



智能可视建模与仿真实验室
Intelligent Visual Modeling & Simulation (iGame) Lab

第五讲 等几何分析中的计算域参数化II

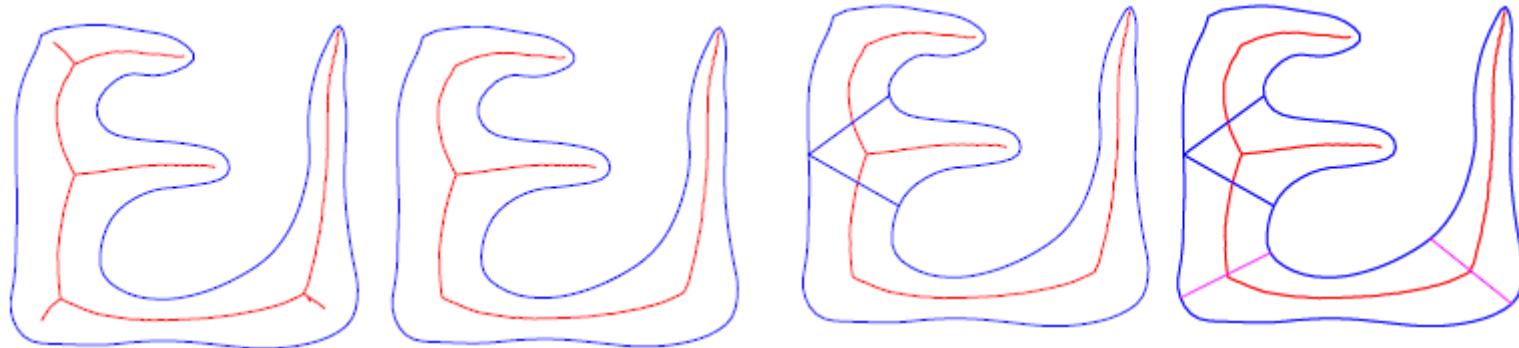
徐 岗

杭州电子科技大学计算机学院
浙江省离散行业工业互联网重点实验室

Email : gxu@hdu.edu.cn

基于骨架的平面区域参数化

- 利用区域的骨架，将区域先剖分后参数化。
 - 简化骨架；
 - 根据分支长短剖分；
 - 根据曲率或骨架圆半径的变化剖分。



《Two-dimensional domain decomposition based on skeleton computation for parameterization and isogeometric analysis》, Jinlan Xu, Falai Chen, Jiansong Deng, CMAME, 2015.

骨架简化

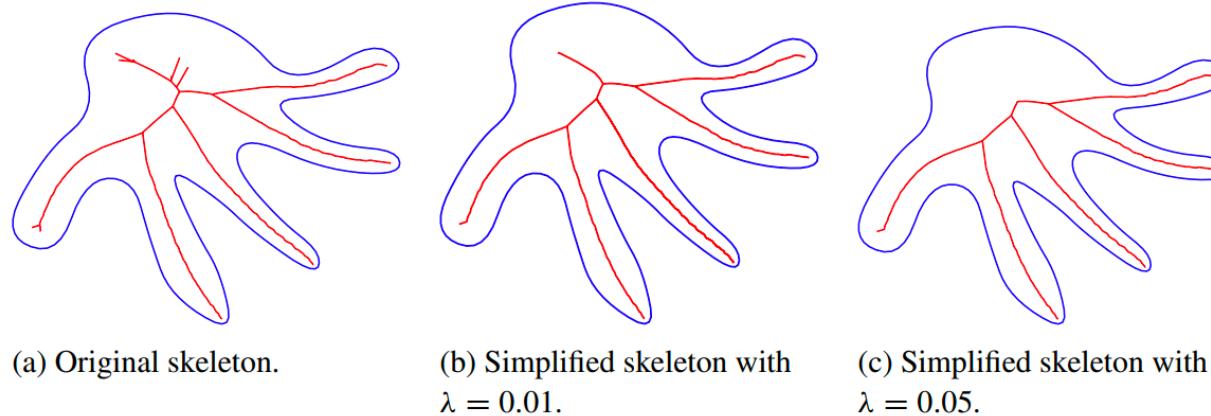


Fig. 8. Skeleton of a hand shape.

Definition 3.1. For a given 2D domain D , the **skeleton** of D is defined by the point set

$$S(D) = \{ p | D(p, B) = \|p - q_i\| = \|p - q_j\|, q_i \neq q_j, q_i, q_j \in B \},$$

where B is the boundary of domain D . The skeleton is the locus of the centers of circles that are tangent to the boundary curve B in two or more points, where all such circles are contained in D . The circles are called **skeleton circles**.

基于骨架的区域分解

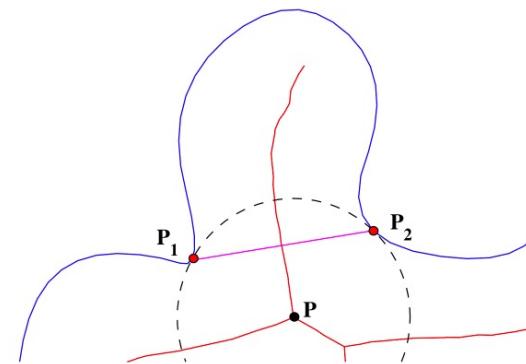
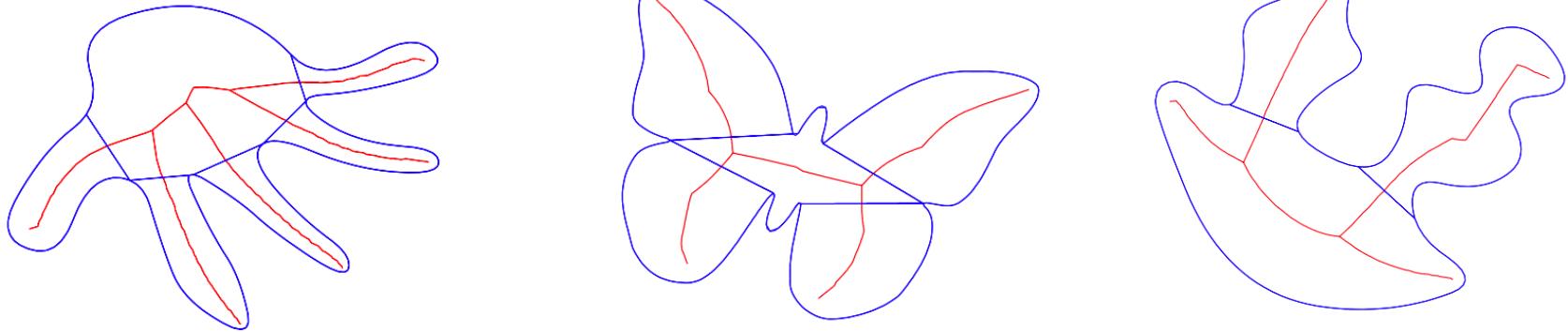
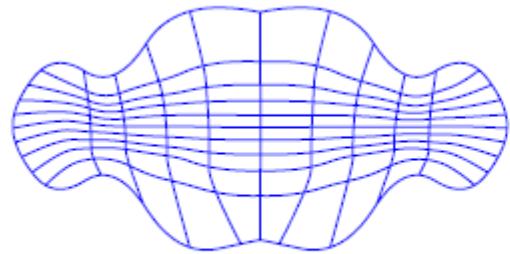


