



智能可视建模与仿真实验室
Intelligent Visual Modeling & Simulation (iGame) Lab

基于体细分的等几何建模与仿真优化一体化框架

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2023年6月26日

提纲

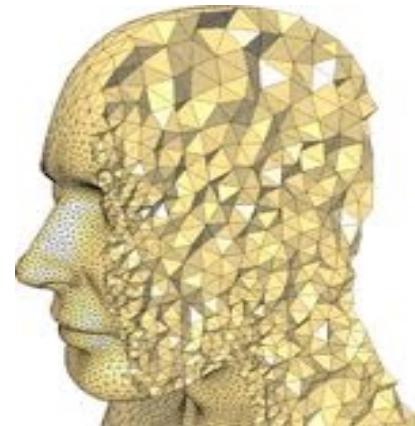
- **曲体建模**: 基于体细分的复杂曲体建模
- **物理仿真**: 基于体细分的高精度IGA物理仿真
- **设计优化**: 基于体细分的IGA形状/拓扑优化方法
- **总结展望**

提纲

- 曲体建模：基于体细分的复杂曲体建模
- 物理仿真：基于体细分的高精度IGA物理仿真
- 设计优化：基于体细分的IGA形状/拓扑优化方法
- 总结与展望

IGA-meshing

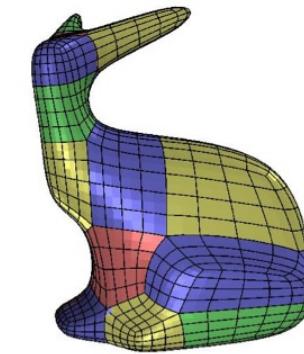
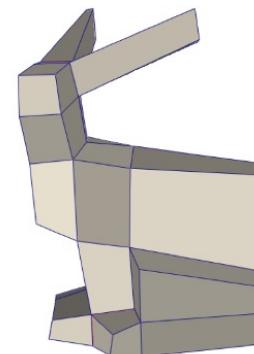
- IGA is a spline-version of FEA
- Mesh generation in FEA
- CAD models usually define only the boundary of a solid, but the application of isogeometric analysis requires a volumetric representation
- As it is pointed by Cotrell et al., the most significant challenge facing isogeometric analysis is developing three-dimensional spline parameterizations from boundary information



IGA为CAGD提出了复杂曲体建模的需求

- 曲线曲面→曲体
- 复杂曲体建模问题：

给定一个复杂拓扑六面体控制网格，构造相应的样条曲体表示



预处理---- 基于加权排序的六面体网格简化（CAD2021）

输入：六面体网格 M ;

目标网格数 N_s ;

奇异结构简化率

用户指定的 Hausdorff 距离比阀值 HR ;

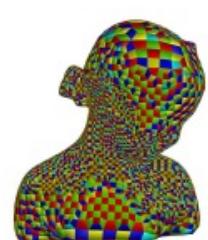
输出：满足上述要求、拓扑有效、外形特征保持良好的六面体简化网格



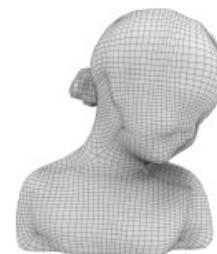
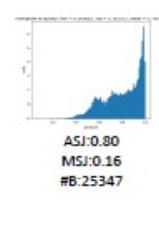
初始网格



奇异结构



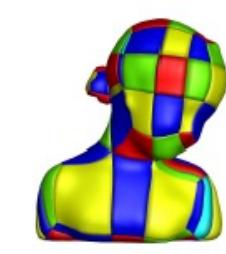
Base-complex



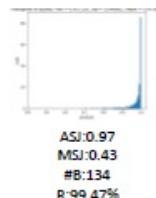
输出网格



奇异结构



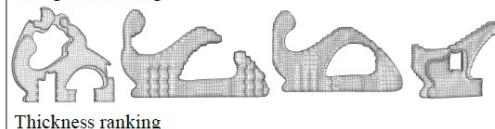
Base-complex



预处理---- 基于加权排序的六面体网格简化 (CAD2021)

HR:0%

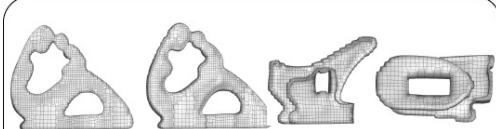
Weighted ranking



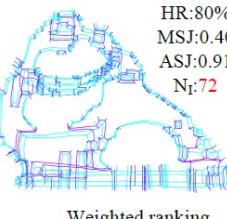
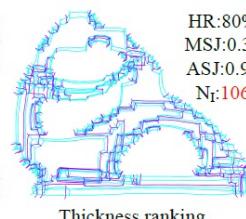
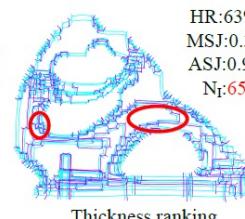
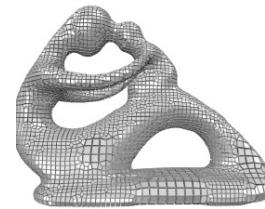
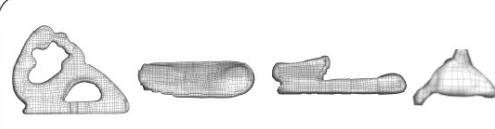
Thickness ranking



HR:60%



HR:80%

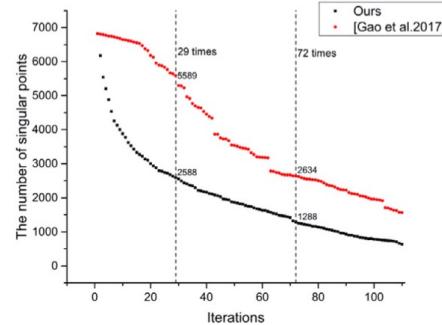
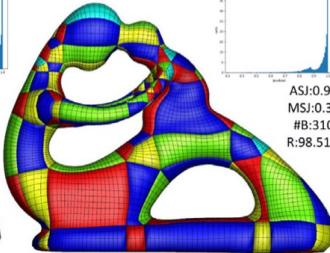
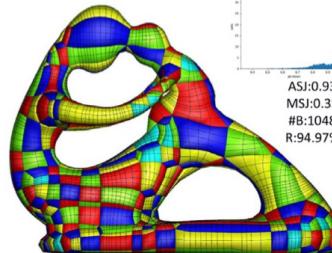


Thickness ranking

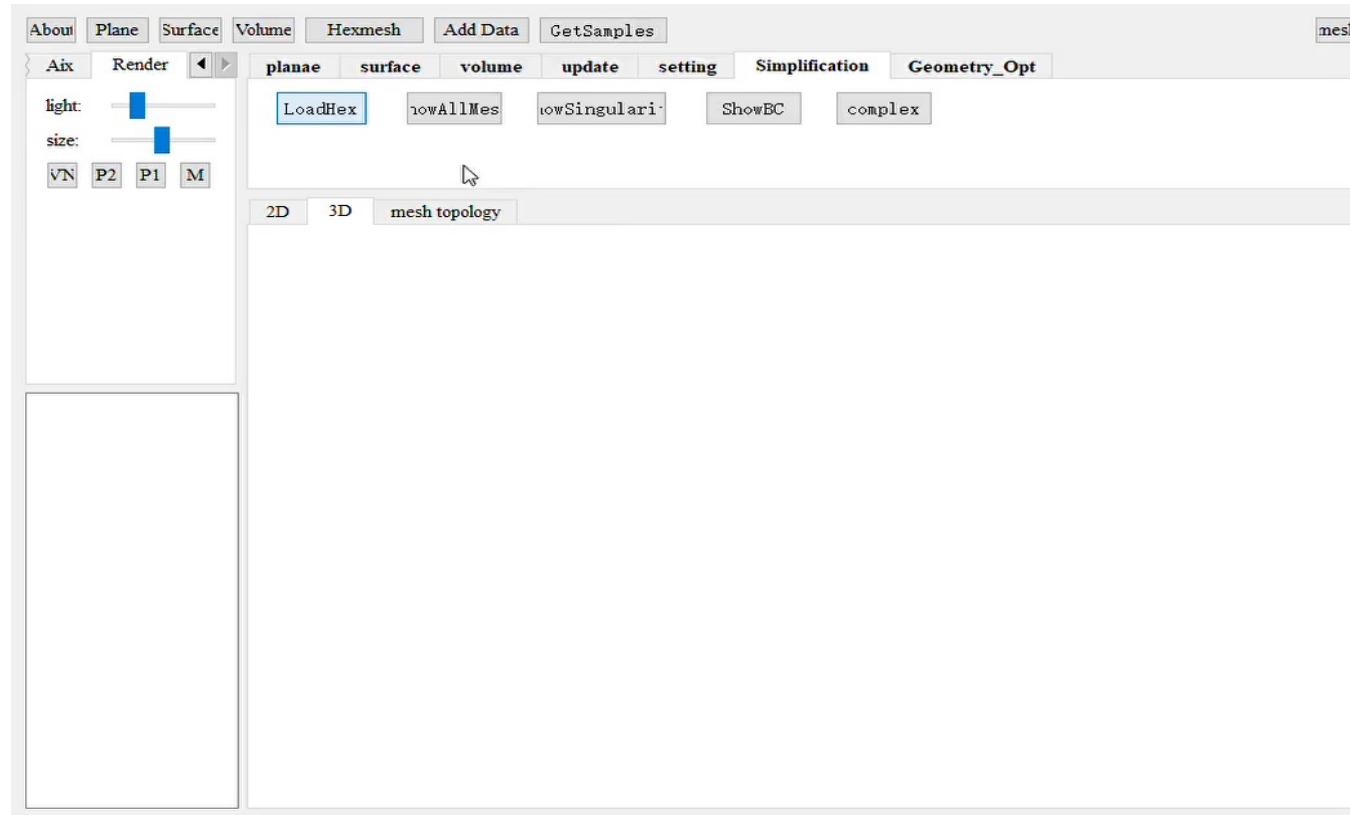
Weighted ranking

Thickness ranking

Weighted ranking



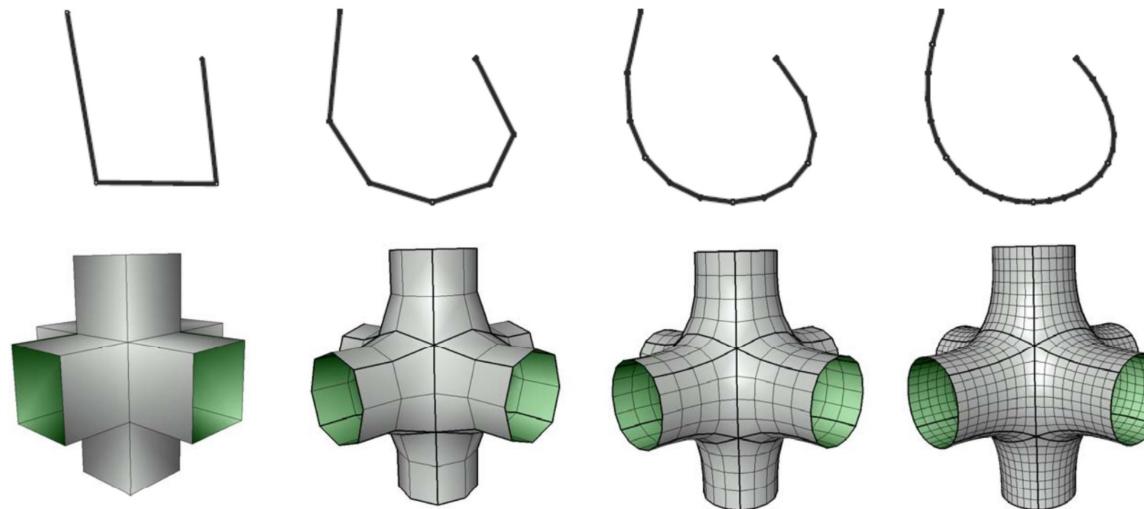
预处理---- 基于加权排序的六面体网格简化（CAD2021）



解决方案

- 细分曲线曲面→细分曲体（体细分）

E. Catmull and J. Clark, Recursively Generated B-spline Surfaces On Arbitrary Topological Meshes, CAD, 1978



Limit point formula of CCSS

Given an original mesh M^0 , M^1 is the mesh derived by subdividing with one C-C subdivision step. For V_i^0 , assume that its corresponding vertex at M^1 is V_i^1 , and the new edge point and face point around V_i^1 are $E_1^1, E_2^1, \dots, E_n^1$ and $F_1^1, F_2^1, \dots, F_n^1$ respectively.

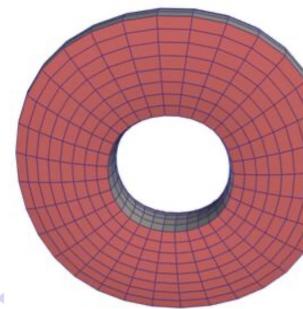
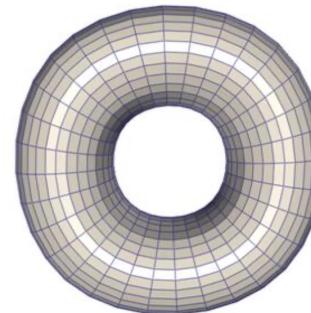
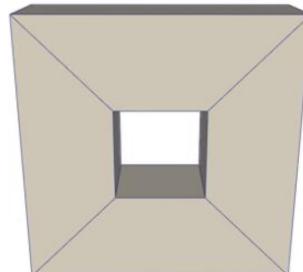
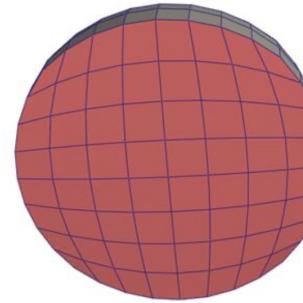
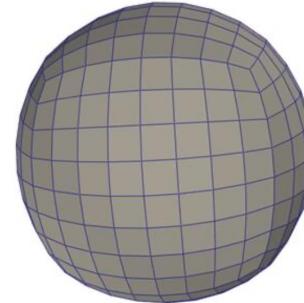
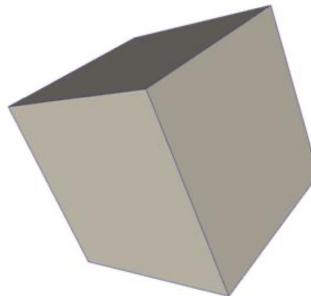
Halstead et al. showed that the explicit limit point formula of CCSS is :

$$V_i^\infty = \frac{n^2 V_i^1 + 4 \sum_{j=1}^n E_j^1 + \sum_{j=1}^n F_j^1}{n(n+5)}$$

Catmull-Clark Subdivision Volumes

MacCracken, R. and Joy, K. Free-form deformations of solid primitives with constraints. Siggraph 1996.

Bajaj, C., Schaefer, S., Warren, J., and Xu, G. A subdivision scheme for hexahedral meshes. The Visual Computer, 2002.



