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix}$$

Continuum Mechanics (Local)

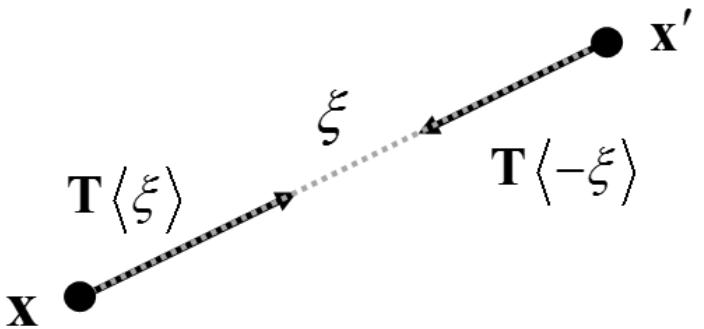


$$\mathbf{T} = \mathbf{T}(\mathbf{Y}, \Lambda)$$

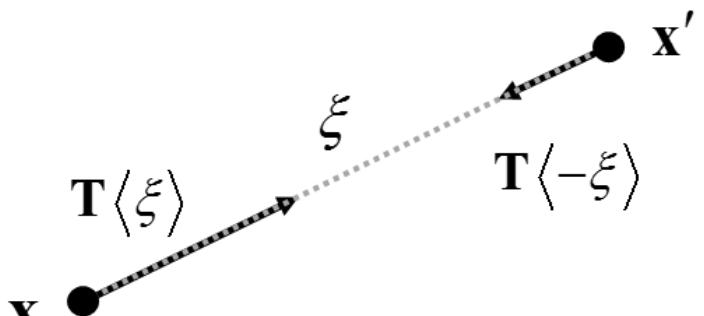
Peridynamics (Nonlocal)

近场动力学 (Peridynamics) 基础

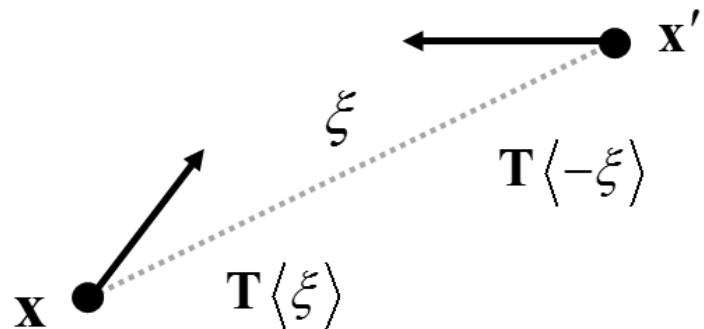
- 三种典型的非局部应力



Bond based



Ordinary state based



Non-ordinary state based

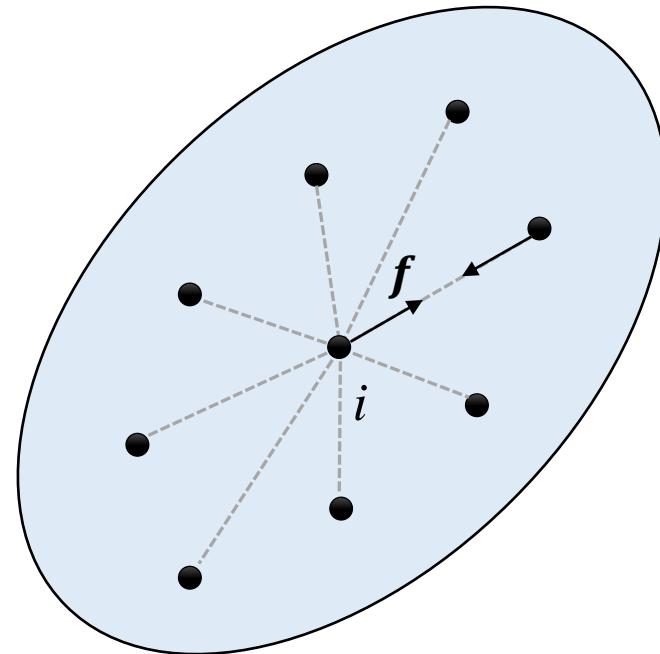
近场动力学 (Peridynamics) 基础

- 动量方程

$$\rho \ddot{\mathbf{y}} = \int_{\mathcal{H}_x} \left\{ \mathbf{T}[\mathbf{x}, t] \langle \xi \rangle - \mathbf{T}[\mathbf{x}', t] \langle -\xi \rangle \right\} dV_\xi + \mathbf{g}$$



$$\rho \ddot{\mathbf{y}}_i = \sum_j \left(\mathbf{T}_i[\mathbf{x}, t] \langle \xi \rangle - \mathbf{T}_j[\mathbf{x}', t] \langle -\xi \rangle \right) V_j + \mathbf{g}$$



$$f = \mathbf{T}[\mathbf{x}, t] \langle \xi \rangle - \mathbf{T}[\mathbf{x}', t] \langle -\xi \rangle$$

投影近场动力学 (Projective Peridynamics)

- 约束优化问题

$$\mathbf{y}^{n+1} = \mathbf{y}^n + h\mathbf{v}^{n+1}$$

$$\mathbf{v}^{n+1} = \mathbf{v}^n + h\mathbf{M}^{-1} \left(\mathbf{f}_{int}(\mathbf{y}^{n+1}) + \mathbf{f}_{ext} \right)$$



$$\min_{\mathbf{y}} \frac{1}{2h^2} \left\| \mathbf{M}^{\frac{1}{2}} (\mathbf{y} - \mathbf{s}^n) \right\|^2 + \sum_i W_i(\mathbf{y}), \quad \mathbf{s}^n = \mathbf{q}^n + h\mathbf{v}^n + h^2 \mathbf{M}^{-1} \mathbf{f}_{ext}$$

半隐式连续迭代法 (SISSM)

- 牛顿法 (Newton's method)

$$f(x) = 0$$

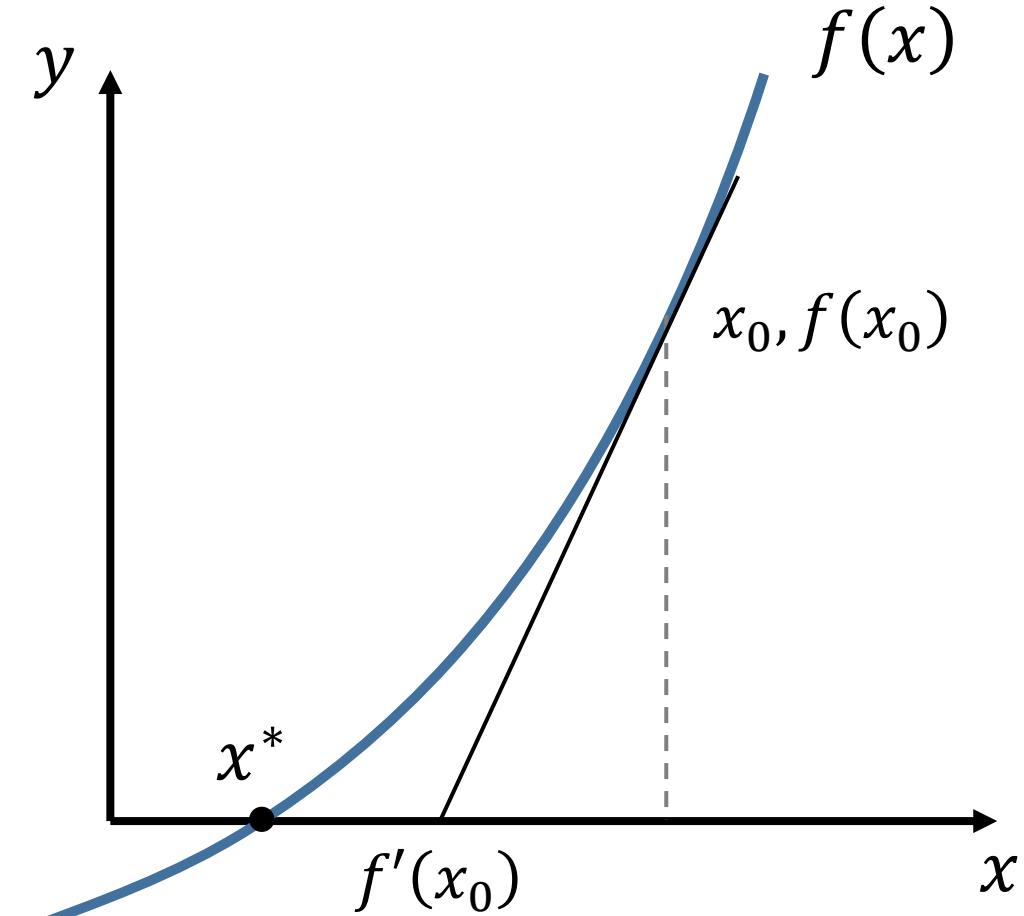


$$f(x_0) + f'(x_0)(x - x_0) = 0$$



$$x_1 = x_0 - \frac{f(x_0)}{f'(x)}$$

$f'(x)$ 的逆计算困难



半隐式连续迭代法 (SISSM)