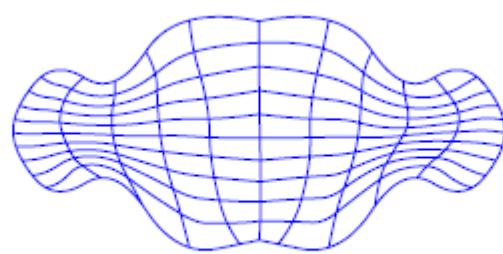
Fig. 9. For the branch point P , two contact points P_1 and P_2 of the skeleton circle at point P with the boundary curves are chosen as segmentation points.



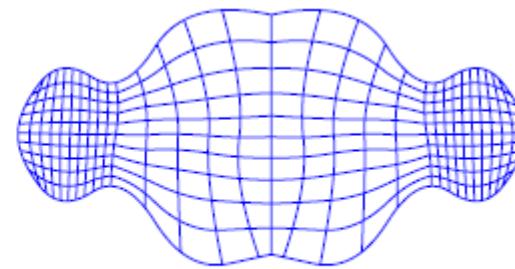
基于骨架的平面区域参数化



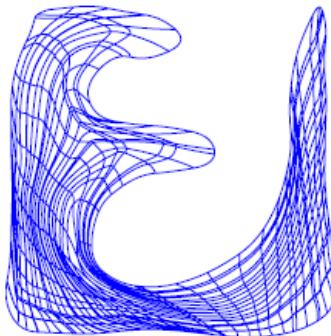
(a) Harmonic map.



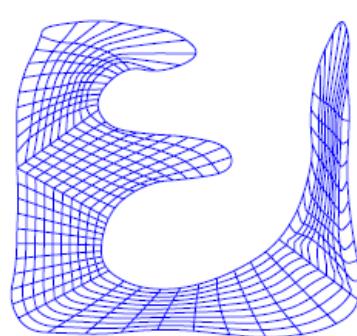
(b) Nonlinear optimization.



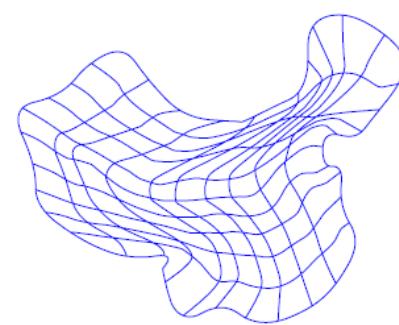
(c) Our method.



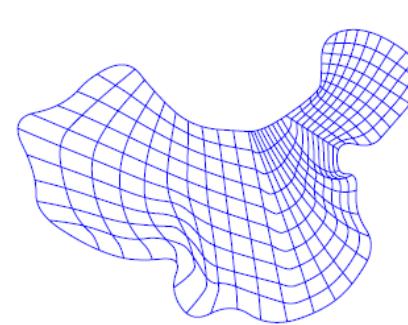
(b) Nonlinear optimization.



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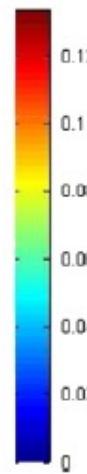
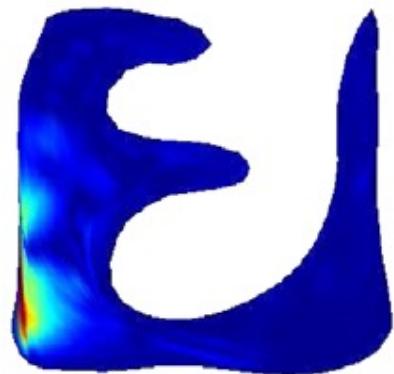


(a) Nonlinear optimization.

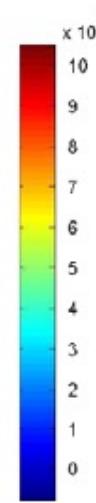
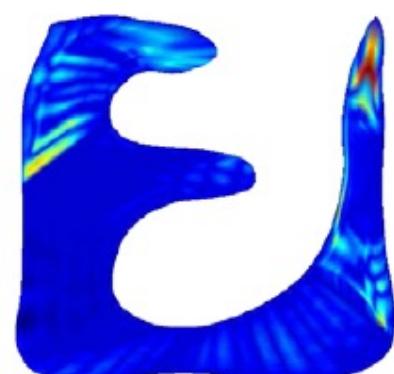


(b) Our method.

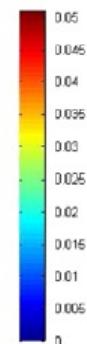
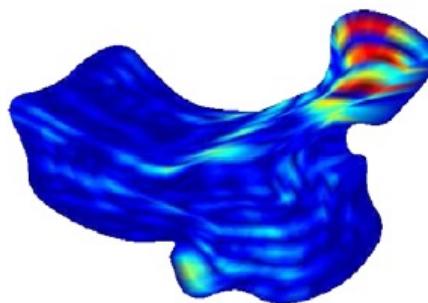
基于骨架的平面区域参数化



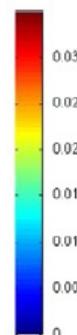
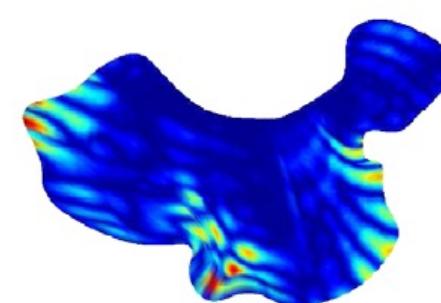
(a) Nonlinear optimization.