Catmull-Clark Subdivision Volumes

- **Cell point C :** Insert a cell point for each hexahedral cell, which is the average of all the points contained in this cell.
- **Face point F_{new} :** Insert a face point for each face, which is derived from the following formula

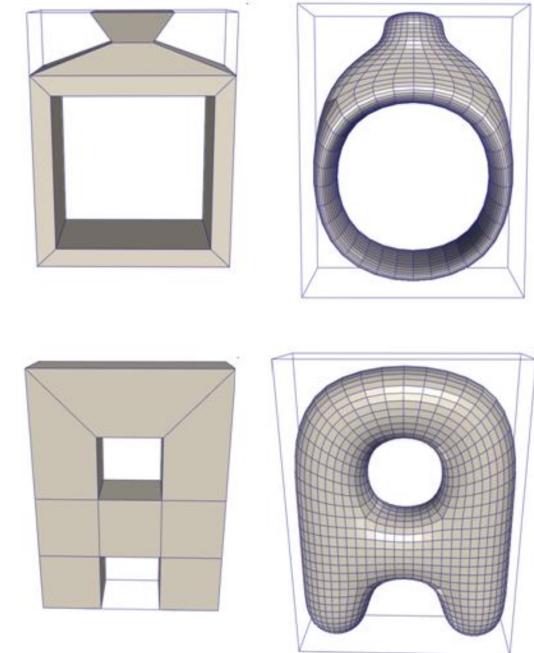
$$F_{new} = \frac{C_1 + C_2 + 2A}{4}$$

where C_1 and C_2 are the cell points of two adjacent cells of this face, and A is the centroid of this face.

- **Edge point E_{new} :** Insert an edge point for each edge, which is derived from the following formula

$$E_{new} = \frac{C_{avg} + 2F_{avg} + M}{4}$$

where C_{avg} is the average of the cell points of all the incident cells of this edge, F_{avg} is the average of the centroids of all the incident faces of this edge, and M is the midpoint of the edge.



Catmull-Clark Subdivision Volumes

- For each hexahedral cell, connect the cell points with all the face points and connect all the face points with all incident edge points in this cell respectively. Then a hexahedral cell is divided into 8 hexahedral cells.
- Vertex V_{new} : update each original vertex according to the following formula

$$V_{new} = \frac{C_{avg} + 3F_{avg} + 3E_{avg} + V_{old}}{8}$$

where $C_{avg}, F_{avg}, E_{avg}$ are the averages of the cell, face and edge points of all the adjacent cells, faces and edges, and V_{old} is the original vertex.

Question

How about the explicit limit point formula of Catmull-Clark subdivision volumes ?

C. Altenhofen et al. Direct Limit Volumes: Constant-Time Limit Evaluation for Catmull-Clark Solids. Pacific Graphics 2018, short paper.
(without explicit limit point formula)

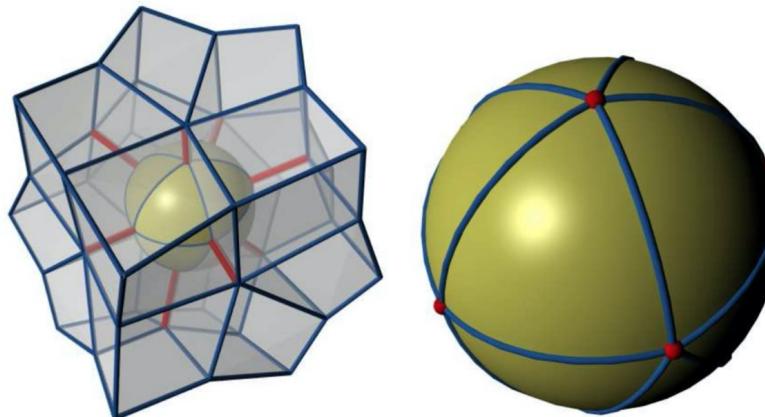
Number of faces and cells around extraordinary points

- E : number of faces around extraordinary point with valence n
- F : number of cells around extraordinary point with valence n
- Euler formula

$$F - E + n = 2 \quad (1)$$

$$3F = 2E \quad (2)$$

- $E = 3(n-2)$, $F = 2(n-2)$



Main result

Limit point formula of Catmull-Clark subdivision volumes

$$v_i^\infty = \frac{16(n-2)v_i^1 + 4 \sum_{j=1}^n m_j e_j^1 + 4 \sum_{j=1}^{3(n-2)} f_j^1 + \sum_{j=1}^{2(n-2)} c_j^1}{30(n-2) + 4 \sum_{j=1}^n m_j}$$

in which n is the valence of v^i , $v_i^1, e_1^1, \dots, e_n^1, f_1^1, \dots, f_{3(n-2)}^1, c_1^1, \dots, c_{2(n-2)}^1$ are the adjacent edge points, face points and cell points of v^i after performing Catmull-Clark subdivision once, m_j is the number of the adjacent faces of e_j^1 .

Application of limit point formula

■ Proof

➤ Subdivision matrix

$$v_i^\infty = \frac{16(n-2)v_i^1 + 4 \sum_{j=1}^n m_j e_j^1 + 4 \sum_{j=1}^{3(n-2)} f_j^1 + \sum_{j=1}^{2(n-2)} c_j^1}{30(n-2) + 4 \sum_{j=1}^n m_j}$$

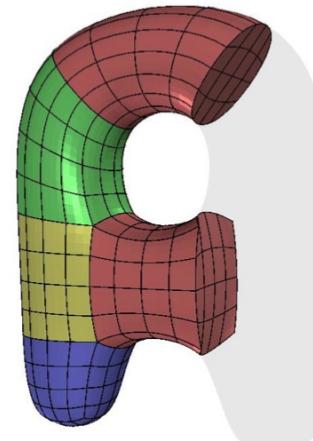
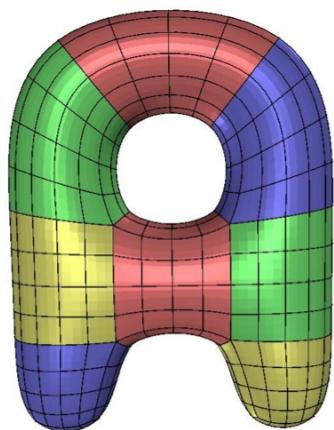
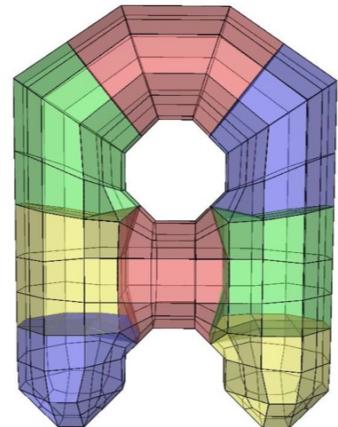
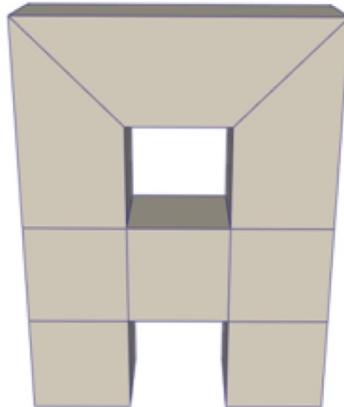
■ Applications

- Spline volume approximation of Catmull-Clark subdivision volume
- Spline volume interpolation of hex-mesh

C-C体细分的Bézier 逼近

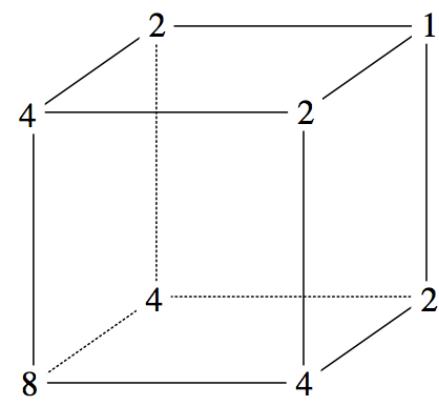
Problem statement

For each cell of a given hexahedral mesh H , construct a control lattice such that the corresponding Bézier volume is an approximation of C-C subdivision volume generated from H .

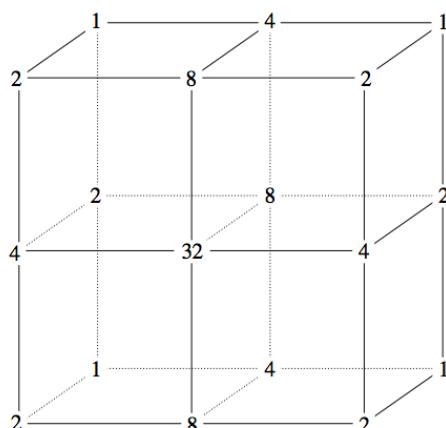


Mask for the regular part

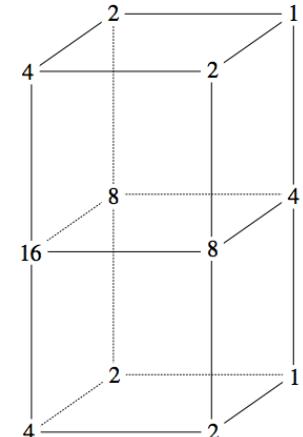
interior point



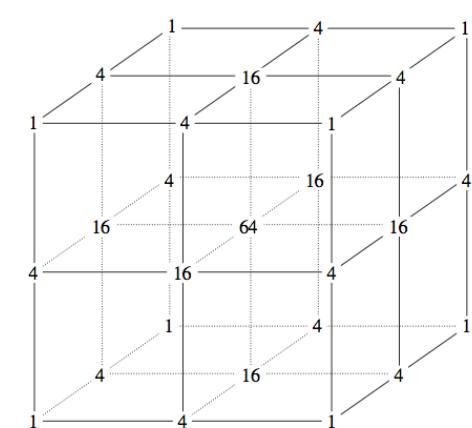
edge point



face point

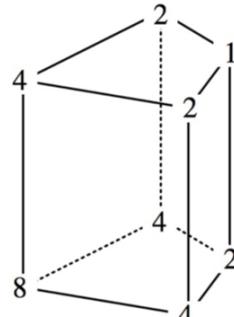


corner point

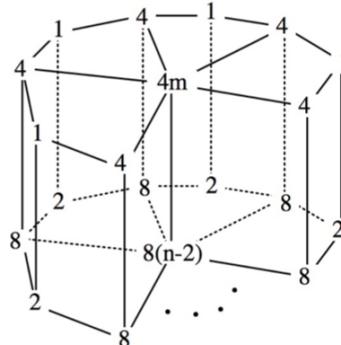


Mask for the irregular part

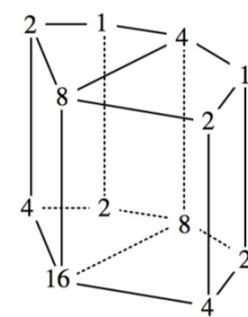
- According to the limit point formula of CCSV, the mask for interior point can be derived as shown in the picture, in which m is the valence of an adjacent edge of the point.
- For regular case, $n = 6$, $m = 4$.



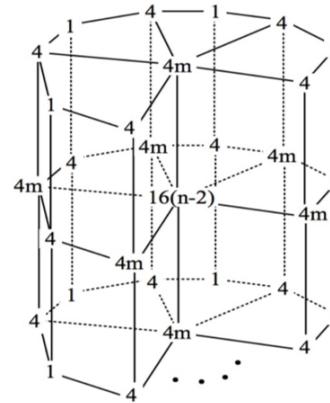
(a)



(b)



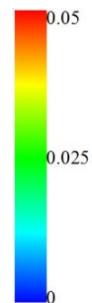
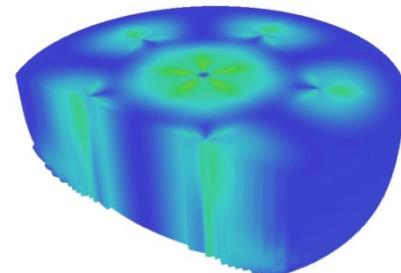
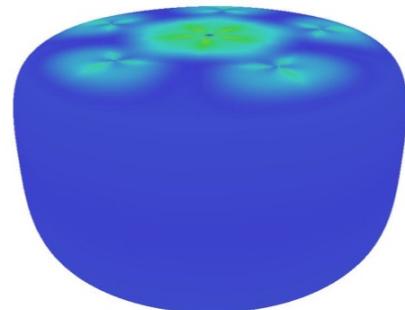
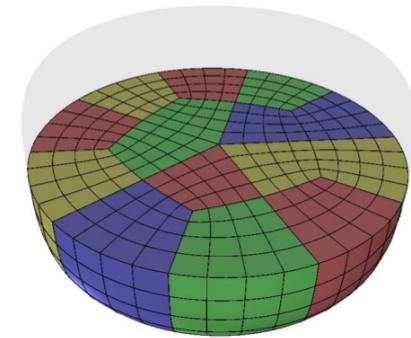
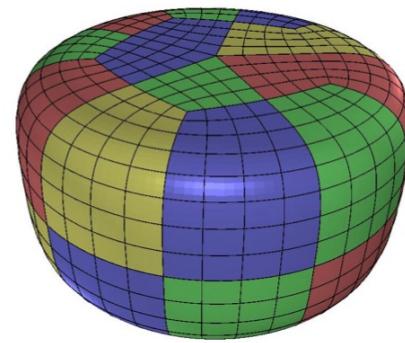
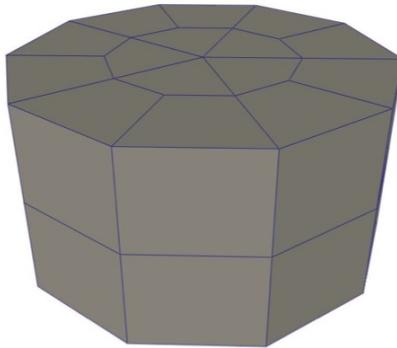
(c)



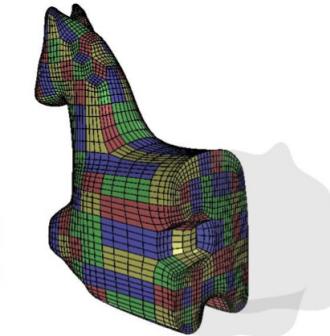
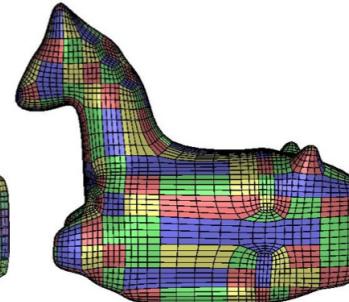
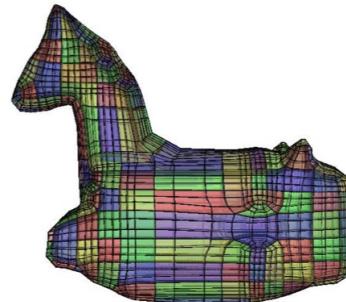
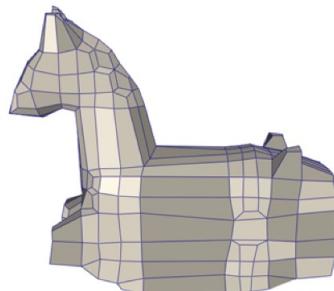
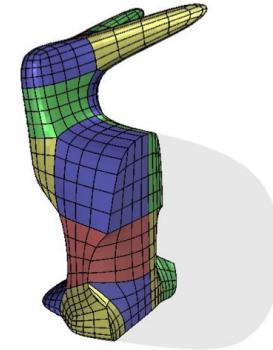
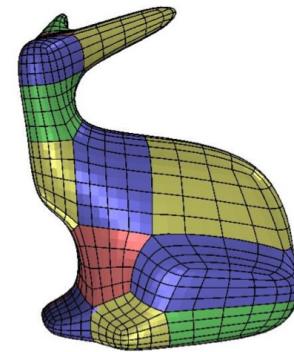
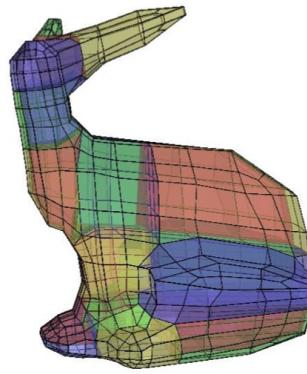
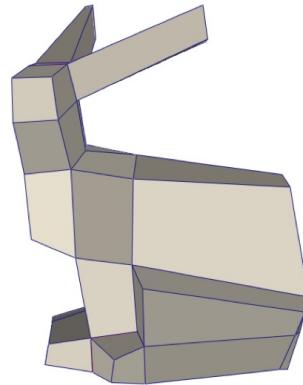
(d)

Fig. 5. Generalized masks for interior, edge, face and corner points.