- 牛顿法 (Newton's method)

Energy Hessian:

$$\frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} = \sum_{k,l} \frac{\partial \mathbf{f}_i}{\partial \mathbf{F}_{kl}} \frac{\partial \mathbf{F}_{kl}}{\partial \mathbf{x}_j} = \sum_{k,l} \frac{\partial \left(\mathbf{U} \text{diag} \left(\frac{\partial W}{\partial \lambda_0}, \frac{\partial W}{\partial \lambda_1}, \frac{\partial W}{\partial \lambda_2} \right) \mathbf{V}^T \right)}{\partial \mathbf{F}_{kl}} - V \mathbf{d}_i \frac{\partial \mathbf{F}_{kl}}{\partial \mathbf{x}_j}$$

constant

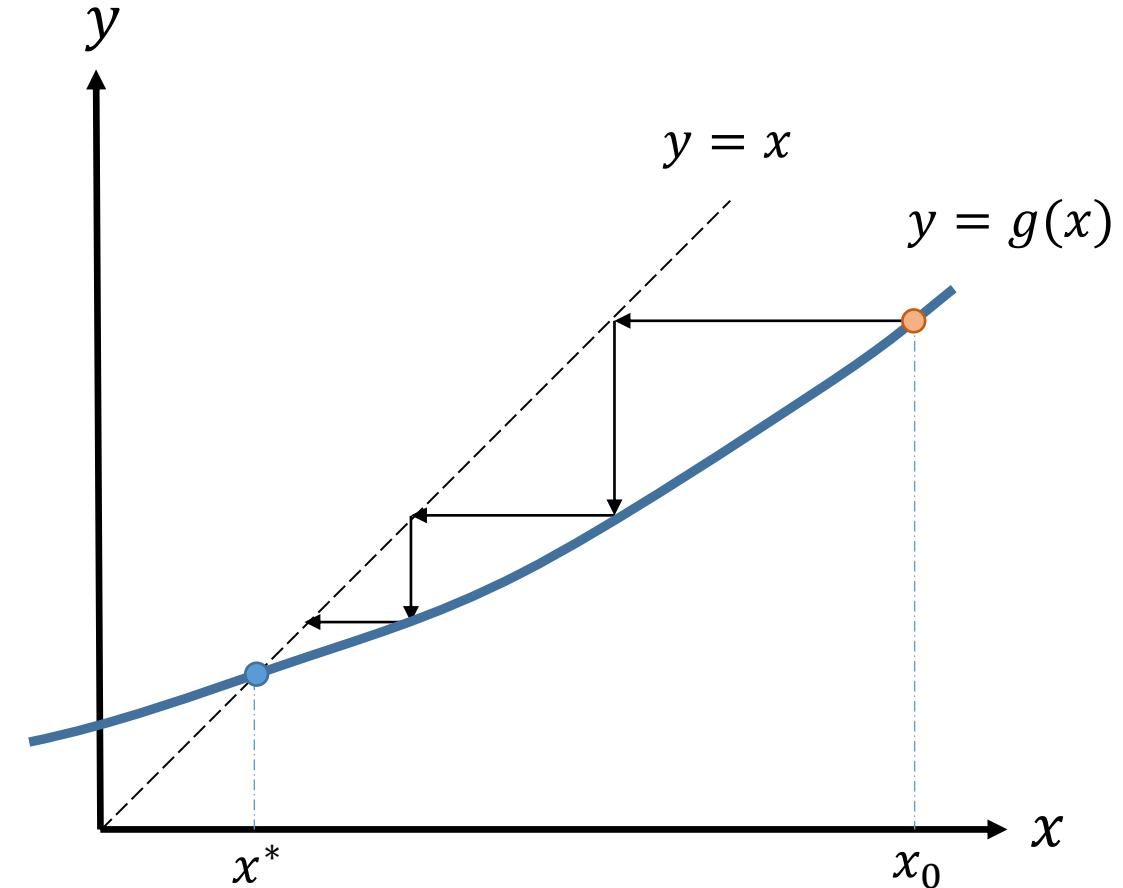
半隐式连续迭代法 (SISSM)

- 连续迭代法 (Successive Substitution Method)

$$\begin{aligned} f(x) &= 0 \\ &\downarrow \\ x = g(x) &\leftrightarrow f(x) = 0 \end{aligned}$$



$$x = g(x_0)$$



收敛条件: $|g'(x)| < 1$

半隐式连续迭代法 (SISSM)

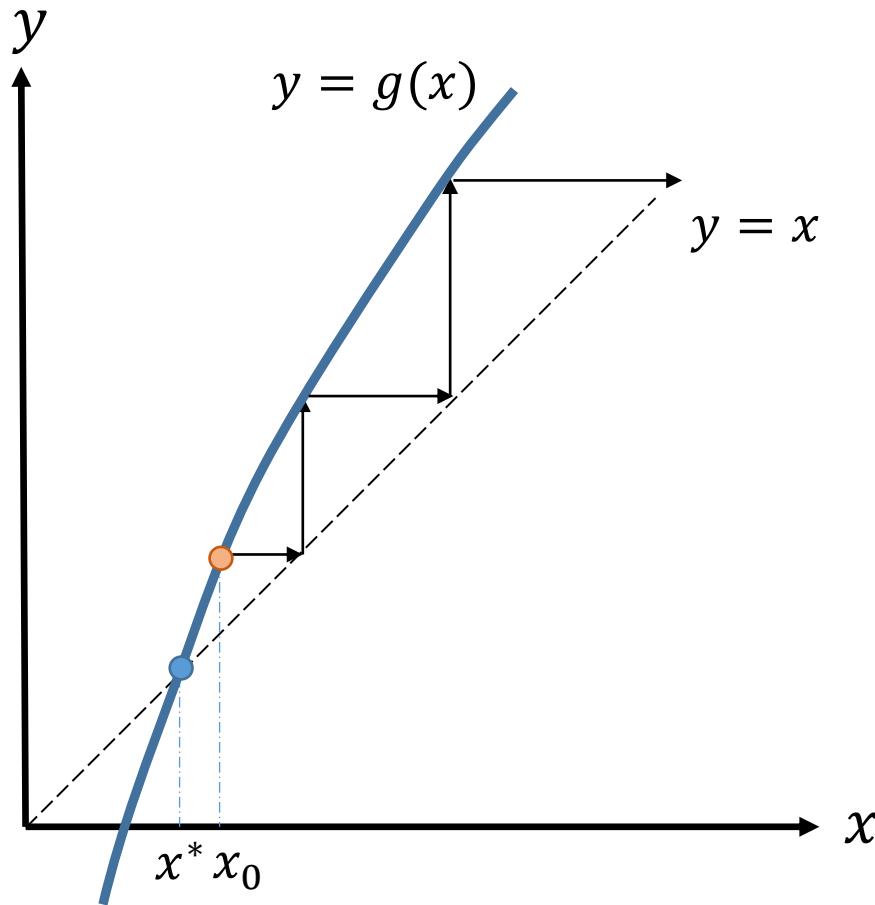
- 连续迭代法 (Successive Substitution Method)

- 验证以下函数什么时候收敛?