(b) Our method.



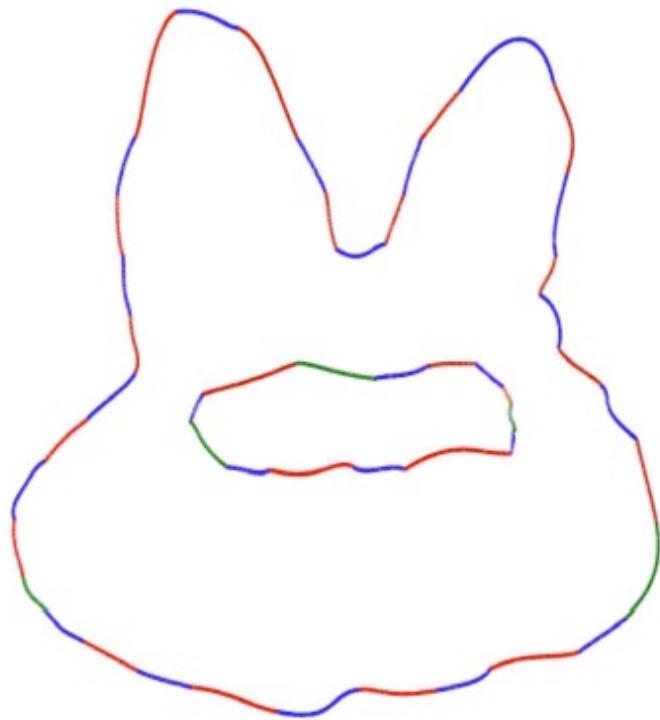
(a) Nonlinear optimization.



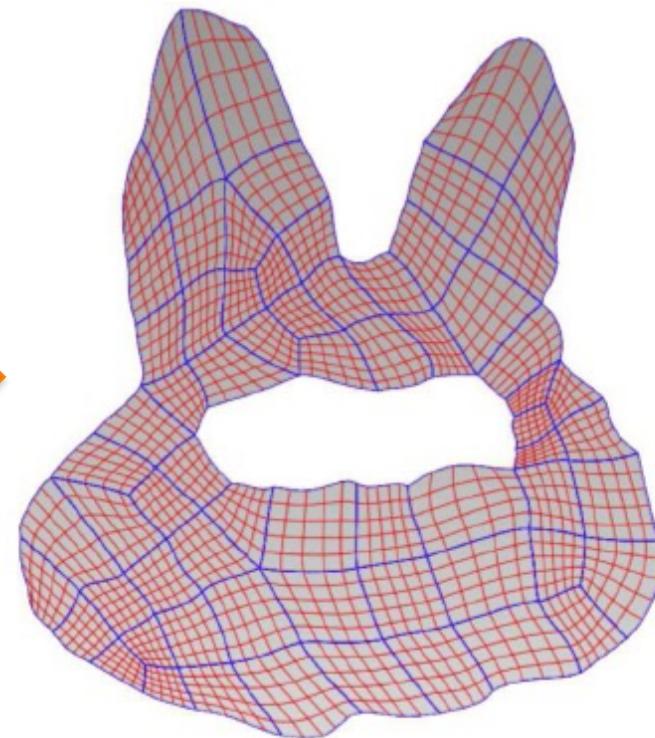
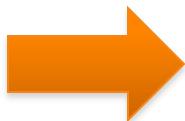
(b) Our method.

Planar domain with arbitrary topology (CMAME 2018)

- Given the boundary spline curves of a planar domain with arbitrary topology, construct the patch structure and control points to obtain IGA-suitable parameterization



(a) boundary Bézier curves



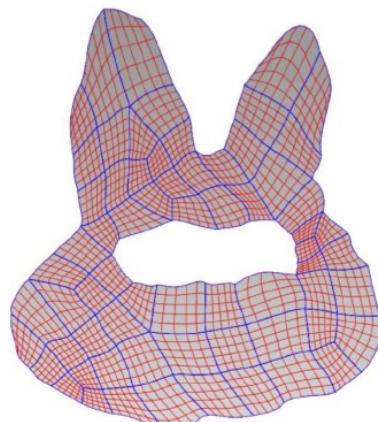
(e) parameterization result

Desired parameterization method

- Boundary-preserving
- Automatic continuity imposition
(\neq high-order meshing with C^0)
- Automatic construction of segmentation curve
- Injective
- Uniform patch size
- Orthogonal iso-parametric structure