逼近误差



曲体建模实例



Interpolatory Catmull-Clark Volumetric Subdivision over Unstructured Hexahedral Meshes for Modeling and Simulation Applications

Jin Xie^a, Jinlan Xu^a, Zhenyu Dong^a, Gang Xu^{a,*}, Chongyang Deng^b,
Bernard Mourrain^c, Yongjie Jessica Zhang^d

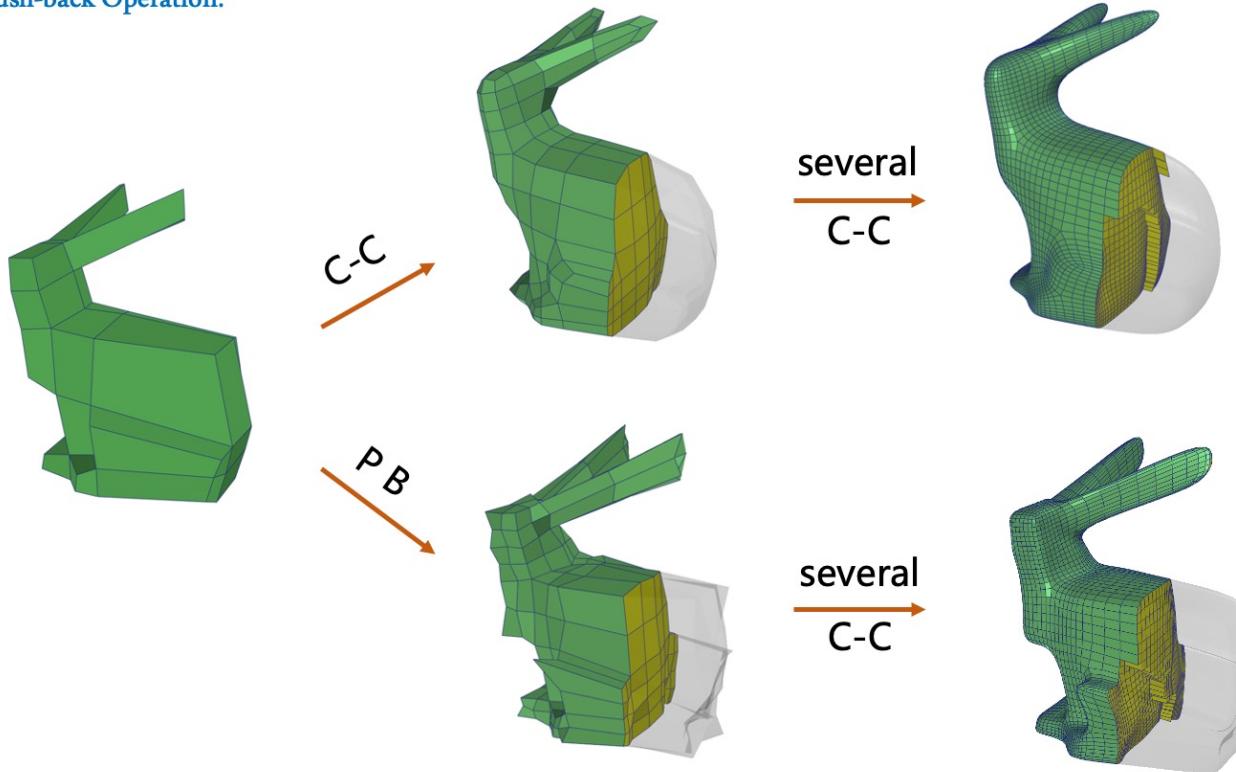
Problem statement

Given a hex-mesh M^0 as input, we would like to construct a Catmull-Clark subdivision volume to interpolate the vertices of M^0 .

Part I: Interpolatory Catmull-Clark Volumetric Subdivision with Push-back Operation

C-C: Catmull-Clark Volumetric Subdivision Operation;

P B: Push-back Operation.



Process of Catmull-Clark
volumetric subdivision

Process of interpolatory
volumetric subdivision
with push-back operation

Part I: Interpolatory Catmull-Clark Volumetric Subdivision with Push-back Operation

Push-back Operation:

$$\text{Step1: } \Delta_i^0 = V_i^0 - V_i^1$$

$$\text{Step2: } E' = \frac{1}{2}(V_i^0 + V_j^0) + \lambda_{ij}(\Delta_i^0 + \Delta_j^0), \quad 0 < \lambda_{ij} < 1$$

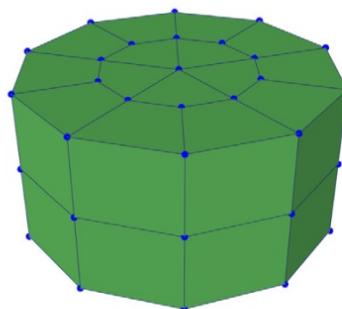
$$\text{Step3: } F' = \frac{\sum_{j=1}^4 V_j^0}{4} + \varrho \mu_F \sum_{j=1}^4 \Delta_j^0, \quad 0 < \mu_F < 1$$

$$\text{Step4: } C' = \frac{\sum_{j=1}^8 V_j^0}{8} + \varrho \gamma_C \sum_{j=1}^8 \Delta_j^0, \quad 0 < \gamma_C < 1$$

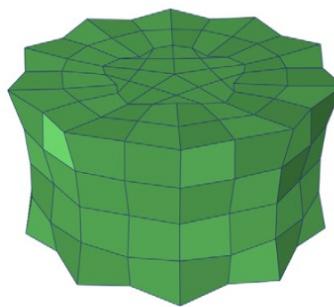
$$\text{Step5: } V' = \frac{[30(n-2) + 4 \sum_{j=1}^n m_j(V_i^0) - 4 \sum_{j=1}^n m_j E'_j - 4 \sum_{j=1}^{3(n-2)} F'_j - \sum_{j=1}^{2(n-2)} C'_j]}{16(n-2)}$$

Part I: Interpolatory Catmull-Clark Volumetric Subdivision with Push-back Operation

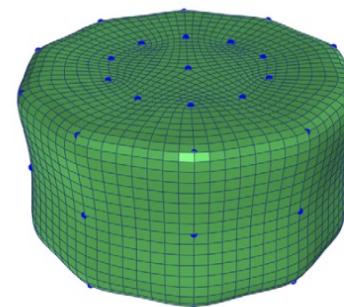
Volumetric Geometry Interpolation:



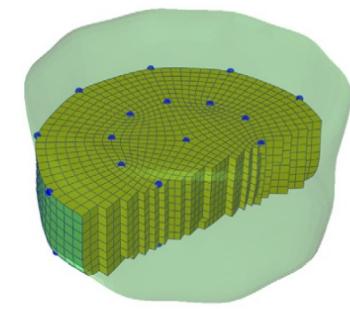
original mesh



new control lattices
constructed by the
push-back operation



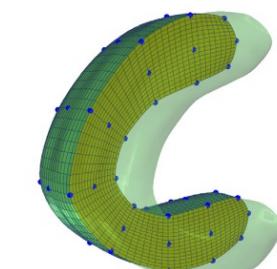
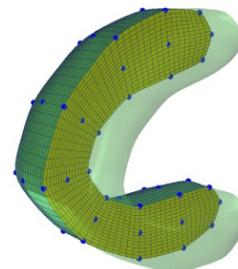
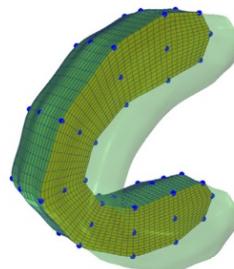
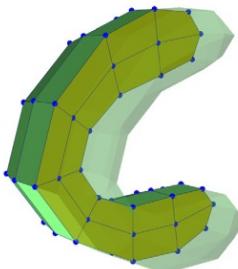
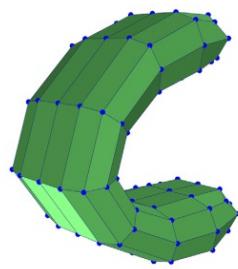
subdivision result



interior view of the result

Part I: Interpolatory Catmull-Clark Volumetric Subdivision with Push-back Operation

Different Shapes due to Different Parameters:



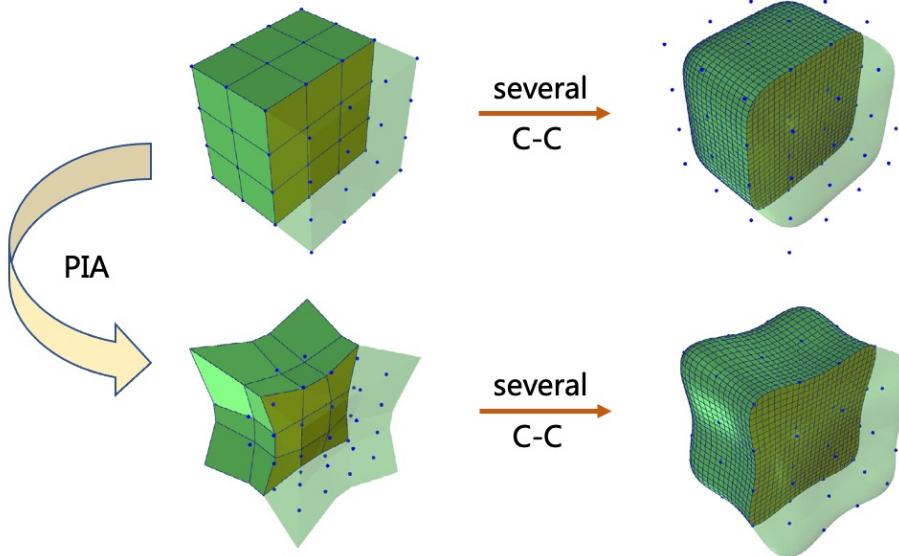
original mesh

subdivision result with
 $\lambda_{ij} = 0.05$, $\mu_F = 0.05$ and $\gamma_C = 0.05$

subdivision result with
 $\lambda_{ij} = 0.25$, $\mu_F = 0.125$ and $\gamma_C = 0.0625$

subdivision result with
 $\lambda_{ij} = 0.25$, $\mu_F = 0.25$ and $\gamma_C = 0.25$

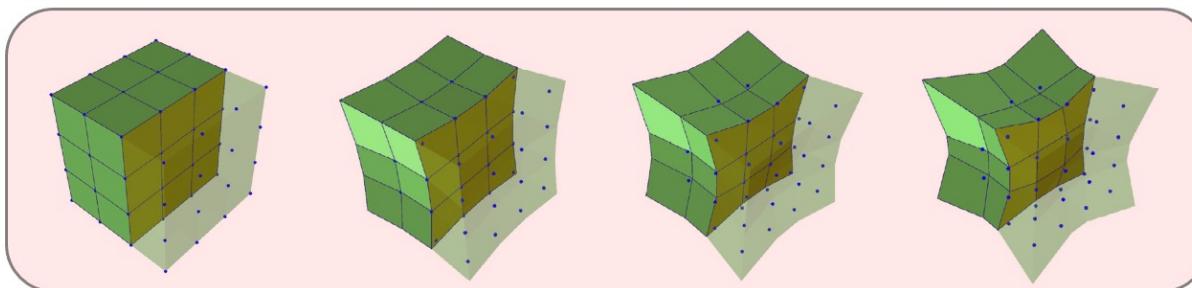
Part II: Progressive interpolation using Catmull-Clark volume subdivision



Process of Catmull-Clark
volumetric subdivision

Process of interpolatory
volumetric subdivision
with PIA

Iteration of PIA:



C-C: Catmull-Clark Volumetric
Subdivision Operation;

PIA : Progressive-Iterative Approximation.