$$g(x) = x^{0.5} \longrightarrow g'(x) = 0.5x^{-0.5}$$

$$g(x) = x^2 \longrightarrow g'(x) = 2x$$

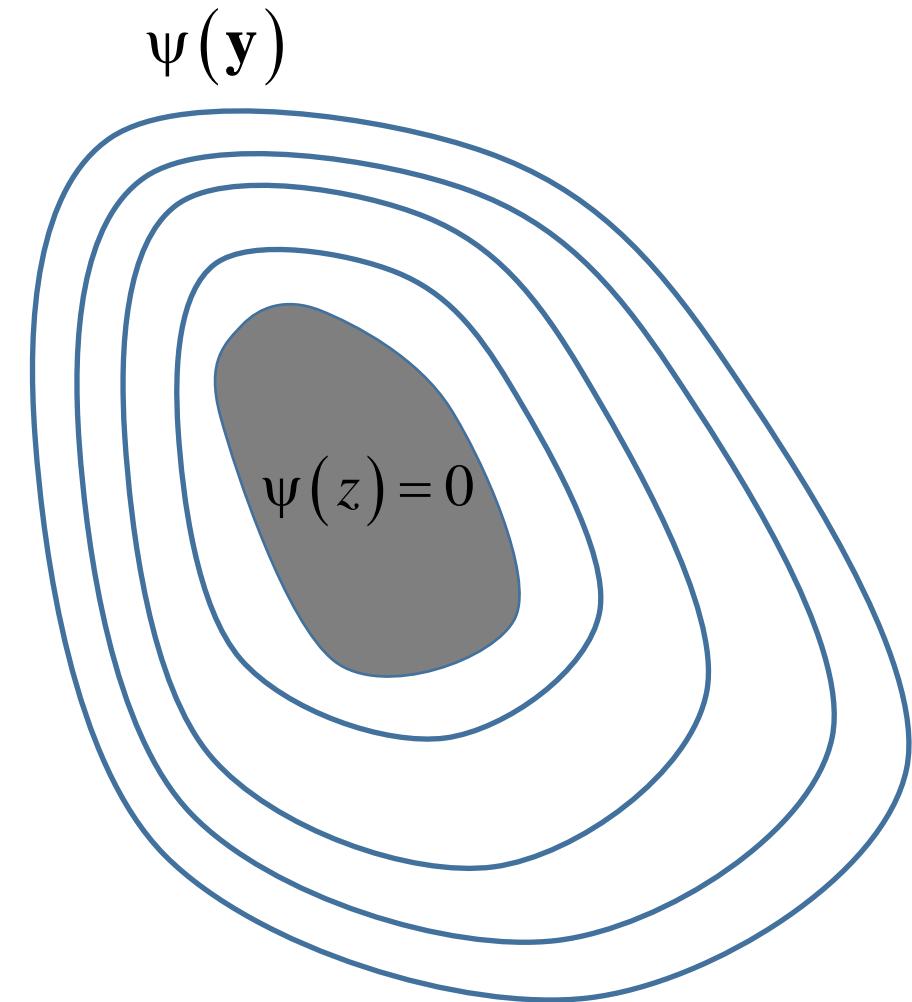
$$g(x) = x^3 \longrightarrow g'(x) = 3x^2$$



投影近场动力学 (Projective Peridynamics)

- 投影动力学 (Projective Dynamics)

$$\min_{\mathbf{y}} \frac{1}{2h^2} \left\| \mathbf{M}^{\frac{1}{2}} (\mathbf{y} - \mathbf{s}^n) \right\|^2 + \sum_i W_i(\mathbf{y})$$



[Bouaziz et al. 2014]

$$\psi(\mathbf{y}) = \sum_i W_i(\mathbf{y})$$

投影近场动力学 (Projective Peridynamics)

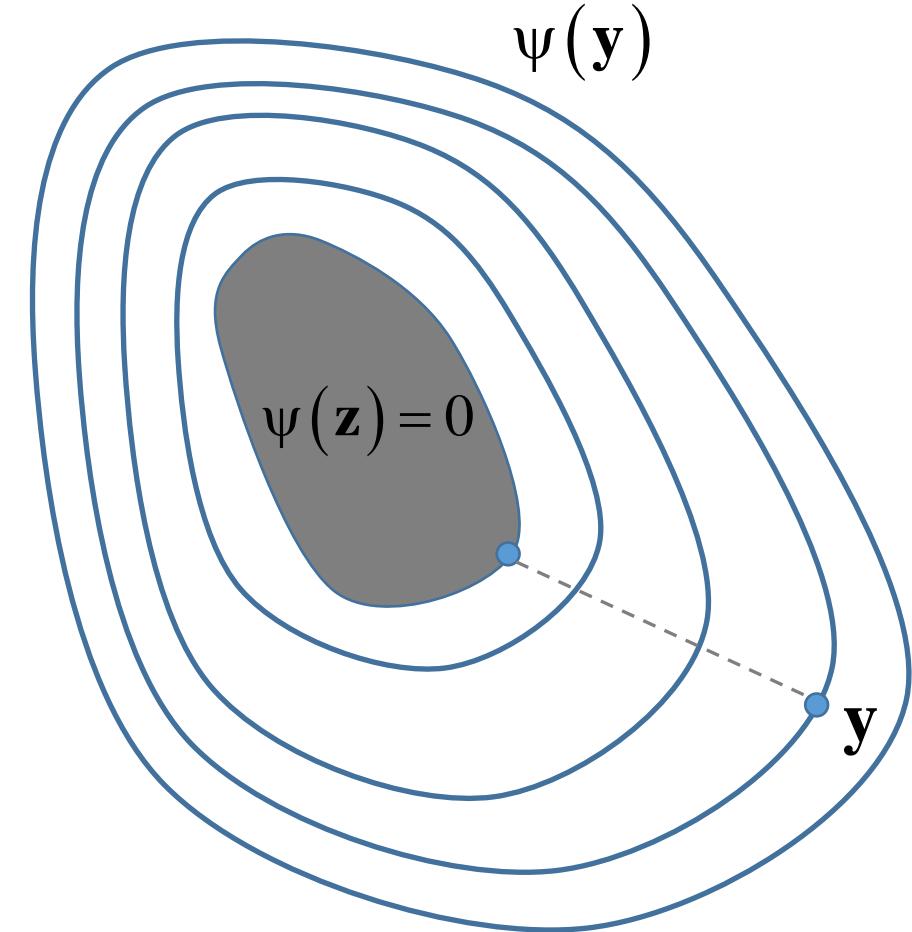
- 投影动力学 (Projective Dynamics)

Global:

$$\min_{\mathbf{y}} \frac{1}{2h^2} \left\| \mathbf{M}^{\frac{1}{2}} (\mathbf{y} - \mathbf{s}^n) \right\|^2 + \frac{w}{2} \left\| \mathbf{A}\mathbf{y} - \mathbf{B}\mathbf{z} \right\|_F^2 + \delta_C(\mathbf{z})$$

Local:

$$\min_{\mathbf{z}} \frac{w}{2} \left\| \mathbf{A}\mathbf{y} - \mathbf{B}\mathbf{z} \right\|_F^2 + \delta_C(\mathbf{z})$$



$$\psi(\mathbf{y}) = \sum_i W_i(\mathbf{y})$$

投影近场动力学 (Projective Peridynamics)

- Limitation with projective dynamics
 - Global solve cannot handle nonlinear problem
 - Local solve cannot handle the diversity in problem stiffness

$$\arg \min_{\mathbf{y}} \frac{1}{2h^2} \left\| \mathbf{M}^{\frac{1}{2}} (\mathbf{y} - \mathbf{y}^*) \right\|_F^2 + \Psi(\mathbf{y}) + B(\mathbf{y})$$

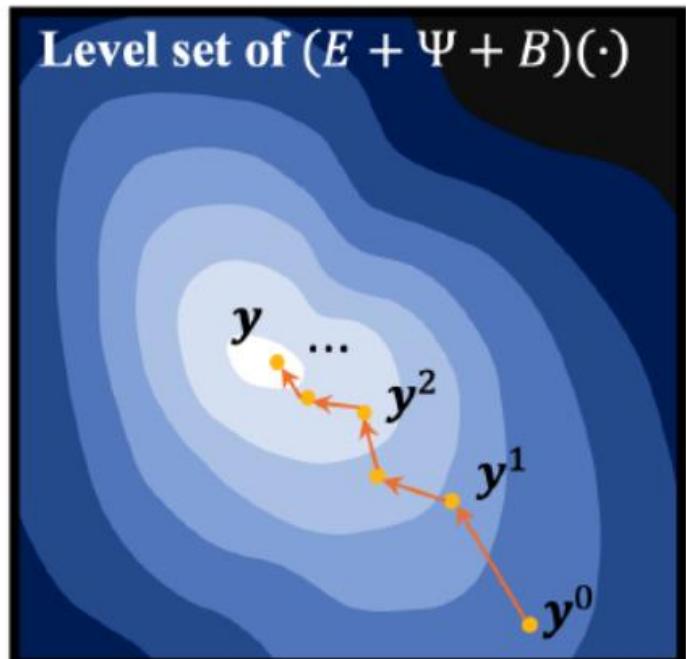
$$B(d) = \begin{cases} -(d - \hat{d})^2 \log\left(\frac{d}{\hat{d}}\right), & 0 < d < \hat{d} \\ 0, & d \geq \hat{d} \end{cases}$$



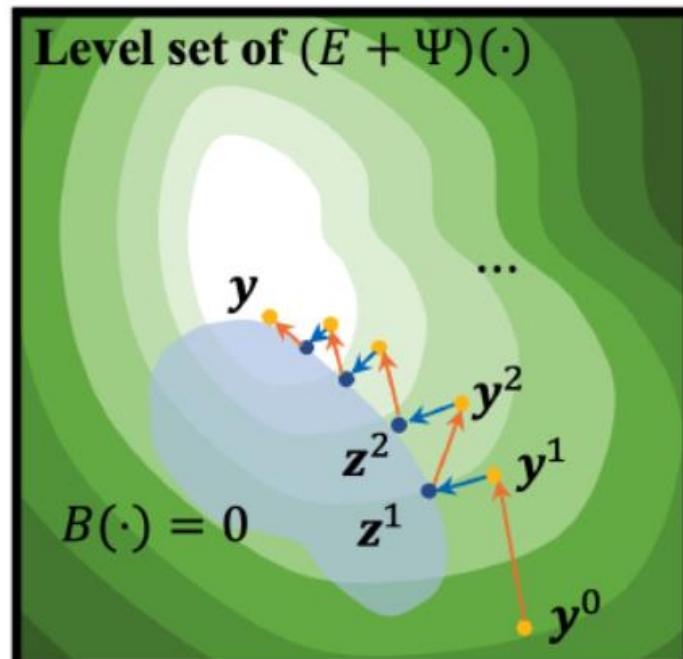
投影近场动力学 (Projective Peridynamics)

- 投影近场动力学

$$\min_{\mathbf{y}} \frac{1}{2h^2} \left\| \mathbf{M}^{\frac{1}{2}} (\mathbf{y} - \mathbf{y}^*) \right\|_F^2 + \Psi(\mathbf{y}) + \frac{1}{2} \|\mathbf{z} - \mathbf{y}\|_2^2, \quad B(\mathbf{z}) = 0$$



(a) Solve original problem.