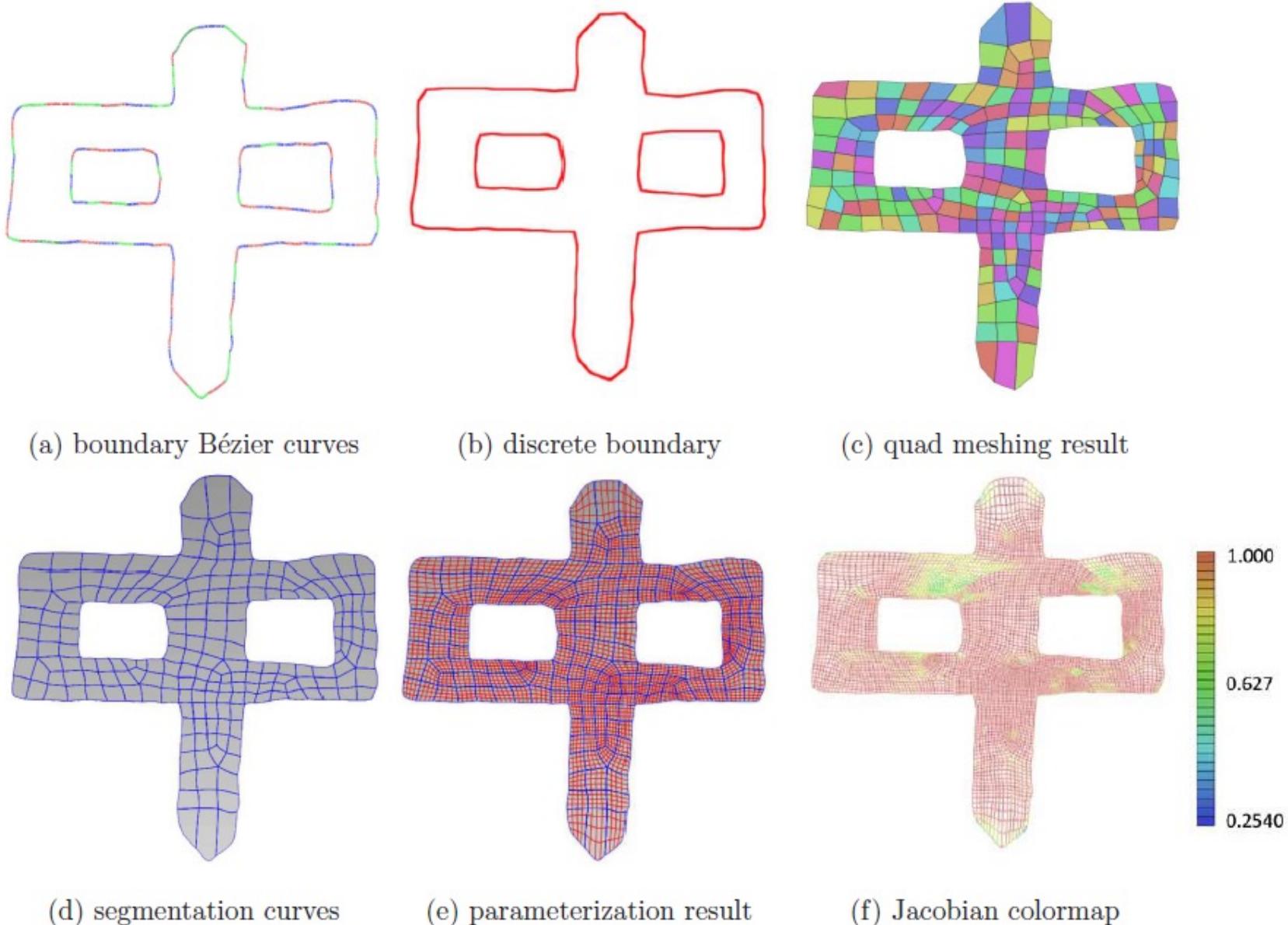
Proposed framework

1. Pre-processing for high-quality parameterization
2. Topology information generation of quadrilateral decomposition
3. Quadrilateral patch partition by global optimization
4. High-quality patch parameterization by local optimization



(e) parameterization result

Framework Overview



Pre-processing of input boundary curves

- Bézier extraction

$$\mathbf{N}(t) = \mathbf{CB}(t)$$

$$\mathbf{P} = \mathbf{CQ}$$

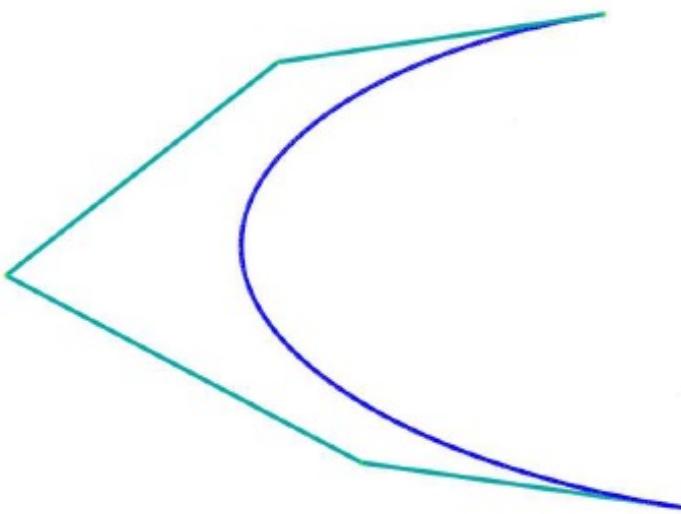
- Bézier subdivision

$$\Gamma \geq \log_4 \frac{\sqrt{3}n(n-1)\eta}{8L_{ave}}$$

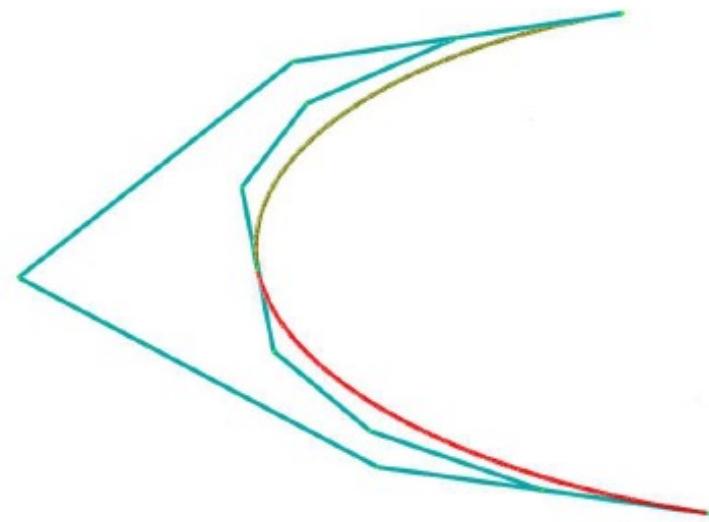
$$\eta = \max_{0 \leq i \leq n-2} \{|s_{i,k}^x - 2s_{i+1,k}^x + s_{i+2,k}^x|, |s_{i,k}^y - 2s_{i+1,k}^y + s_{i+2,k}^y|\}$$