Part II: Progressive interpolation using Catmull-Clark volume subdivision

Progressive-Iterative Approximation:

Step1: $v_i^\infty = \frac{16(n-2)v_i^1 + 4\sum_{j=1}^n m_j e_j^1 + 4\sum_{j=1}^{3(n-2)} f_j^1 + \sum_{j=1}^{2(n-2)} c_j^1}{30(n-2) + 4\sum_{j=1}^n m_j}$

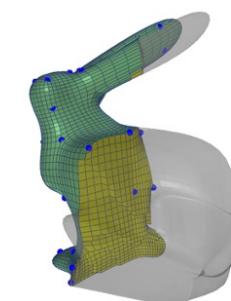
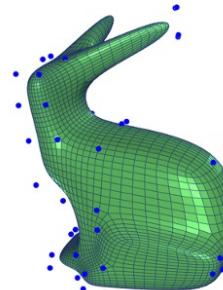
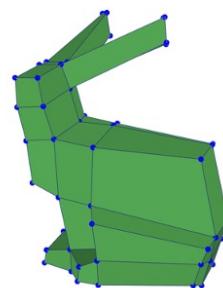
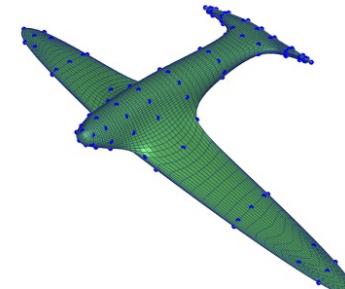
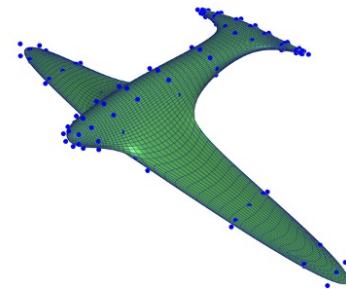
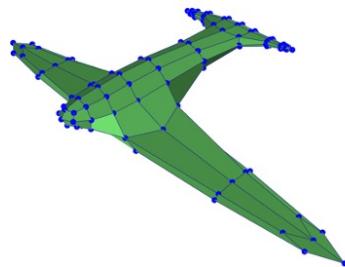
Step2: $d^k = v^0 - v_\infty^k$

Step3: *If $d^k < \varepsilon$, end; else continue.*

Step4: $v^{k+1} = v^k + d^k$ *Goto Step1.*

Part II: Progressive interpolation using Catmull-Clark volume subdivision

Volumetric Geometry Interpolation:

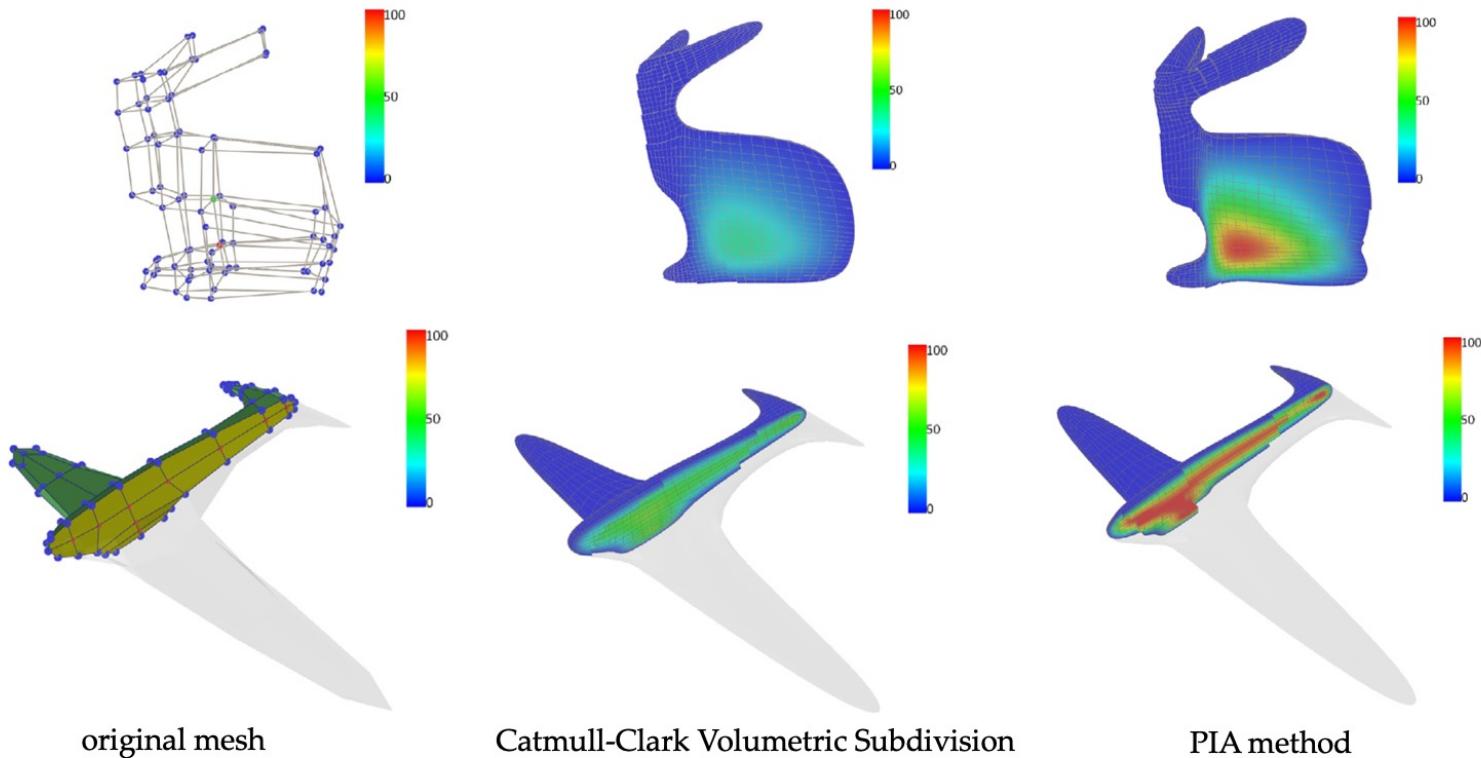


Catmull-Clark Volumetric Subdivision

PIA method

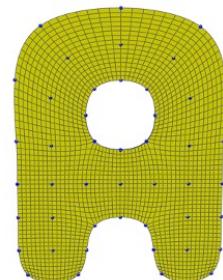
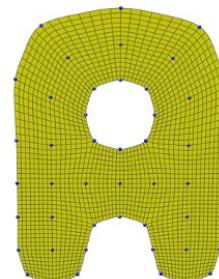
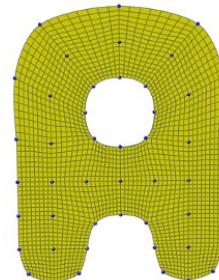
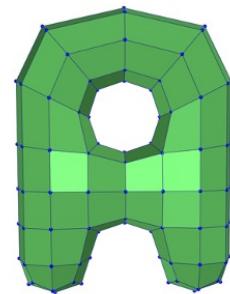
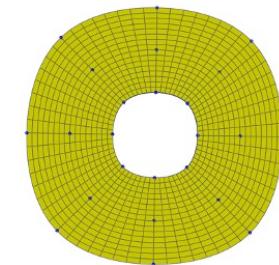
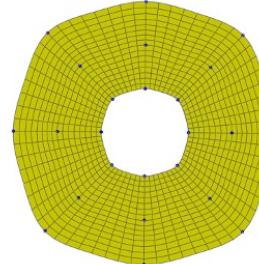
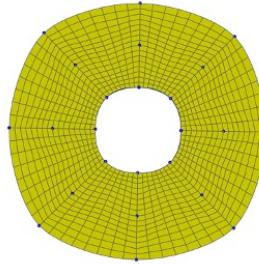
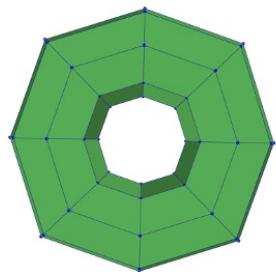
Part II: Progressive interpolation using Catmull-Clark volume subdivision

Volumetric Material Interpolation:



Part III: Comparisons and Application

Comparisons of different interpolatory subdivision schemes



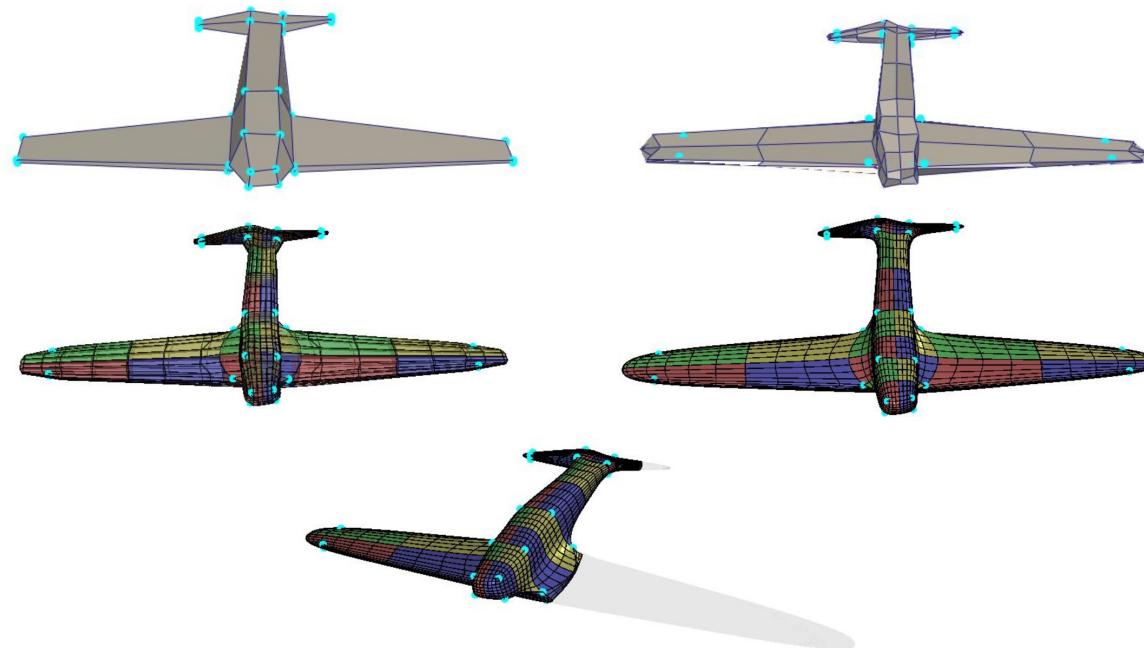
butterfly method

Push-Back method

PIA method

Problem statement

Given a hex-mesh M^0 as input, construct a set of Bézier volumes to interpolate the vertices of M^0 with continuity constraints.

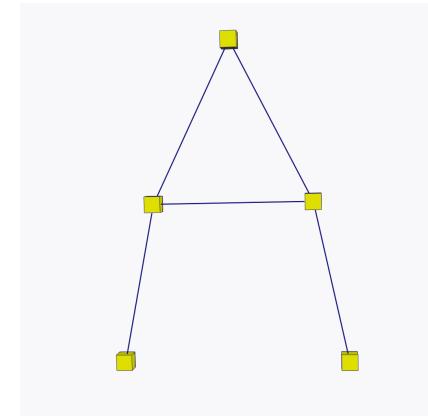
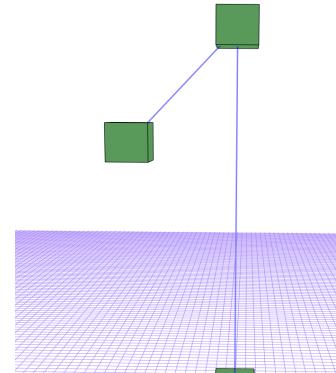
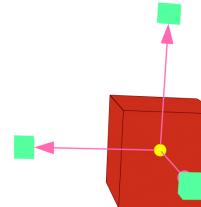
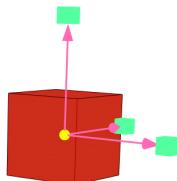
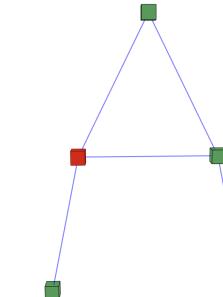
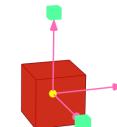
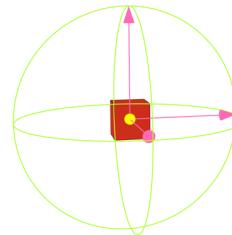
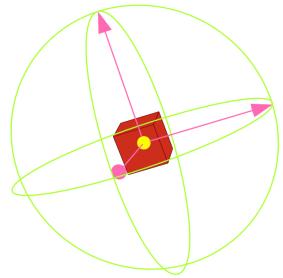


基于体细分的复杂六面体网格模型交互式构造方法

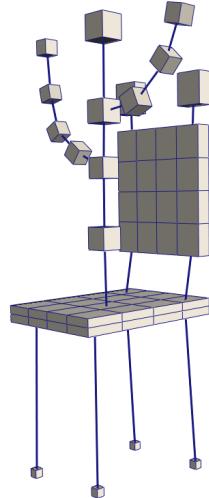
整体技术框架



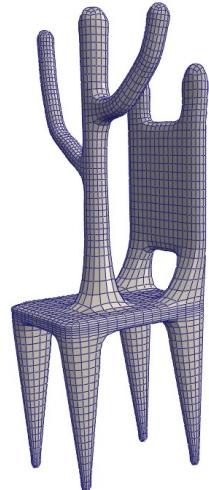
六面体节点的编辑方式



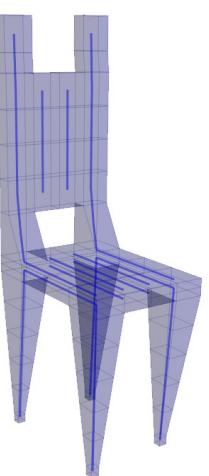
骨架节点的拼接与检查



多个骨架拼接显示



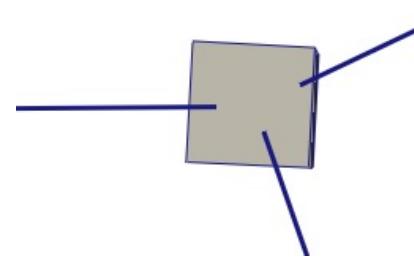
骨架结构检查模型



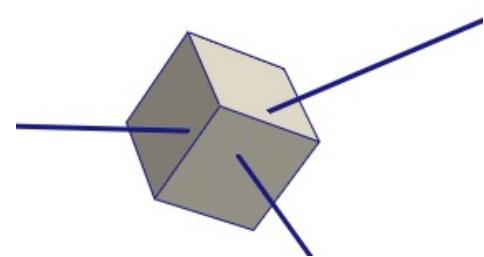
角度自动调整

节点如果使用普通的方式布局可以会产生较多的分裂，这些分裂可能会有点复杂而难以处理，本文使用了文献[1]提出的方法进行节点旋转以减少节点的分裂，文献[1]所提出的对于立方体旋转产生的最小化目标函数如下所示。

$$f_{\epsilon}(\mathbf{U}_i, \mathbf{V}_i, \mathbf{W}_i) = \sum_{j=1}^{k_i} \sqrt{(d_j^i \cdot \mathbf{U}_i)^2 + \epsilon} + \sqrt{(d_j^i \cdot \mathbf{V}_i)^2 + \epsilon} + \sqrt{(d_j^i \cdot \mathbf{W}_i)^2 + \epsilon}$$