(b) With constraint projection.

半隐式连续迭代法 (SISSM)

- Global step (假设 $m = 1, h = 1$)

$$E_i = \frac{1}{2} \left\| \mathbf{y}_i^{n+1} - \mathbf{y}_i^* \right\|^2 + \psi(r_{ij}^{n+1}), \quad r_{ij}^{n+1} = \left\| \mathbf{y}_i^{n+1} - \mathbf{y}_j^{n+1} \right\|$$

$$\frac{\partial E_i}{\partial \mathbf{y}_i^{n+1}} = (\mathbf{y}_i^{n+1} - \mathbf{y}_i^*) + \sum_j \psi'(r_{ij}^{n+1}) \frac{\mathbf{y}_i^{n+1} - \mathbf{y}_j^{n+1}}{\|r_{ij}^{n+1}\|} = 0$$

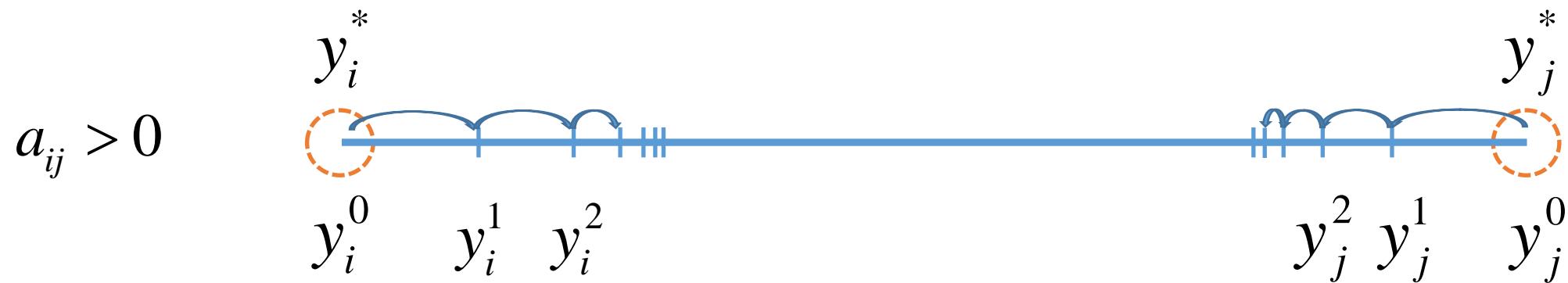
$$\mathbf{y}_i^{n+1} = \mathbf{y}_i^* + \sum_j \psi'(r_{ij}^{n+1}) \frac{\mathbf{y}_j^{n+1} - \mathbf{y}_i^{n+1}}{\|r_{ij}^{n+1}\|}$$

$$= \mathbf{y}_i^* + \sum_j a_{ij}^t (\mathbf{y}_j^{n+1} - \mathbf{y}_i^{n+1}) \quad a_{ij}^n = \frac{\psi'(r_{ij}^n)}{\|r_{ij}^n\|}$$

半隐式连续迭代法 (SISSM)

- Global step (假设 $m = 1, h = 1$)

$$\mathbf{y}_i^{n+1} = \frac{1}{\left(1 + \sum_j a_{ij}^n\right)} \mathbf{y}_i^* + \sum_j \frac{a_{ij}^n}{\left(1 + \sum_j a_{ij}^n\right)} \mathbf{y}_j^{n+1}, \quad a_{ij}^n = \frac{\psi'(r_{ij}^n)}{\|r_{ij}^n\|}$$



半隐式连续迭代法 (SISSM)

- Global step (假设 $m = 1, h = 1$)

$$\mathbf{y}_i^{n+1} = \frac{1}{\left(1 + \sum_j a_{ij}^n\right)} \mathbf{y}_i^* + \sum_j \frac{a_{ij}^n}{\left(1 + \sum_j a_{ij}^n\right)} \mathbf{y}_j^{n+1}, \quad a_{ij}^n = \frac{\psi'(r_{ij}^n)}{\|r_{ij}^n\|}$$



半隐式连续迭代法 (SISSM)

- Global step (假设 $m = 1, h = 1$)

$$\mathbf{y}_i^{n+1} = \mathbf{y}_i^* + \sum_j a_{ij}^n (\mathbf{y}_j^{n+1} - \mathbf{y}_i^{n+1}) \quad a_{ij}^n = \frac{\psi'(r_{ij}^n)}{\|r_{ij}^n\|}$$

$$= \mathbf{y}_i^* + \sum_j (a_{ij}^n)^+ (\mathbf{y}_j^{n+1} - \mathbf{y}_i^{n+1}) + \sum_j (a_{ij}^n)^- (\mathbf{y}_j^{n+1} - \mathbf{y}_i^{n+1})$$

$$= \mathbf{y}_i^* + \underbrace{\sum_j (a_{ij}^n)^+ (\mathbf{y}_j^{n+1} - \mathbf{y}_i^{n+1})}_{\text{implicit}} + \underbrace{\sum_j (a_{ij}^n)^- (\mathbf{y}_j^n - \mathbf{y}_i^n)}_{\text{explicit}}$$

半隐式连续迭代法 (SISSM)

- Global step (假设 $m = 1, h = 1$)

$$\mathbf{y}_i^{n+1} = \frac{1}{\left(1 + \sum_j (a_{ij}^n)^+\right)} \mathbf{y}_i^* + \sum_j \frac{(a_{ij}^n)^+}{\left(1 + \sum_j (a_{ij}^n)^+\right)} \mathbf{y}_j^{n+1} + \sum_j \frac{(a_{ij}^n)^-}{\left(1 + \sum_j (a_{ij}^n)^+\right)} (\mathbf{y}_j^n - \mathbf{y}_i^n),$$

半隐式连续迭代法 (SISSM)