Pre-processing of input boundary curves

- Subdivision of a Bézier curve with concave shape



(a) original Bézier curve



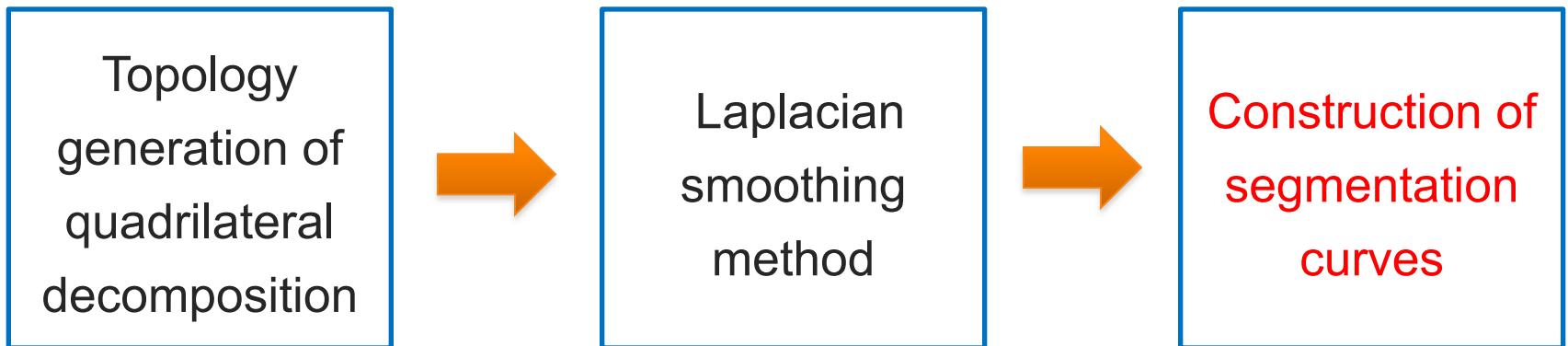
(b) Bézier subdivision

Proposed framework

1. Pre-processing for high-quality parameterization
2. Topology information generation of quadrilateral decomposition
3. Quadrilateral patch partition by global optimization
4. High-quality patch parameterization by local optimization

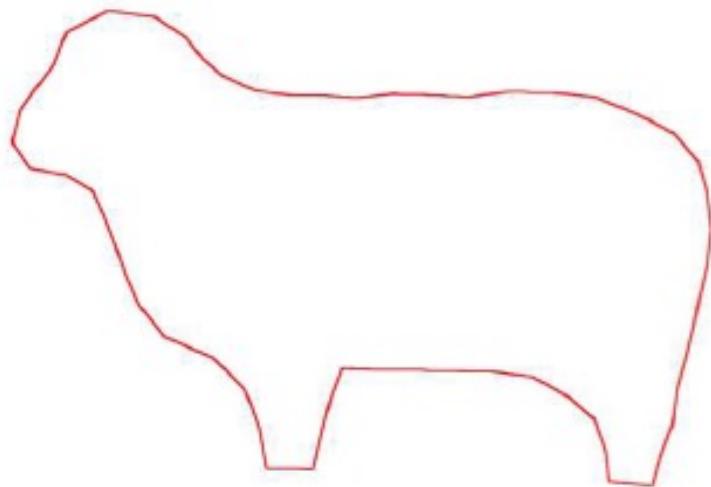
Global optimization method

- Propose a global optimization method to construct the four-sided curved partition of the computational domain



Topology generation of quadrilateral decomposition

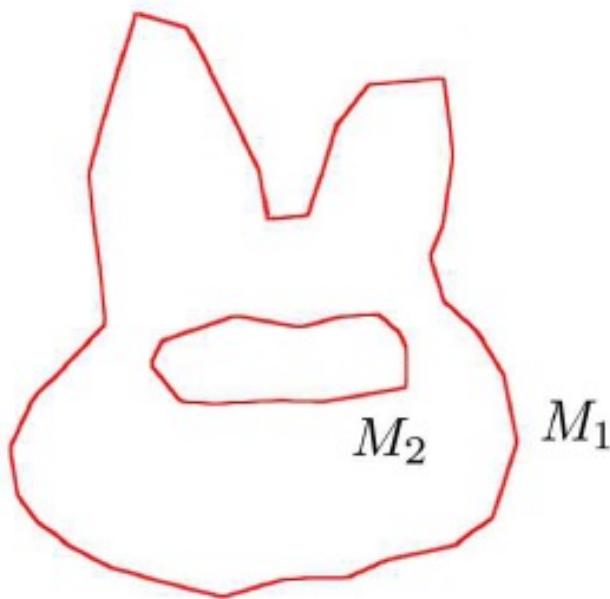
- **Step. 1:** Construct the discrete boundary by connecting the endpoints of the extracted Bézier curves.



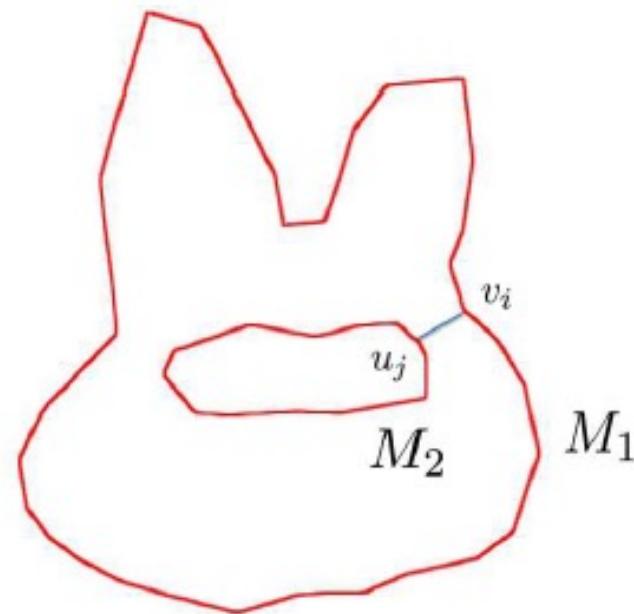
(a)input discrete boundary

Step. 2

- Multiply-connected region → simply-connected region.