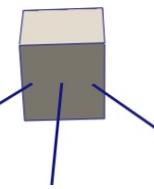


(a) 没有进行旋转优化的六面体网格

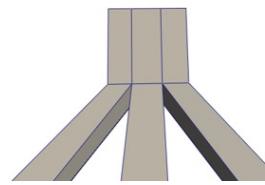


(b) 进过旋转优化的六面体网格

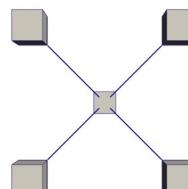
中间体单元的连接与构造



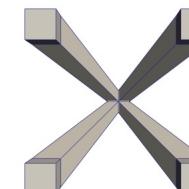
(a) 未进行中间连接体的连接



(b) 中间连接体的生成



(c) 出现重叠的情况



(d) 重叠情况中间连接体的生成



(a) 两个六面体正对的效果



(b) 上方的六面体绕其局部坐标系的
x轴旋转了45度

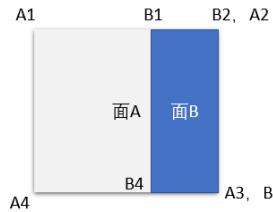


(c) 绕局部坐标系x轴, z轴
分别旋转了45度

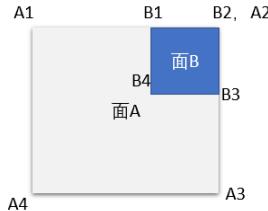


(d) 绕x轴, 绕y轴, 绕z轴分别
旋转了45度

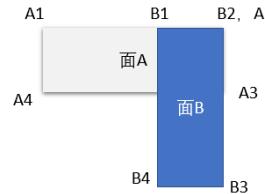
拓扑分裂



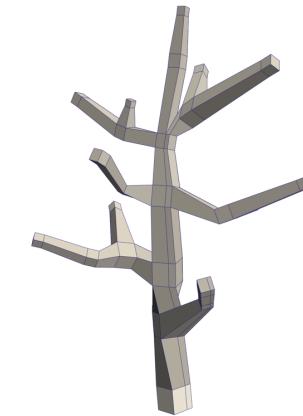
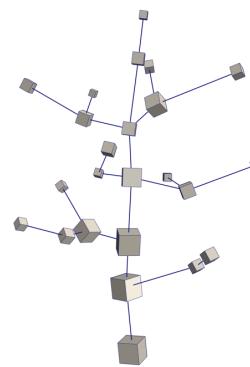
(a) 一个大的面对应一个小的面同时有两个相同的顶点



(b) 一个大的面对应一个小的面同时有1个相同的顶点

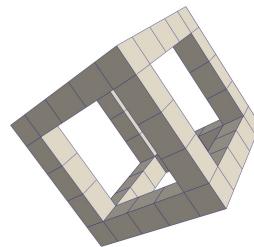


(c) 两个面相互交错同时有1个公共的顶点

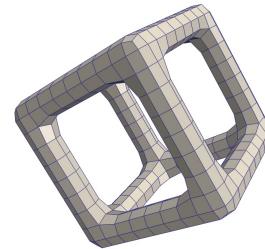


树的拓扑分裂结果

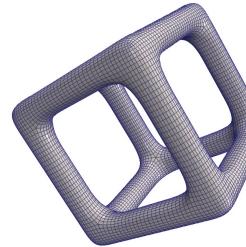
体细分与尖锐特征保持



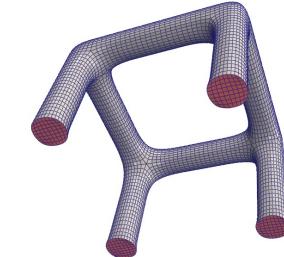
(a) 初始六面体模型



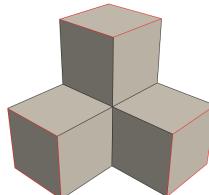
(b) 1次CC体细分



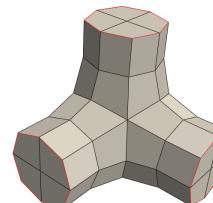
(c) 3次CC体细分



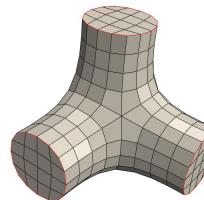
(d) 为 (c) 剖面图



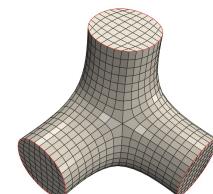
(a) 对选定的边进行尖锐边保持 (红色边)



(b) 1次CC体细分

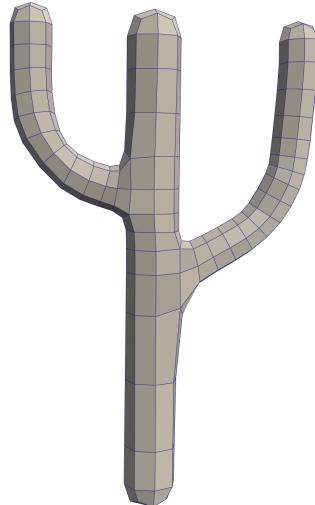


(c) 2次CC体细分

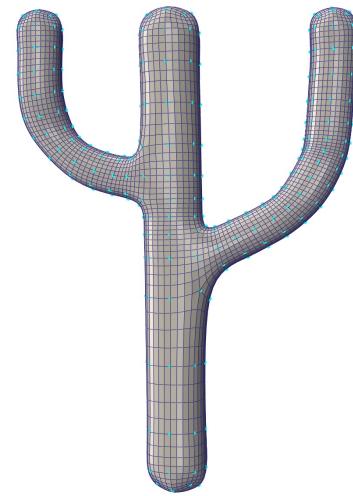


(d) 3次CC体细分

PIA插值体细分



(a) 原模型



(b) 经过细分后的模型，
插值与原控制网格

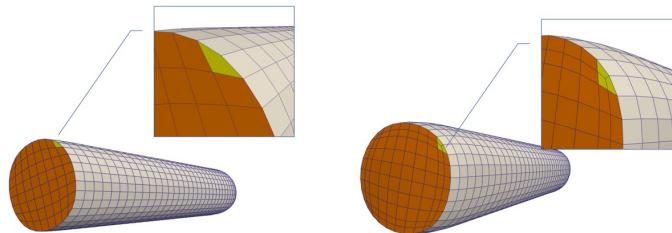
$$d^k = v^0 - v_k^\infty$$

原控制顶点和控制顶点的极限细分点的差距计算

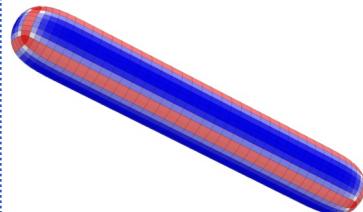
$$v^{k+1} = v^k + d^k$$

将偏移叠加到控制顶点上

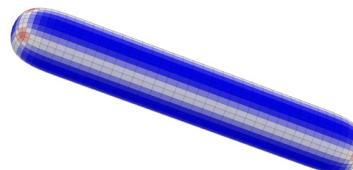
六面体单元质量提升



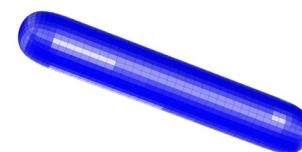
插入填充层示意图，左图没有插入填充层，右图显示了填充层的插入后的效果



(a) Catmull-Clark体细分后模型质量图



(b) 新增填充层之后的模型质量图



(c) 经过六面体网格质量优化程序[2]优化之后的六面体网格模型

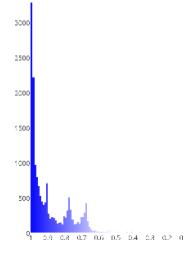
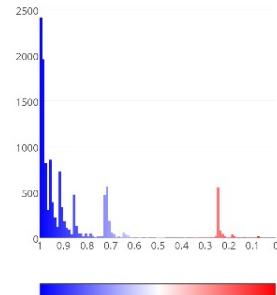
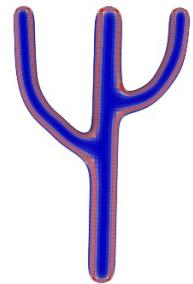
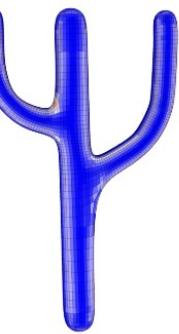
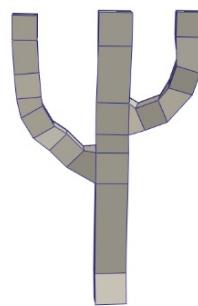
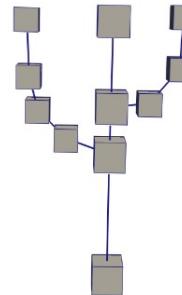
MSJ 最小雅可比值

ASJ 平均雅可比值

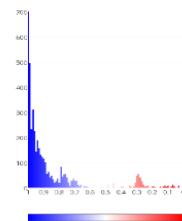
如果一个模型的最小雅可比值小于0，那么说明这个模型不能用于有限元分析

平均雅可比值越大，说明模型的整体质量越好

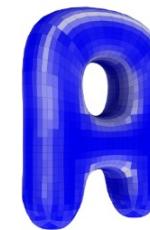
		模型优化前				模型优化后			
模型	Vertices	Hexahedron	MSJ	ASJ	Vertices	Hexahedron	MSJ	ASJ	
椅子	38 880	31 744	0.053	0.903	52 448	45 312	0.263	0.911	
蚂蚁	36 801	29 696	0.053	0.833	50 243	43 136	0.224	0.847	
卡通	21 465	16 896	0.070	0.831	30 043	25 472	0.261	0.859	
圆口	98 073	86 528	0.070	0.979	120 987	109 440	0.360	0.978	
方口	98 073	86 528	0.070	0.984	120 987	109 440	0.225	0.981	
仙人	13 689	10 752	0.070	0.867	19 195	16 256	0.301	0.897	
树枝	38 097	30 720	0.031	0.857	52 051	44 672	0.209	0.870	



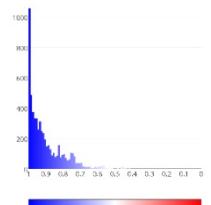
文献[3]生成的模型



六面体网格质量分布图



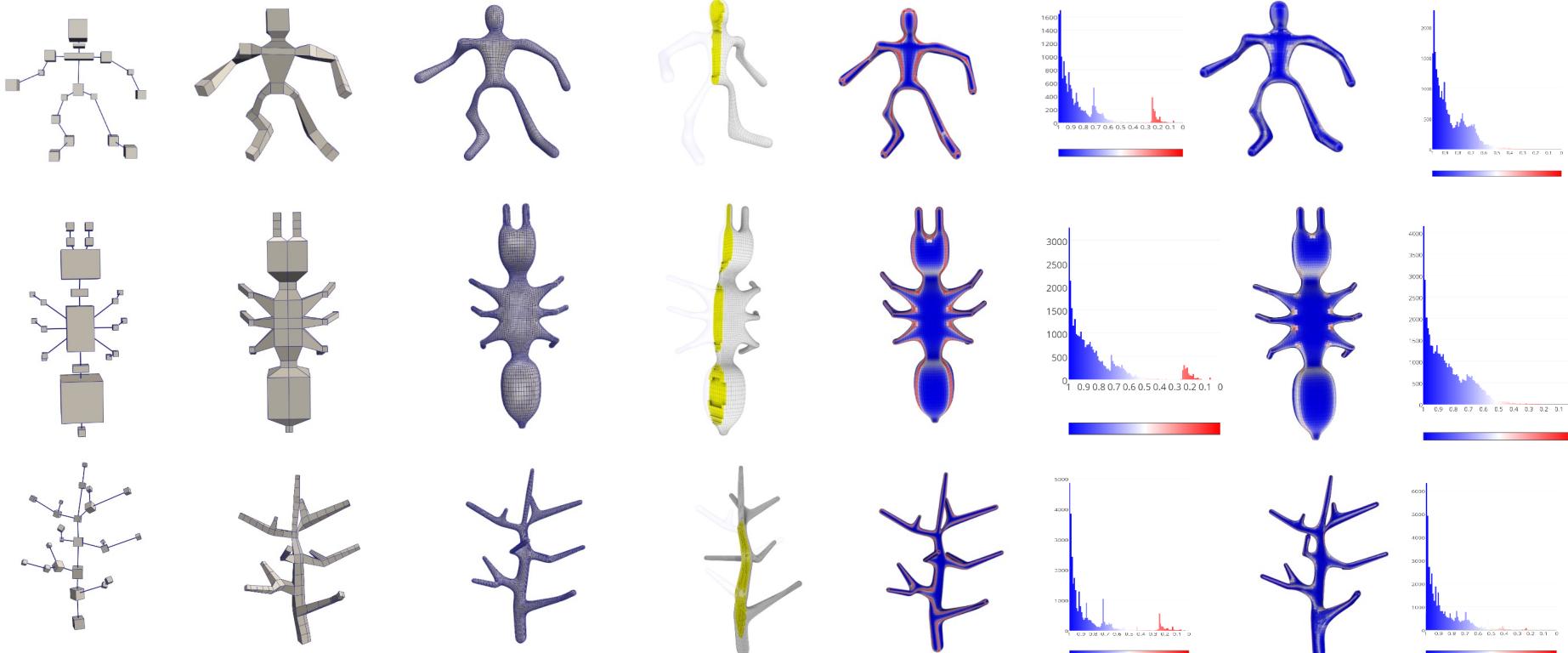
经过优化后的模型



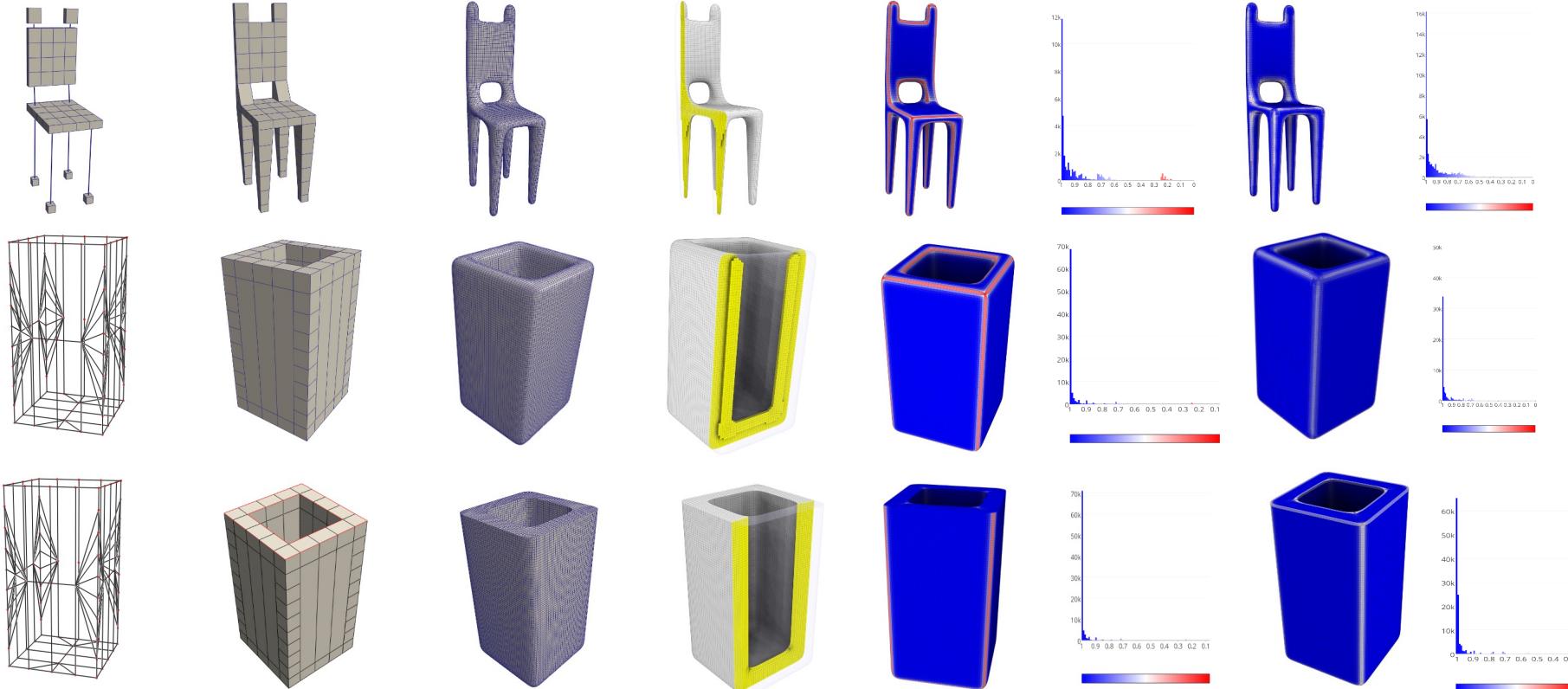
六面体网格质量分布图

[3] Xie J, Xu J, Dong Z, et al. Interpolatory Catmull-Clark volumetric subdivision over unstructured hexahedral meshes for modeling and simulation applications[J]. Computer Aided Geometric Design. 2020, 80: 101867.