- 非线性一维弹簧

$$x = (x - x_1 - 1)^2$$

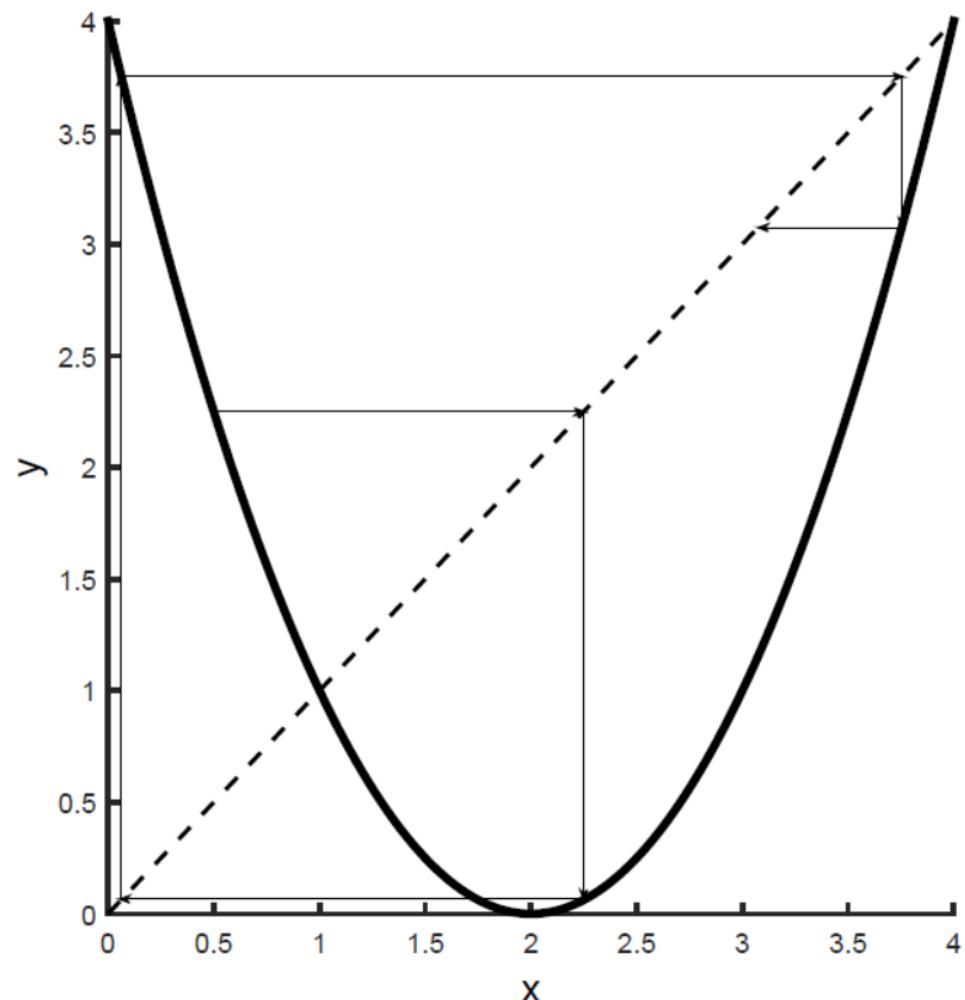
$$x = (x - 2)^2$$

$$x^0 = 0.5$$

$$x^1 = (x^0 - 2)^2 = 2.25$$

$$x^2 = (x^1 - 2)^2 = 0.0625$$

$$x^3 = \dots$$



半隐式连续迭代法 (SISSM)

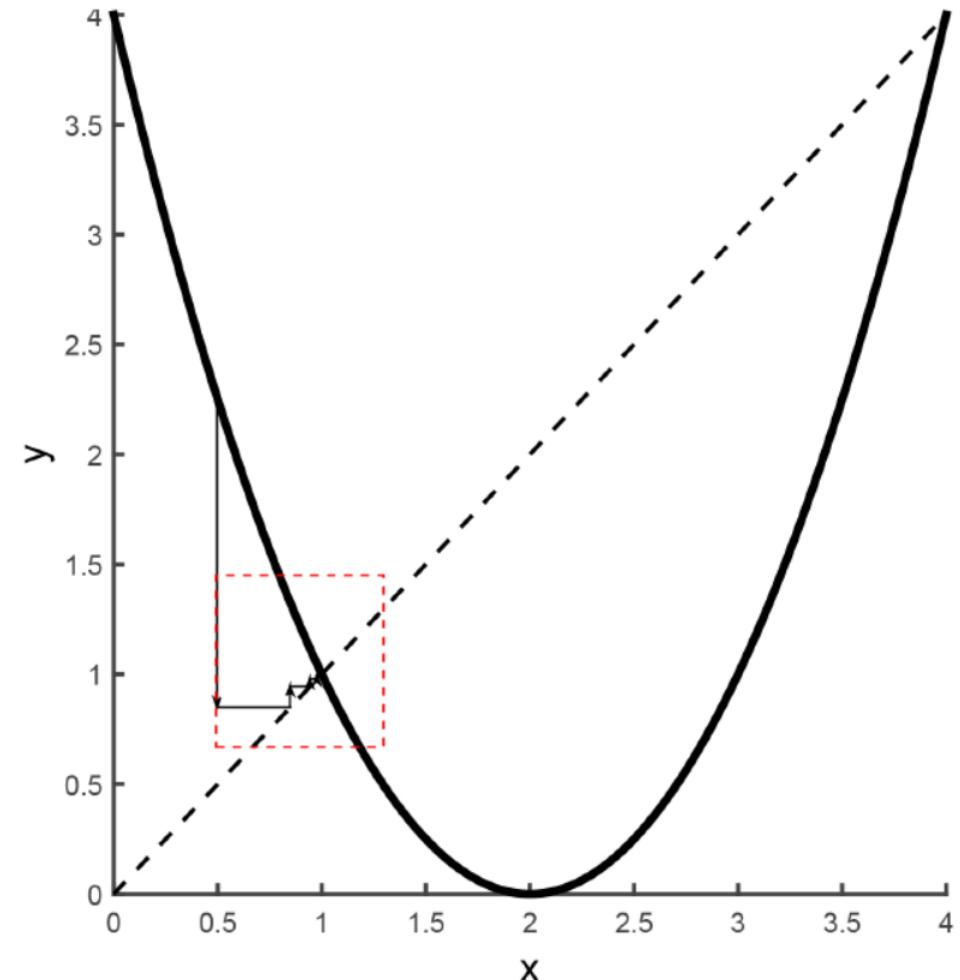
- 非线性一维弹簧

$$x = (x - 2)^2$$

$$x = \begin{matrix} x^2 \\ \text{explicit} \end{matrix} - \begin{matrix} 4x \\ \text{implicit} \end{matrix} + 4, \quad x \in [0, 1]$$

$$x^{k+1} = (x^k)^2 - 4x^{k+1} + 4, \quad x \in [0, 1]$$

$$x^{k+1} = \frac{1}{5}(x^k)^2 + \frac{4}{5}, \quad x \in [0, 1]$$



半隐式连续迭代法 (SISSM)

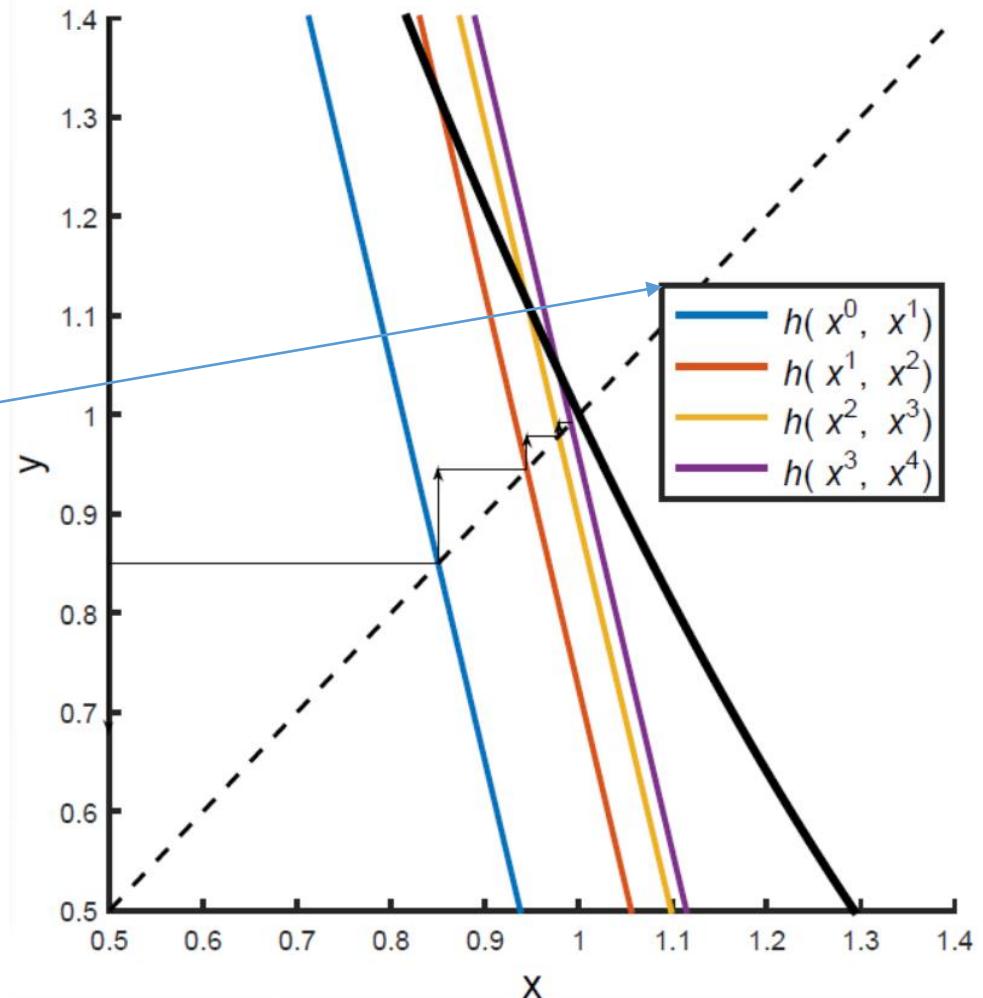
- 非线性一维弹簧

$$x = (x - 2)^2$$

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$$x^{k+1} = \boxed{(x^k)^2 - 4x^{k+1} + 4}, \quad x \in [0, 1]$$