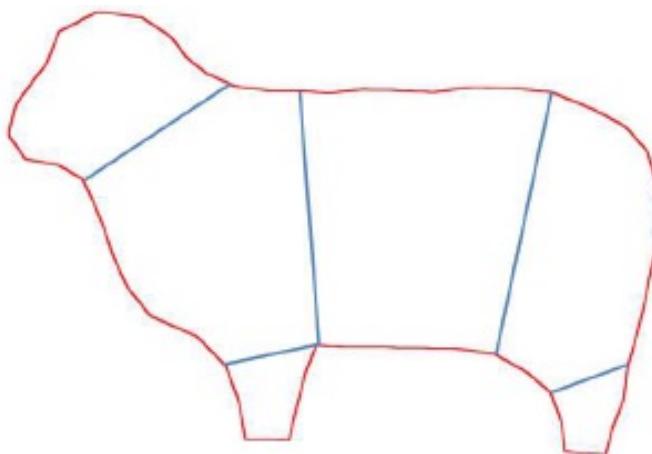
(a) multiply-connected domain



(b) simply-connected domain

Step. 3

- Approximate convex decomposition of the simply-connected regions



(b) quasi-convex polygon decomposition

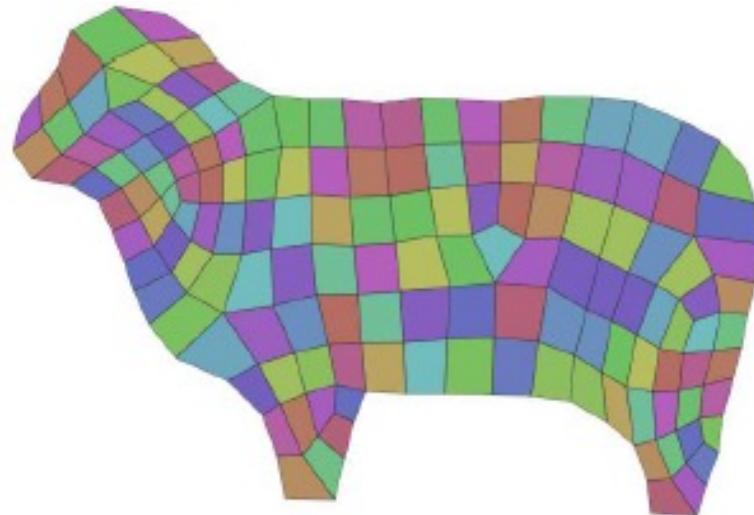
J.M. Lien, N.M. Amato, Approximate convex decomposition of polygons,
Comput. Geom. 35 (1–2) (2016) 100–123

Step. 4

- For each quasi-convex polygon obtained in Step.3, generate the quadrangulation topology information
- Only introduce irregular vertices with valence 3 or 5, which guarantees the solution existence for G1 planar parameterization around the irregular vertex

Reference:

K.Takayama,D.Panozzo,O.
Sorkine-Hornung Pattern-based
quadrangulation for N-sided
patches. CGF, 2015



(c) quad-meshing result by our method with 147 elements and 14 irregular vertices

Laplacian smoothing

- We adapt an iterative Laplacian smoothing method to improve the quality of the quad mesh.

$$x_i^k = \frac{\sum_{j=1}^{N_i} x_j^{k-1}}{N_i}, \quad y_i^k = \frac{\sum_{j=1}^{N_i} y_j^{k-1}}{N_i}$$

Termination rules:

$$\frac{\left[\sum_{i=1}^m [(x_i^k - x_i^{k-1})^2 + (y_i^k - y_i^{k-1})^2] \right]^{1/2}}{\left[\sum_{i=1}^m [(x_i^{k-1})^2 + (y_i^{k-1})^2] \right]^{1/2}} < \delta$$

Construction of segmentation curves

- The segmentation curves should interpolate two vertices on the quad mesh $Q(V,E)$.
- **Global optimization** method to construct the optimal shape of segmentation curves.

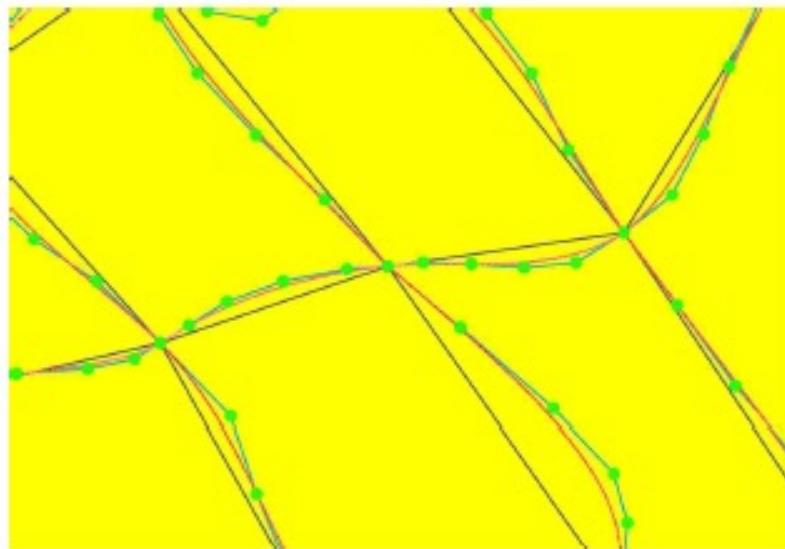


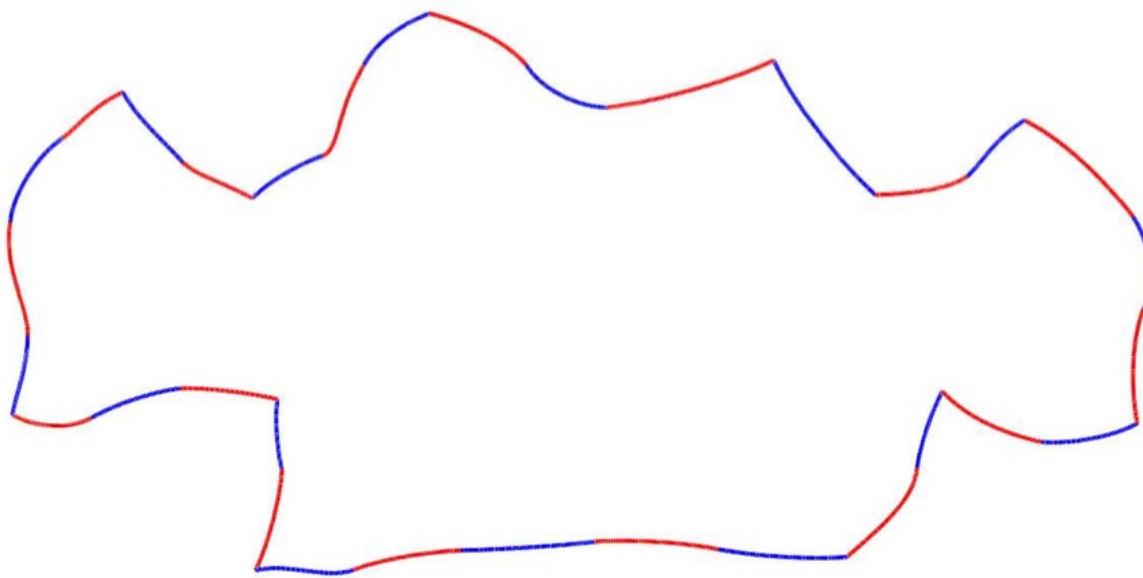
Fig.5(a) segmentation curves(red) and quad edges(black)