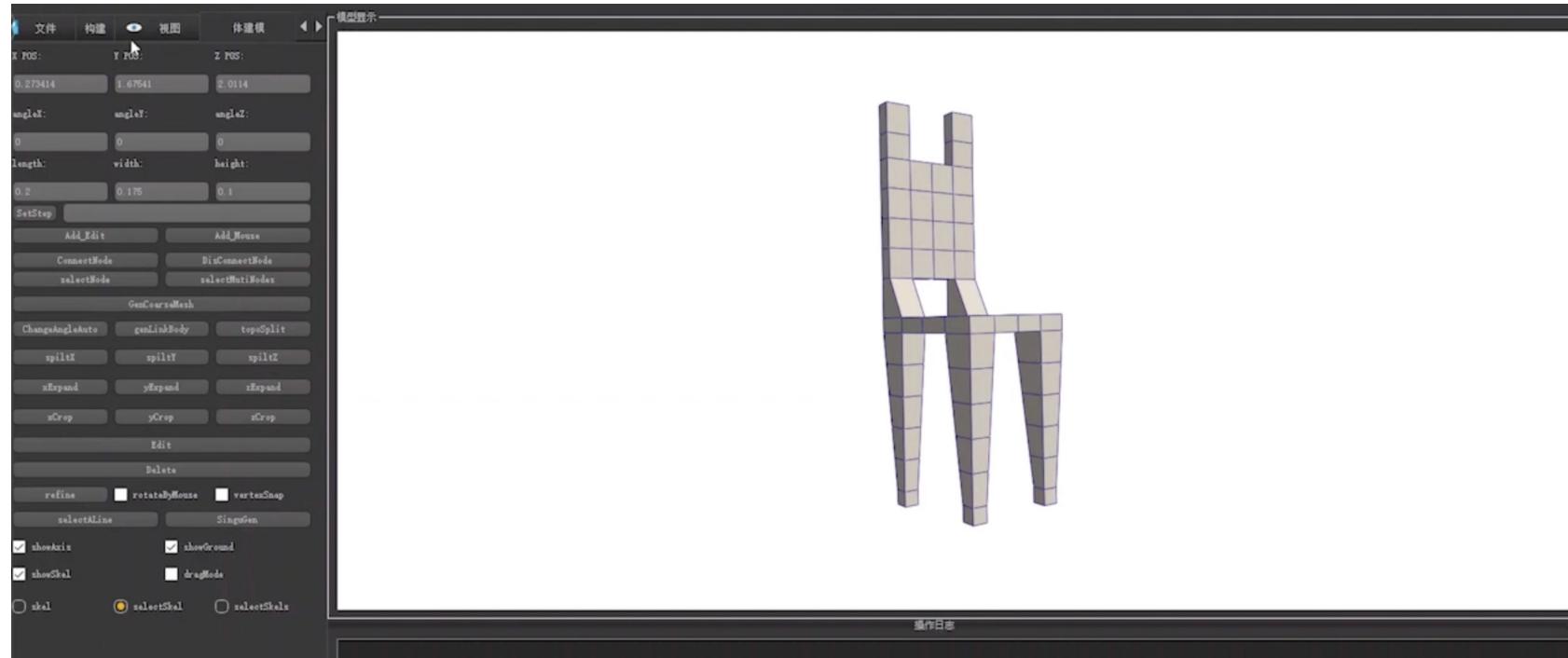
实验结果



实验结果



基于体细分的复杂样条曲体模型交互式构造方法



基于体细分的复杂样条曲体模型交互式构造方法

椅子建模

提纲

- 曲体建模：基于体细分的复杂曲体建模
- 物理仿真：基于体细分的高精度IGA物理仿真
- 设计优化：基于体细分的IGA形状/拓扑优化
- 总结展望

三维线弹性问题

弹性力学的理论建立在几何方程、平衡方程和本构方程三组方程基础之上。其中线弹性问题的表达式为：

$$\boldsymbol{\epsilon}(\boldsymbol{u}) = \frac{1}{2}(\nabla \boldsymbol{u} + \nabla^T \boldsymbol{u})$$

$$\nabla \cdot \boldsymbol{\sigma}(\boldsymbol{u}) + \boldsymbol{f} = \mathbf{0} \quad \text{in } \Omega$$

$$\boldsymbol{\sigma}(\boldsymbol{u}) = \boldsymbol{C} : \boldsymbol{\epsilon}(\boldsymbol{u})$$

狄利克雷约束和诺依曼约束为

$$\boldsymbol{u} = \mathbf{0} \quad \text{on } \Gamma_D$$

$$\boldsymbol{\sigma}(\boldsymbol{u})\boldsymbol{n} = \bar{\boldsymbol{t}} \quad \text{on } \Gamma_N$$

三维线弹性问题的IGA离散表达式

根据最小势能原理，线弹性问题的离散表达形式为：

$$\mathbf{K}\mathbf{u} = \mathbf{f}.$$

单元刚度矩阵为：

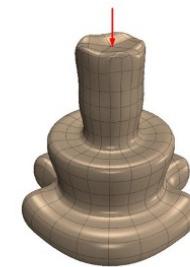
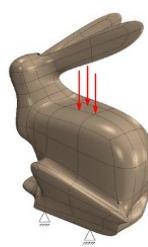
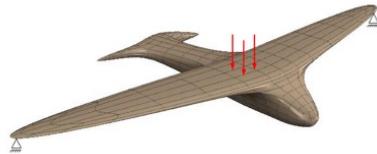
$$\mathbf{K}_e = \int_{V_e} \mathbf{B}^T \mathbf{D} \mathbf{B} dV$$

式中B为应变矩阵，D为本构方阵

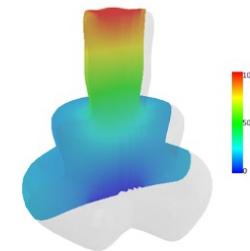
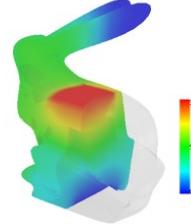
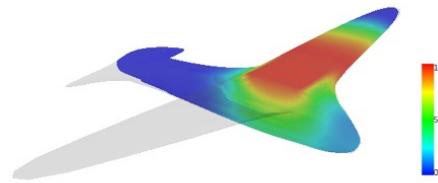
根据IGA等参单元思想，其中涉及到物理空间、参数空间、基准空间的雅可比变换，对于积分式采用高斯积分法进行求解刚度矩阵。

基于体细分的等几何分析求解器

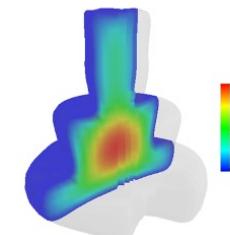
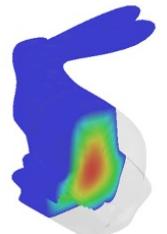
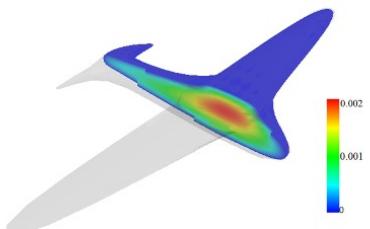
boundary condition for linear elasticity simulation



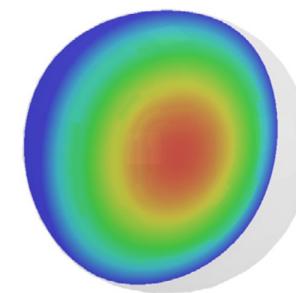
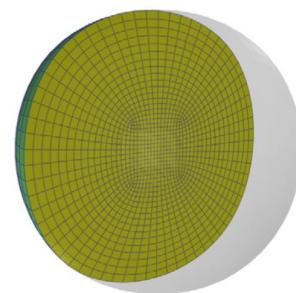
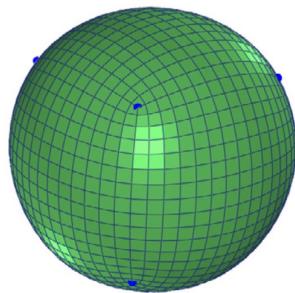
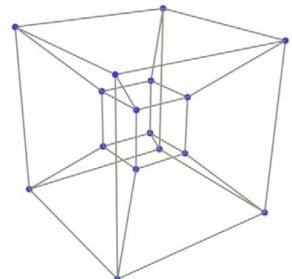
linear elasticity simulation by IGA



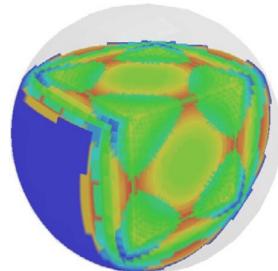
heat conduction simulation by IGA



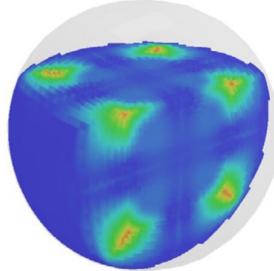
收敛性



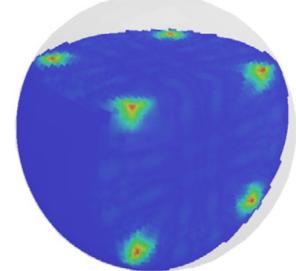
1.094
0.547
0



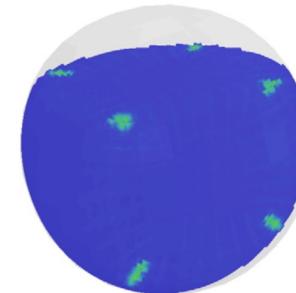
0.0824
0.0412
0



0.0392
0.0196
0



0.0121
0.0060
0



0.0050
0.0025
0

提纲

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形状优化的目标函数

结构优化问题的一般数学公式可以表述为：

$$\begin{cases} \min_{\alpha_s} & f(\mathbf{u}(\alpha), \alpha) \\ \text{s.t.} & h_i(\mathbf{u}(\alpha), \alpha) = 0, \quad i = 1 \text{ to } n_h, \\ & g_j(\mathbf{u}(\alpha), \alpha) \leq 0, \quad j = 1 \text{ to } n_g, \\ & \alpha_{min_s} \leq \alpha_s \leq \alpha_{max_s}, \quad s = 1 \text{ to } n_{eq} \end{cases}$$

设置优化目标为达到结构**最小柔度**时的表达形式为：

$$J(\Omega) = \int_{\Gamma_N} \mathbf{g} \cdot \mathbf{u} \, d\Gamma$$

式中 \mathbf{g} 为外力矢量， \mathbf{u} 为线弹性问题的位移解

采用拉格朗日表达引入体积约束的目标函数表达式即为：

$$\min_{\Omega \in \mathbb{R}^n} L(\Omega, \beta), \quad L(\Omega, \beta) = J(\Omega) + \beta \left(\int_{\Omega} d\Omega - V_0 \right)$$

式中 β 为大于0的拉格朗日正因子

形状灵敏度的计算

目标函数的拉格朗日表达式的**形状导数**表示为：

$$dL(\Omega, \beta; V) = \int_{\Gamma} \left(\beta - (2\mu |\epsilon(\mathbf{u})|^2 + \lambda |div \mathbf{u}|^2) \right) \mathbf{v} \cdot \mathbf{n} d\Gamma$$

式中 μ 、 λ 为材料常数， \mathbf{v} 为形状位移

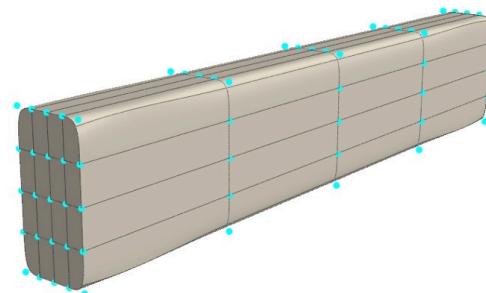
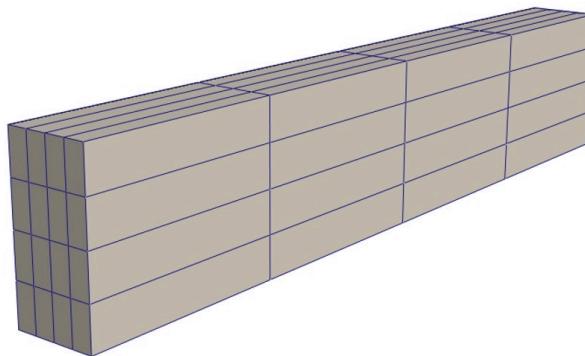
为了达到结构柔度下降的目标方向，**形状位移** \mathbf{v} 表示为：

$$\mathbf{v} = \begin{cases} (2\mu |\epsilon(\mathbf{u})|^2 + \lambda |div \mathbf{u}|^2 - \beta) \mathbf{n} & \text{on } \Gamma \\ 0 & \text{on } \Gamma_D \cup \Gamma_N \end{cases}$$

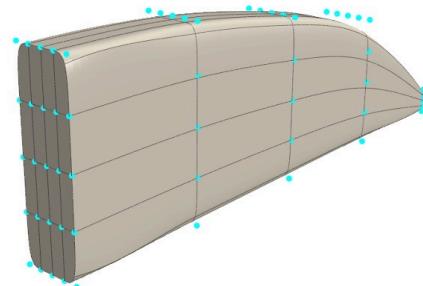
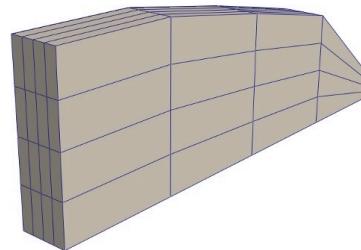
IGA形状优化求解框架

1. 读入样条几何模型: 基函数信息与控制点信息
2. 确定初始模型 (几何信息与拓扑信息)
3. 进行IGA求解, 计算结构位移解 $U^{(k)}$
4. 计算形状梯度, 进行IGA求解 , 计算形状位移解 $V^{(k)}$
5. 设置初始下降步长 $t^{(k)}$
6. 迭代进行线性搜索
7. 计算更新的控制点坐标 : $X^{(k+1)} = X^{(k)} + t^{(k)}V^{(k)}$
8. 进行IGA求解 : 计算结构位移解 $U^{(k+1)}$
9. 计算当前目标函数值 (柔度信息值)
10. 如果柔度值降低 , 步长 $t^{(k)}$ 不变 ; 否则步长减小回到步骤7
11. 如果收敛算法结束 , 否则开始执行步骤3进行下一次迭代