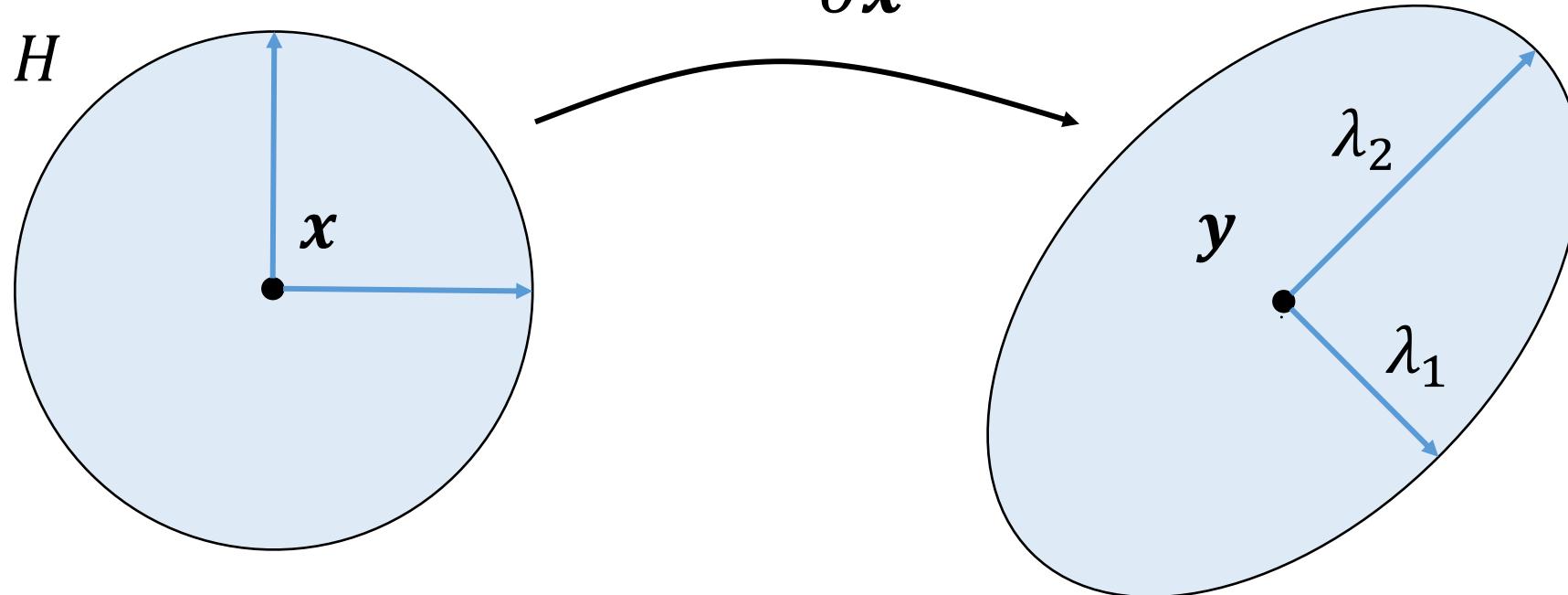
$$x^{k+1} = \frac{1}{5}(x^k)^2 + \frac{4}{5}, \quad x \in [0, 1]$$



半隐式连续迭代法 (SISSM)

- 超弹性材料

$$F = \frac{\partial y}{\partial x}$$



$$F = RU = VR$$

半隐式连续迭代法 (SISSM)

- 超弹性材料应变能

$$\psi(I_1, I_2, I_3)$$

$$I_1 = \text{trace}(\mathbf{C}) = \lambda_1^2 + \lambda_2^2 + \lambda_3^2 \quad \mathbf{C} = \mathbf{F}^T \mathbf{F}$$

$$I_2 = \mathbf{C} : \mathbf{C} = \lambda_1^4 + \lambda_2^4 + \lambda_3^4$$

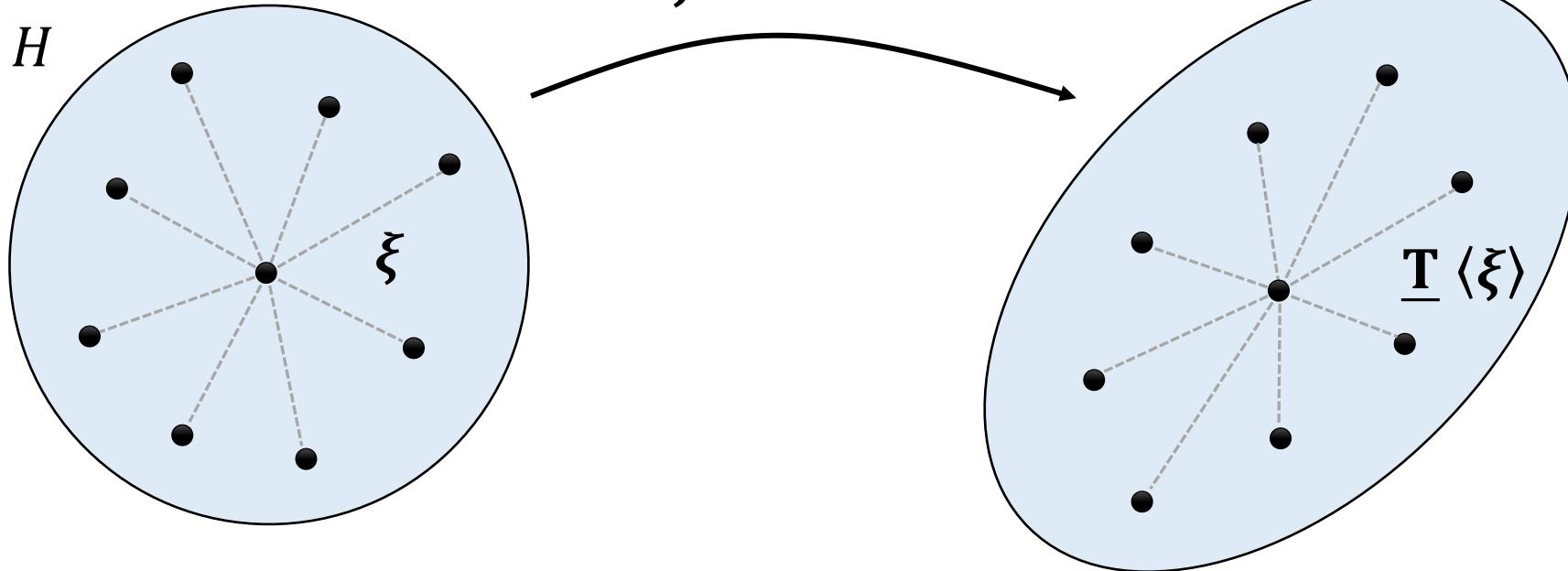
$$I_3 = \mathbf{C} : \mathbf{C} = \lambda_1^2 \lambda_2^2 \lambda_3^2$$

半隐式连续迭代法 (SISSM)

- 应变向量态

[Silling et al. 2007]

$$\underline{\mathbf{T}} \langle \xi \rangle = \frac{\partial \psi}{\partial \xi} = \omega_{ij} \mathbf{P}(\mathbf{F}) \mathbf{K}^{-1} \xi$$



$$\mathbf{K}_i = \mathbf{x} * \mathbf{x} = \sum_j \omega_{ij} (\mathbf{x}_j - \mathbf{x}_i) (\mathbf{x}_j - \mathbf{x}_i)^T \quad \mathbf{F} = \mathbf{y} * \mathbf{x} = \left(\sum_j \omega_{ij} (\mathbf{y}_j - \mathbf{y}_i) (\mathbf{x}_j - \mathbf{x}_i)^T \right) \mathbf{K}_i^{-1}$$

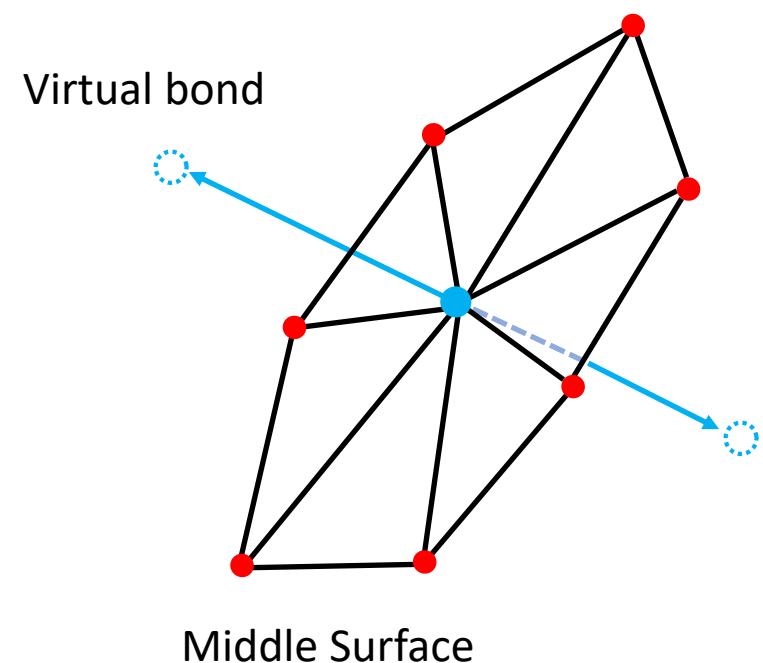
半隐式连续迭代法 (SISSM)