Desired parameterization method

- Boundary-preserving
- Automatic continuity imposition
(\neq high-order meshing with C^0)
- Automatic construction of segmentation curve
- Injective
- Uniform patch size
- Orthogonal iso-parametric structure

Uniform patch size

Computing the area of planar region bounded by B-spline curves?



Computing the area of planar region with Bézier boundary

- For the planar region bounded by N pieces of Bézier curves

$$\mathbf{S}_k(t) = (S_k^x(t), S_k^y(t)) = \sum_{i=1}^n (s_{i,k}^x, s_{i,k}^y) B_i^n(t)$$

Then the area $A(\Omega)$ of the planar region is

$$A(\Omega) = \frac{1}{4n} \sum_{k=1}^N \sum_{j=0}^{2n-1} (c_j^k - d_j^k)$$

$$c_j^k = \sum_{r=\max(0, j-n)}^{\min(j, n-1)} \frac{\binom{n}{r} \binom{n-1}{j-r}}{\binom{2n-1}{j}} s_{r,k}^x (s_{j-r+1,k}^y - s_{j-r,k}^y)$$

$$d_j^k = \sum_{r=\max(0, j-n)}^{\min(j, n-1)} \frac{\binom{n}{r} \binom{n-1}{j-r}}{\binom{2n-1}{j}} s_{r,k}^y (s_{j-r+1,k}^x - s_{j-r,k}^x)$$

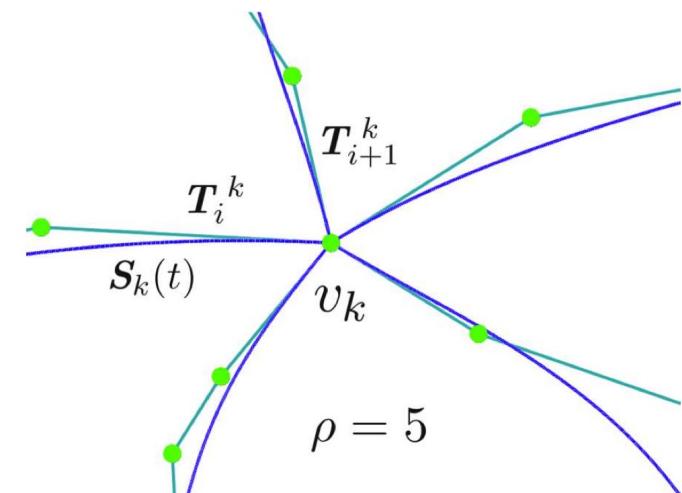
Global optimization method

- Objective functions:

$$F_{\text{uniform}} = \frac{1}{L} \sum_{i=0}^L (A_i - A_{ave})^2$$

$$F_{\text{shape}} = \sum_{k=0}^N \int_0^1 \sigma_1 \|S_k'(t)\|^2 + \sigma_2 \|S_k''(t)\|^2 dt$$

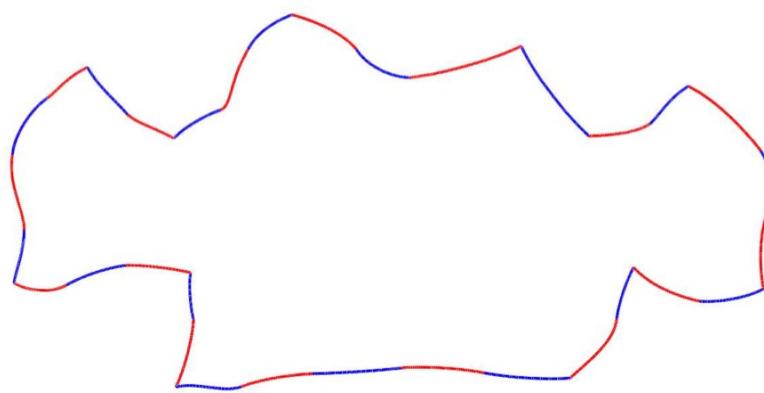
$$F_{\text{tangent}} = \sum_{k=0}^N \sum_{i=1}^{\rho} \left(\frac{\mathbf{T}_i^k \cdot \mathbf{T}_{i+1}^k}{\|\mathbf{T}_i^k\| \|\mathbf{T}_{i+1}^k\|} - \cos \frac{2\pi}{\rho} \right)^2$$



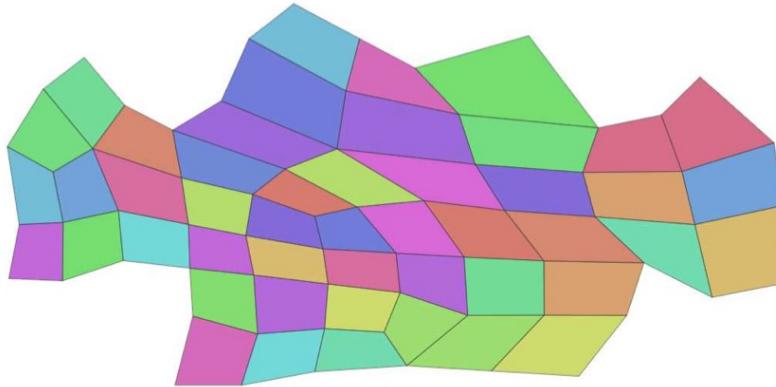
$$F = \omega_1 F_{\text{uniform}} + \omega_2 F_{\text{shape}} + \omega_3 F_{\text{tangent}}$$