数值实例

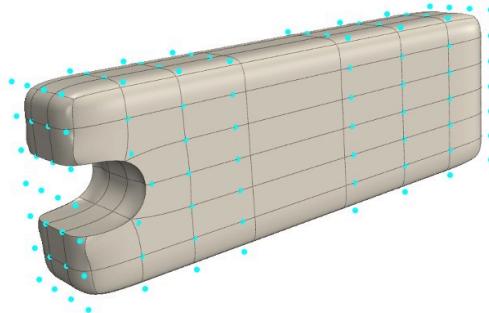
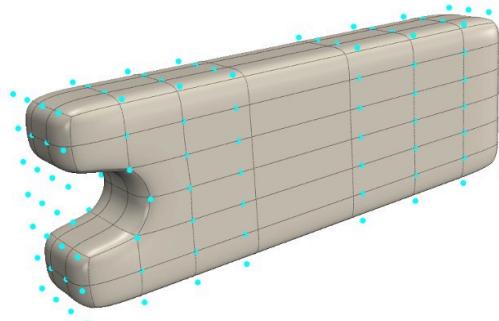
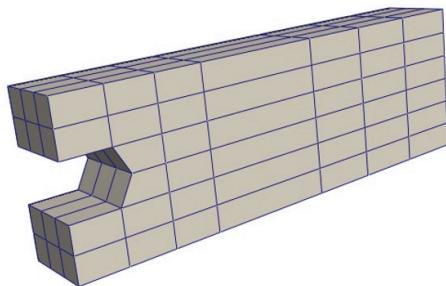


初始模型

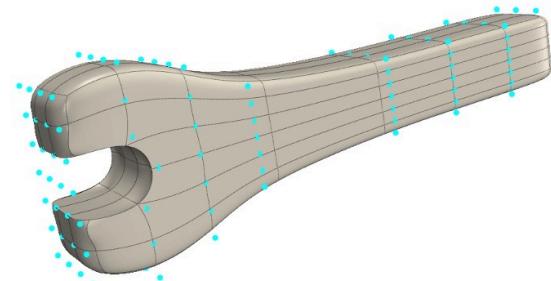
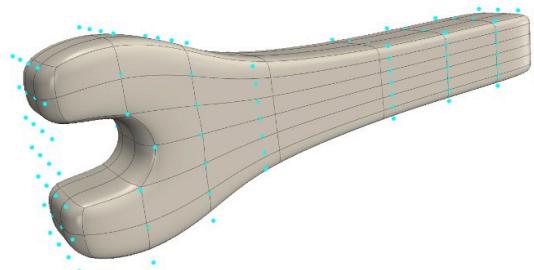
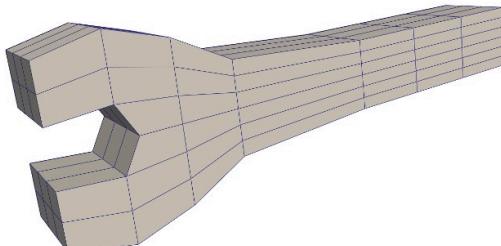


优化结果

数值实例



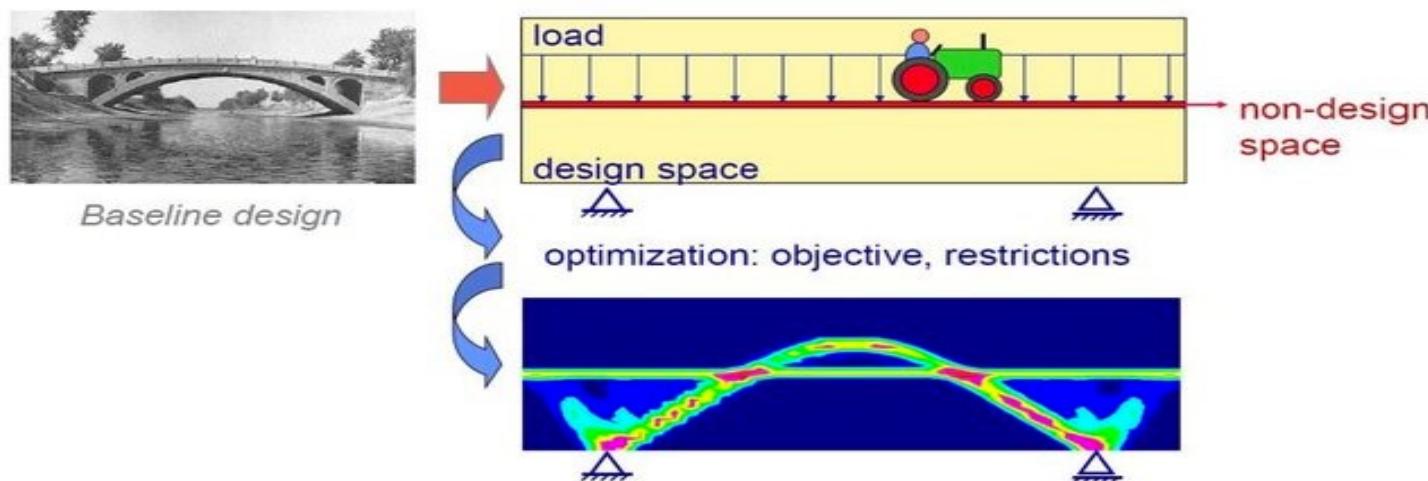
初始模型



优化结果

拓扑优化技术 (Topology optimization)

- 拓扑优化是一种具有创新性的概念设计技术，在产品设计的最初阶段，设计人员确定设计空间、设计目标、设计约束和制造工艺约束等



基于体细分的IGA拓扑优化

$$\text{minimize} : c(\rho) = \frac{1}{2} \mathbf{U}^T \mathbf{K}(\rho) \mathbf{U} \quad (6a)$$

$$s.t : \mathbf{K}(\rho) \mathbf{U} = \mathbf{F} \quad (6b)$$

$$\sum_{i=1}^N \rho_i V_i \leq V^* \quad (6c)$$

$$\rho_i = \rho_{min} \text{ or } 1 \quad (6d)$$

where \mathbf{U} is the global displacement vector. V_i is the volume of an individual block and V^* is the target volume. The binary design variable ρ_i denotes the density of i th element. To avoid singularities of the global stiffness matrix \mathbf{K} , ρ_{min} is set to be 0.0001 .

灵敏度计算

The derivatives of the total strain energy c with respect to the design variable ρ_e^i can be computed as

$$\frac{\partial c}{\partial \rho_e} = -\frac{p}{2}(\rho_e)^{p-1} \mathbf{u}_e^T (\mathbf{K}_e)^0 \mathbf{u}_e \quad (7)$$

Then the sensitivity of density element e can be computed as

$$\alpha_e = -\frac{\partial c}{\partial \rho_e}$$

A large value of sensitivity indicates that the current element has a high impact on the change of the strain energy.

算法流程---多分辨率拓扑优化

- ① Input a ACC subdivision volume model M and set the density ρ of each cell to 1.
- ② Define the BESO parameter such as objective volume V^* , evolutionary ratio ER and penalty exponent p .
- ③ Perform isogeometric analysis, i.e. construct the stiffness matrix K and the right end term f .
- ④ Calculate the displacement u , and perform sensitivity analysis for next level subdivision solid.
- ⑤ Determine the volume V_i to be retained by ER for the current iteration

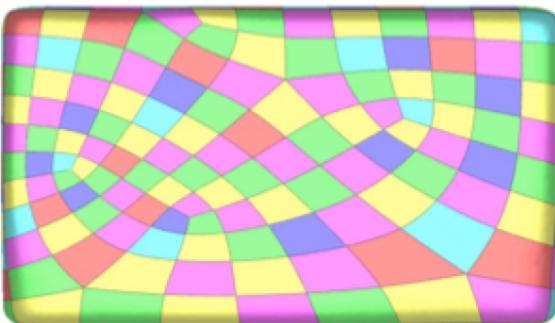
$$V_i = V_{i-1}(1 - ER)$$

- ⑥ Sort all the elements by sensitivity from small to large, and the density of elements to be removed is set to be ρ_{min} .
- ⑦ Repeat 4-7 until the objective volume is achieved i.e. $V_i = V^*$. Then delete the element whose $\rho = \rho_{min}$.

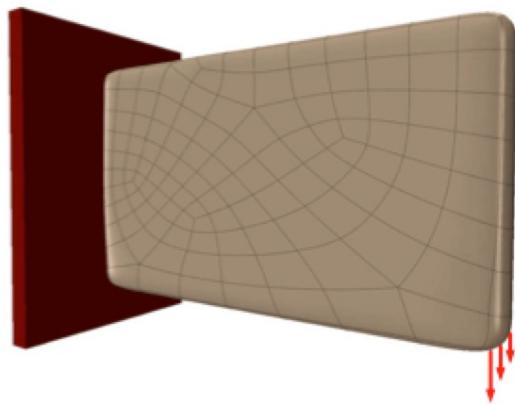
Example I



(a) Input mesh



(b) Approximation Catmull-Clark
subdivision solids of (a)

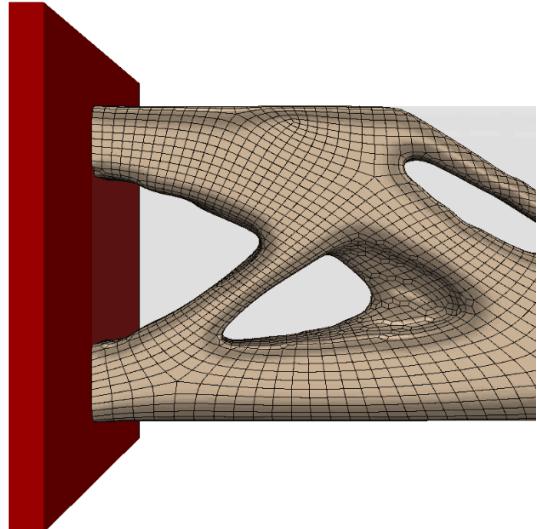


(c) Stress condition

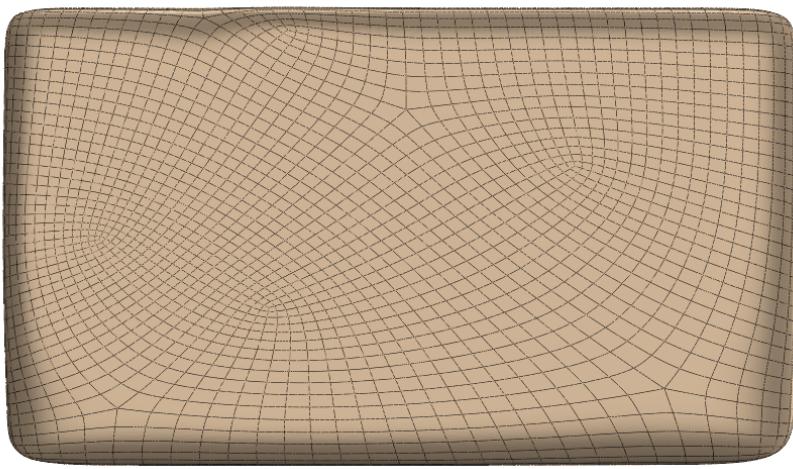


(d) Numerical Solution

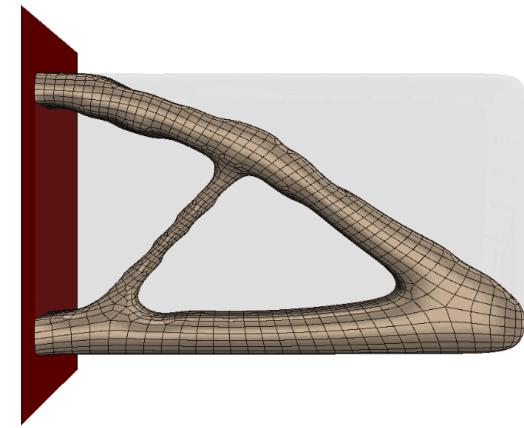
Examples I



Volume fraction: 53.6%

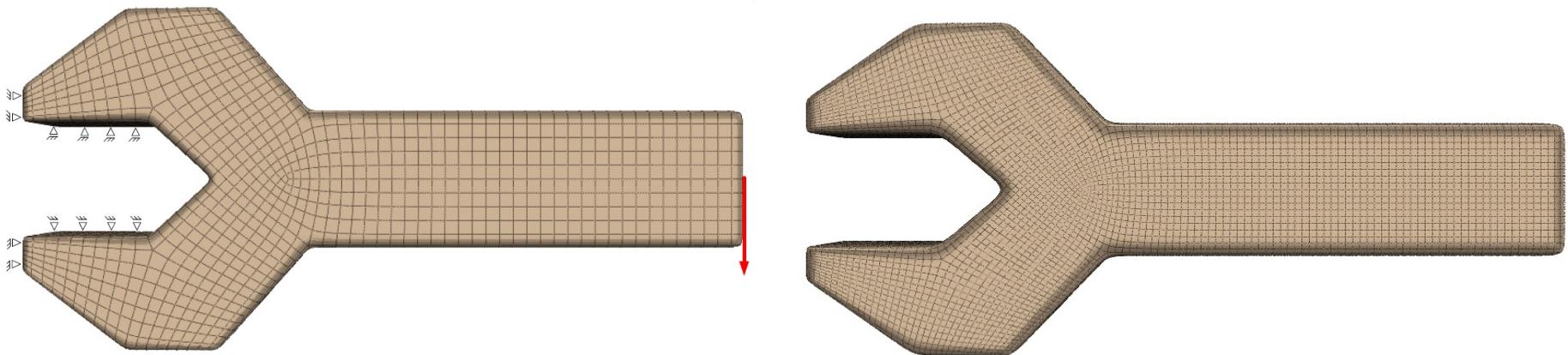


Volume fraction: 63.6%

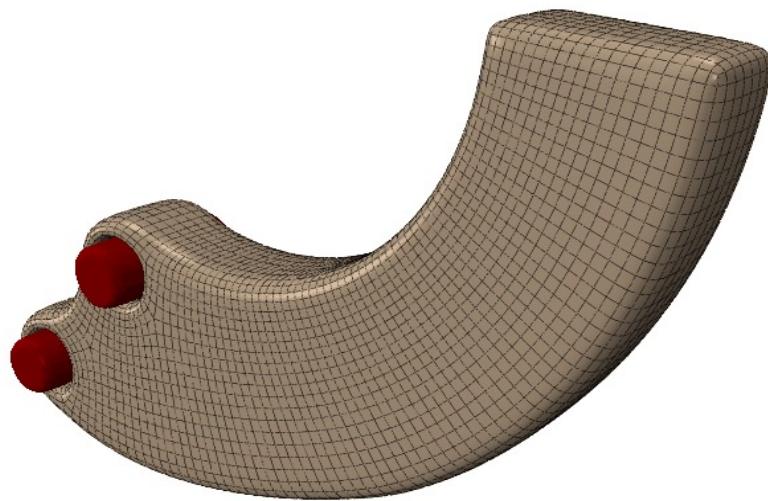
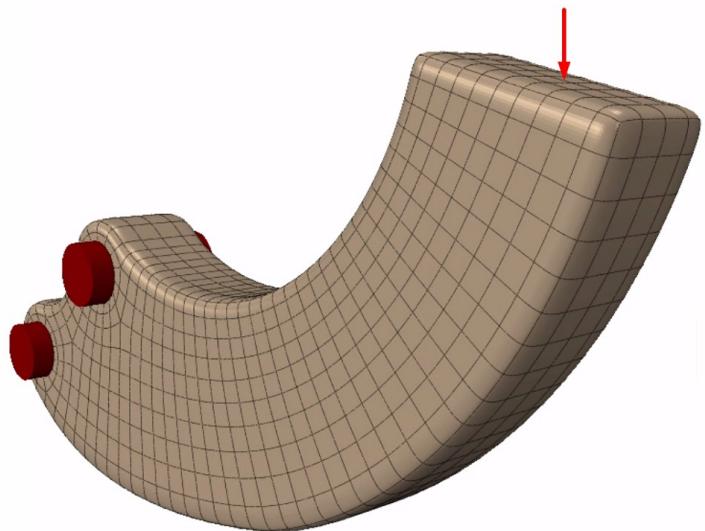