- 应变向量态

$$\mathbf{K}_i = \sum_j \omega_{ij} (\mathbf{x}_j - \mathbf{x}_i)(\mathbf{x}_j - \mathbf{x}_i)^T + \boxed{\mathbf{x}_i^\perp \otimes \mathbf{x}_i^\perp}$$

$$\mathbf{F}_i = \left(\sum_j \omega_{ij} (\mathbf{y}_j - \mathbf{y}_i)(\mathbf{x}_j - \mathbf{x}_i)^T + \boxed{\mathbf{y}_i^\perp \otimes \mathbf{x}_i^\perp} \right) \mathbf{K}_i^{-1}$$

$$\mathbf{x}_i^\perp = \xi \text{norm} \left(\frac{\sum_J \theta_J \mathbf{n}_J}{\sum_J \theta_J} \right), \mathbf{y}_i^\perp = \xi \text{norm} \left(\frac{\sum_J \theta_J^* \mathbf{n}_J^*}{\sum_J \theta_J^*} \right)$$



半隐式连续迭代法 (SISSM)

- 应变向量态（各向同性材料）

$$\underline{\mathbf{T}} \langle \xi \rangle = \omega_{ij} \mathbf{P}(\mathbf{F}) \mathbf{K}^{-1} \xi$$



$$\underline{\mathbf{T}} \langle \xi \rangle = \omega_{ij} \mathbf{U}_i \hat{\mathbf{P}}_i(\hat{\mathbf{F}}_i) \mathbf{V}_i^T \mathbf{K}_i^{-1} \xi$$



$$\underline{\mathbf{T}} \langle \xi \rangle = \omega_{ij} \mathbf{U}_i \hat{\mathbf{P}}_i^+(\hat{\mathbf{F}}_i) \mathbf{V}_i^T \mathbf{K}_i^{-1} \xi + \omega_{ij} \mathbf{U}_i \hat{\mathbf{P}}_i^-(\hat{\mathbf{F}}_i) \mathbf{V}_i^T \mathbf{K}_i^{-1} \xi$$

[Irving et al. 2004]

$$\mathbf{P}_i = \mathbf{U}_i \hat{\mathbf{P}}_i(\hat{\mathbf{F}}_i) \mathbf{V}_i^T$$

$$[\hat{\mathbf{F}}] = \begin{bmatrix} \lambda_0 & 0 & 0 \\ 0 & \lambda_1 & 0 \\ 0 & 0 & \lambda_2 \end{bmatrix}$$

半隐式连续迭代法 (SISSM)

- 应变向量态（各向同性材料）

$$\underline{\mathbf{T}}\langle\xi\rangle = \kappa_{ij} \mathbf{U}_i \widehat{\mathbf{P}}_i^+ (\widehat{\mathbf{F}}_i) \widehat{\mathbf{F}}_i^{-1} \mathbf{U}_i^T (y_j - y_i) \quad \text{----- implicit}$$

$$+ \kappa_{ij} \mathbf{U}_i \widehat{\mathbf{P}}_i^- (\widehat{\mathbf{F}}_i) \mathbf{V}_i^T (x_j - x_i) \quad \text{----- explicit}$$

$$(y_j - y_i) = \mathbf{F}_i (x_j - x_i)$$

$$\kappa_{ij} = \frac{16}{15} \pi \omega_{ij} r_i^5$$

半隐式连续迭代法 (SISSM)

- 运动控制方程

$$\mathbf{y}_i^{k+1} = \left(m_i \mathbf{I} + \mathbf{A}_i^k \right)^{-1} \left(\sum_j \mathbf{A}_{ij}^k \mathbf{y}_j^k + \mathbf{s}_i^k + \mathbf{s}_i^t \right)$$

$$\mathbf{A}_{ij}^k = h^2 V_i \left(\kappa_{ij} \mathbf{U}_i \hat{\mathbf{P}}_i^+ \hat{\mathbf{F}}_i^{-1} \mathbf{U}_i^T + \kappa_{ij} \mathbf{U}_j \hat{\mathbf{P}}_j^+ \hat{\mathbf{F}}_j^{-1} \mathbf{U}_j^T \right) V_j$$

$$\mathbf{s}_i^k = h^2 V_i \sum_j \left(\kappa_{ij} \mathbf{U}_i \hat{\mathbf{P}}_i^- \mathbf{V}_i^T + \kappa_{ij} \mathbf{U}_j \hat{\mathbf{P}}_j^- \mathbf{V}_j^T \right) V_j \cdot (\mathbf{x}_j - \mathbf{x}_i)$$

$$\mathbf{s}_i^t = m_i (\mathbf{y}_i^t + h \mathbf{v}_i^*)$$

$$\mathbf{A}_i^k = \sum_j \mathbf{A}_{ij}^k .$$

半隐式连续迭代法 (SISSM)

- 收敛性

$$\left\| \left(m_i \mathbf{I} + \mathbf{A}_i^k \right)^{-1} \sum_j \mathbf{A}_{ij}^k \right\| < 1, \quad k = 0, 1, \dots.$$

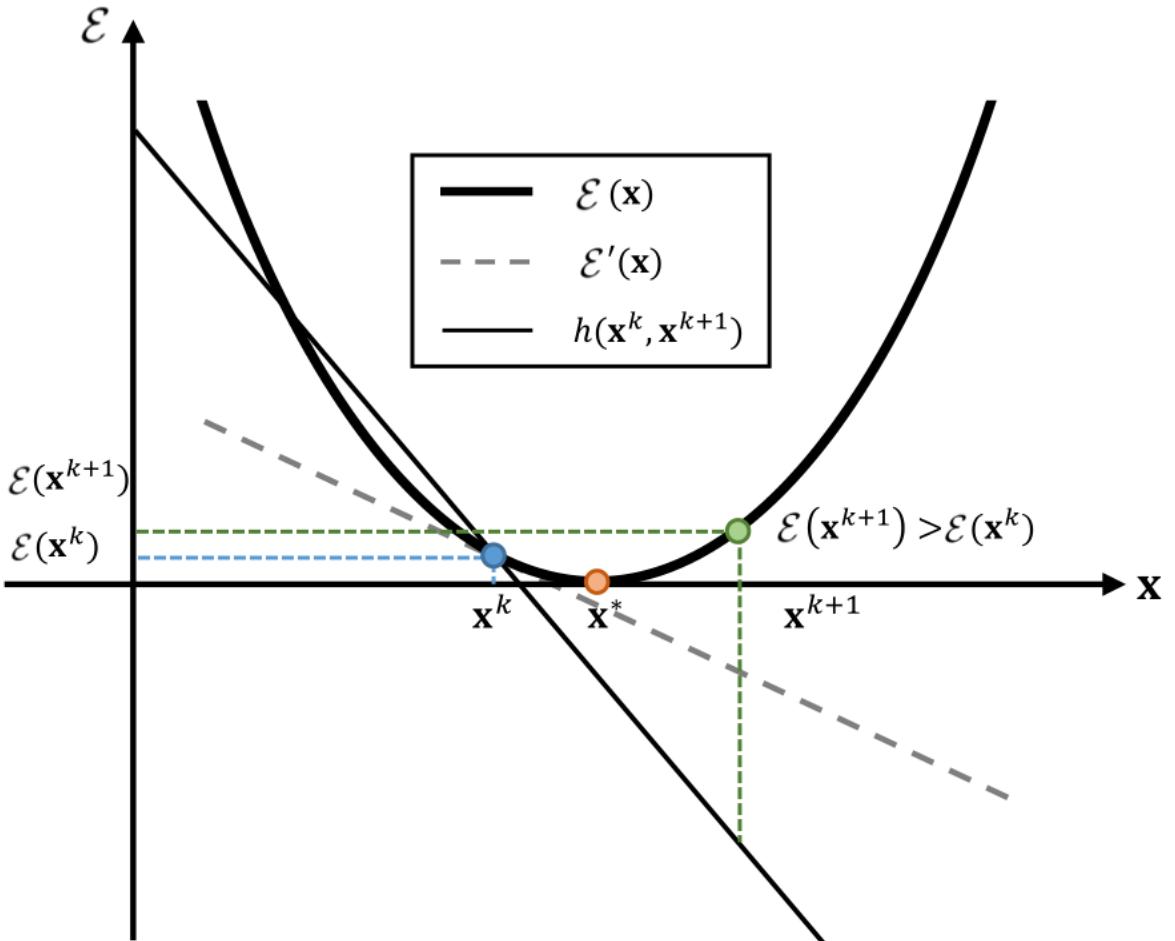
半隐式连续迭代法 (SISSM)