Volume fraction: 79.4%

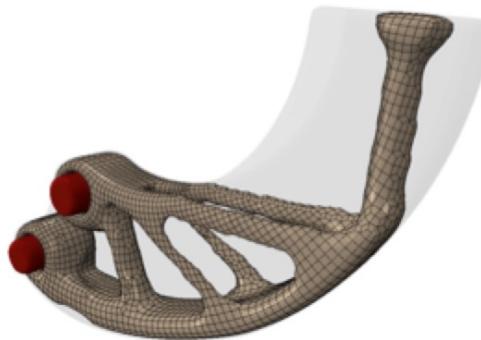
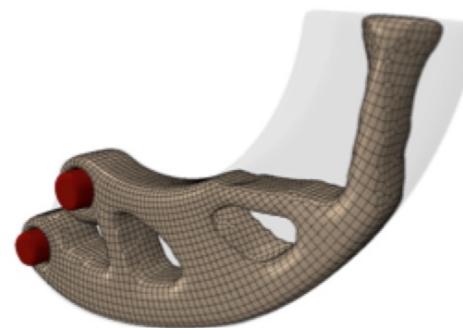
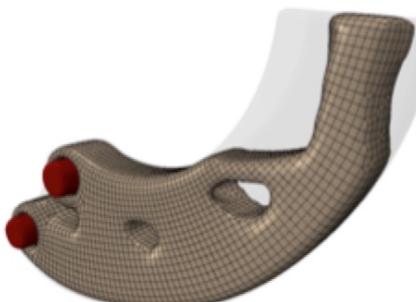
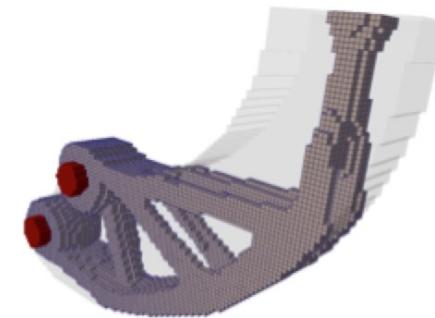
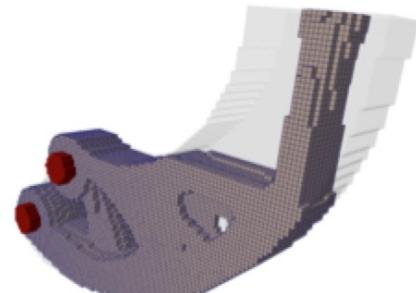
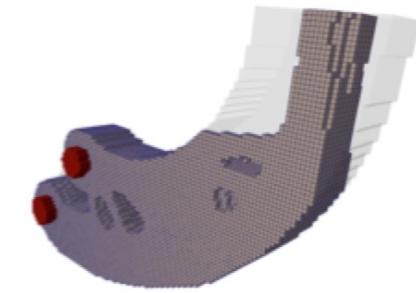
Examples II



Examples III



与Voxel 方法比较

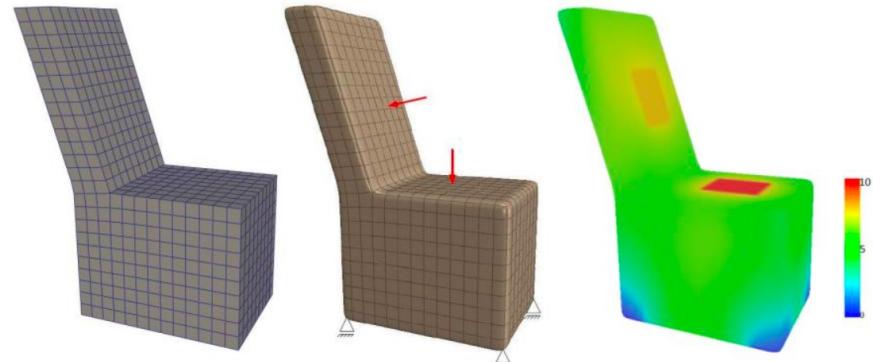


Volume fraction : 33.3%

volume fraction: 53.6%

volume fraction: 75.7%

Examples III



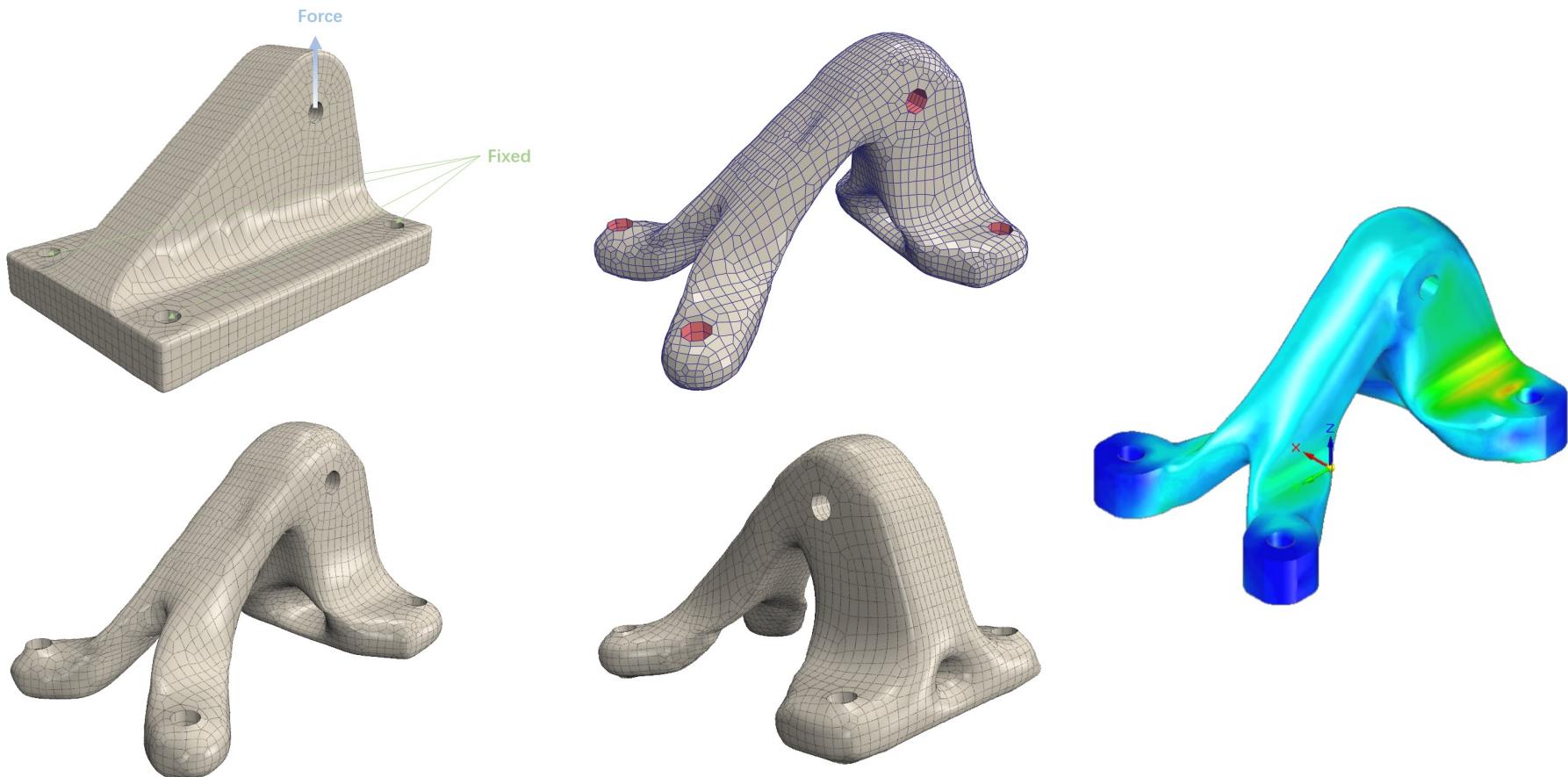
(a) 输入模型

(b) 外力描述

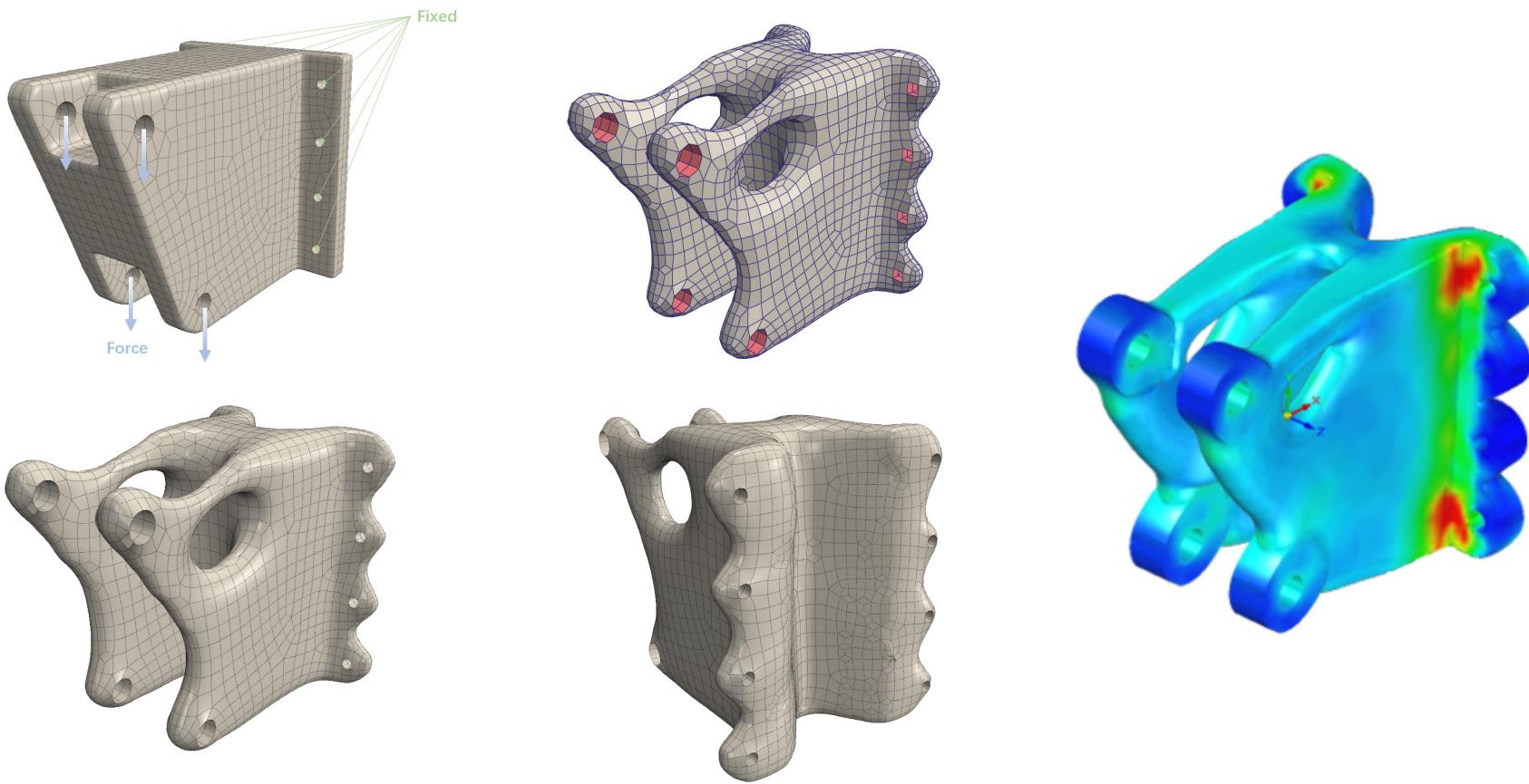
(c) IGA 求解



与商软结果比较 I

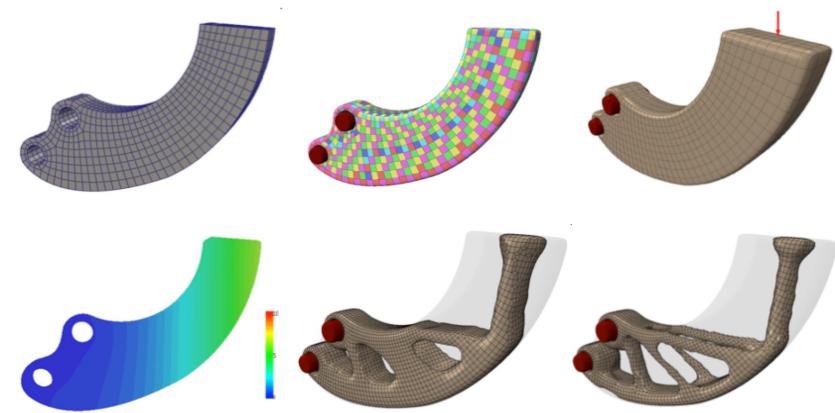


与商软结果比较 II



总结

- **体细分**: 连接复杂曲体建模与等几何分析的桥梁
- **前处理**: 基于体细分的复杂曲体建模
- **求解器**: 基于体细分的高精度仿真计算
- **后处理**: 基于体细分的多分辨率形状/拓扑优化



Advantages of the proposed framework

- Seamless data integration of geometric modeling, physical simulation and shape/topology optimization, which is currently a bottleneck problem in generative design;
- Efficient simulation/optimization framework due to IGA properties;
- Re-mesh-generation is avoided during the shape/topology optimization;
- The final shape/topology optimization results have a spline representation, which can be imported to CAD applications directly;
- Subdivision representation naturally has a multi-resolution scheme, by which computational efficiency can be significantly improved for high-resolution design optimization.

Thank you



智能可视建模与仿真实验室
Intelligent Visual Modeling & Simulation (iGame) Lab