- Over shooting

$$\mathbf{y}_i^{k+1} = \mathbf{y}_i^k + \alpha_i (\mathbf{y}_i^{k+1} - \mathbf{y}_i^k)$$

$$\mathcal{E}(\mathbf{y}) = \mathcal{E}(\mathbf{y}_i^k) + \alpha \frac{\partial \mathcal{E}_i}{\partial \mathbf{y}_i} \cdot (\mathbf{y}_i^{k+1} - \mathbf{y}_i^k) = 0$$

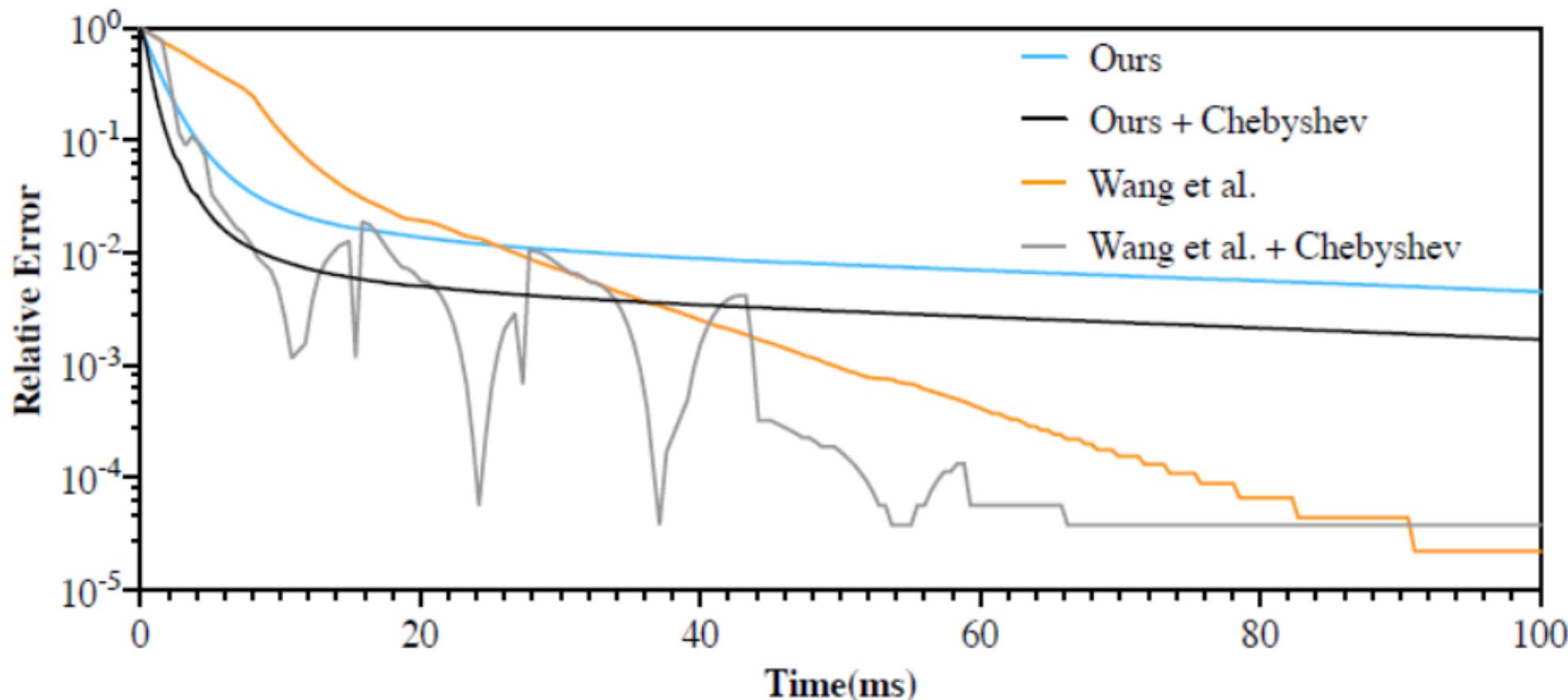
$$\alpha_i = \min \left[-\frac{\mathcal{E}(\mathbf{y}_i^k)}{\frac{\partial \mathcal{E}_i}{\partial \mathbf{y}_i} \cdot (\mathbf{y}_i^{k+1} - \mathbf{y}_i^k)}, 1 \right]$$



半隐式连续迭代法 (SISSM)

- Limitation

- Convergence rate
- Only suitable for problem $x = g(x - x')$

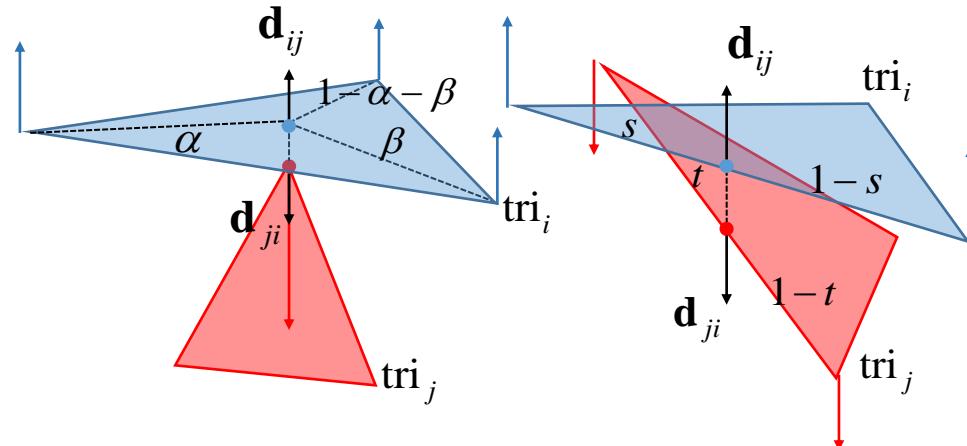


投影近场动力学 (Projective Peridynamics)

- Local step

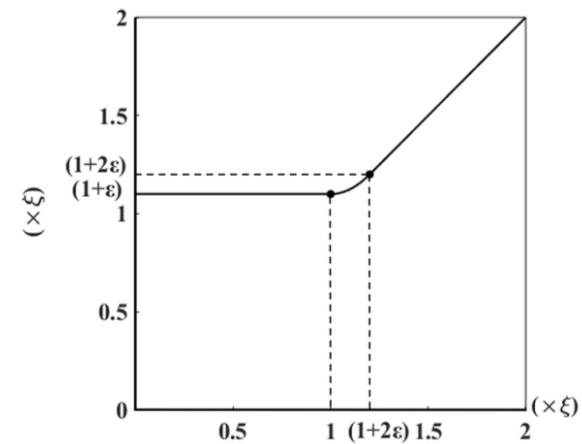
$$\min_{\mathbf{z}} \left(\sum_i \frac{1}{2} \left\| \mathbf{z}_i - \mathbf{y}_i^k \right\|^2 + \mu \sum_k B_k(\mathbf{z}) \right) \quad \rightarrow$$

$$\begin{aligned} \mathbf{g}_i &= -\nabla_{\mathbf{z}_i} \mathcal{B} \\ &= \mu \sum_I \alpha_I^i \left[\frac{(d_{IJ} - \hat{d})^2}{d_{IJ}} + 2(d_{IJ} - \hat{d}) \log \left(\frac{d_{IJ}}{\hat{d}} \right) \right] \\ &\quad \text{norm}(\mathbf{d}_{IJ}) + \mathbf{y}_i^k - \mathbf{z}_i \end{aligned}$$



Additive CCD[Li et al. 2020]

$$\lambda \leq \frac{\varepsilon \xi}{\mu \alpha_0 \lceil |B'(d)| \rceil + \hat{d}}$$



(a) $f(d)$

场景演示

- 多层布料

×3 slow down



1 layer



3 layers



6 layers

场景演示

• 桌布

$s_0 = 8000$
 $k_b = 0$



$s_0 = 8000$
 $k_b = 24$



$s_0 = 8000$
 $k_b = 240$



$s_0 = 8000$
 $k_b = 2400$



#Vertex: 40.0K

#Face: 79.2K

$dt = 1e-3$

Gravity = 1.0 × default

场景演示

- 布料模拟

#Vertex: 32.8K

#Face: 64.8K

dt = 2e-3

Gravity = 0.1 × default

Average single frame

time cost: 1/8 s.



场景演示

- 布料模拟

#Vertex: 14.3K

#Face: 28.3K

dt = 1e-3

Gravity = 1.0 × default

Average single frame

time cost: 1/22 s.



扩展阅读

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Question