$$\arg \min_{s_{i,k}} F$$

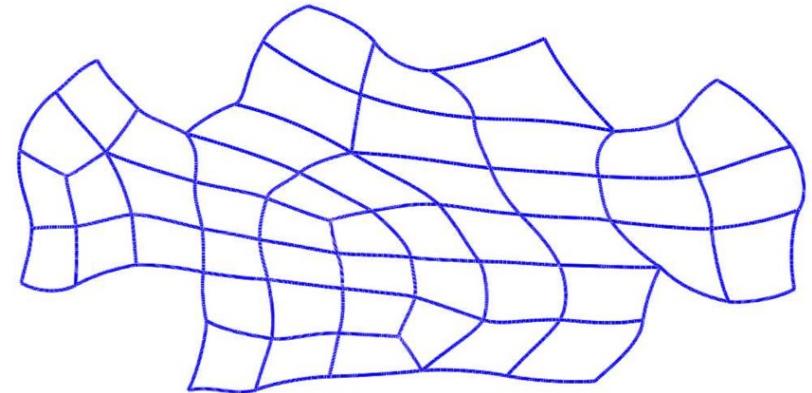
An example



(a) boundary Bézier curves



(e) quad meshing result II



(f) segmentation curves II

Proposed framework

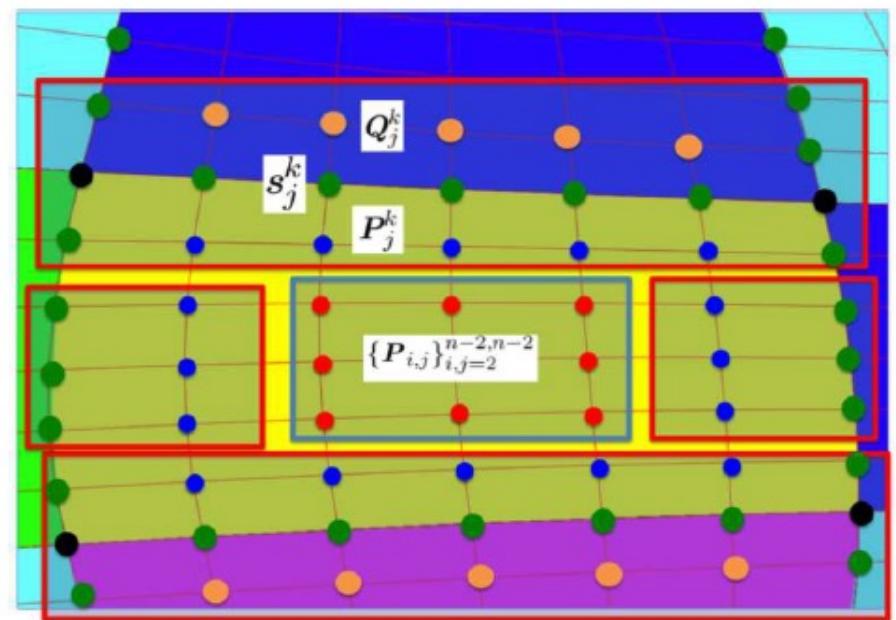
1. Pre-processing for high-quality parameterization
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High-quality patch parameterization

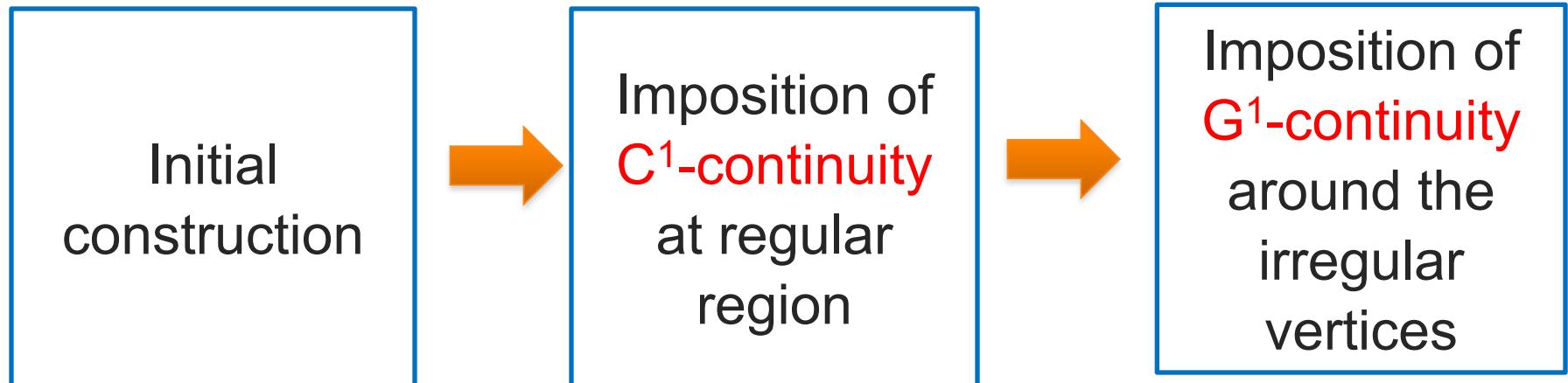
Step 1: Construction of boundary second-layer control points with orthogonality optimization;

Step 2: Local C1 linear-energy-minimizing method for constructing inner control points;

Step 3: Find out the invalid patches on the parameterization, then recover patch validity;



Step. 1: Construction of boundary control points



Initial construction

- Firstly, we will describe the initial construction by orthogonality optimization.

$$\mathbf{P}_{n-1,j}^0 = \mathbf{P}_{n,j} + \frac{(\mathbf{P}_{0,j} - \mathbf{P}_{n,j})}{n}$$

$$\arg \min_{\mathbf{P}_{n-1,j}} \int_0^1 (\langle \mathbf{r}_{1,u}(1, v), \mathbf{r}_{1,v}(1, v) \rangle)^2 dv$$

$$\mathbf{r}_{1,u}(1, v) = n \sum_{j=0}^n B_l^n(v) \Delta^{1,0} \mathbf{P}_{n-1,l},$$

$$\mathbf{r}_{1,v}(1, v) = n \sum_{j=0}^{n-1} B_l^{n-1}(v) \Delta^{0,1} \mathbf{P}_{n,l},$$

Imposition of C¹-continuity by Lagrange Multiplier method

Minimize the change of related control points along the segmentation curves such that they satisfy the C¹-constraints

$$s_j^k - P_j^k = Q_j^k - s_j^k,$$

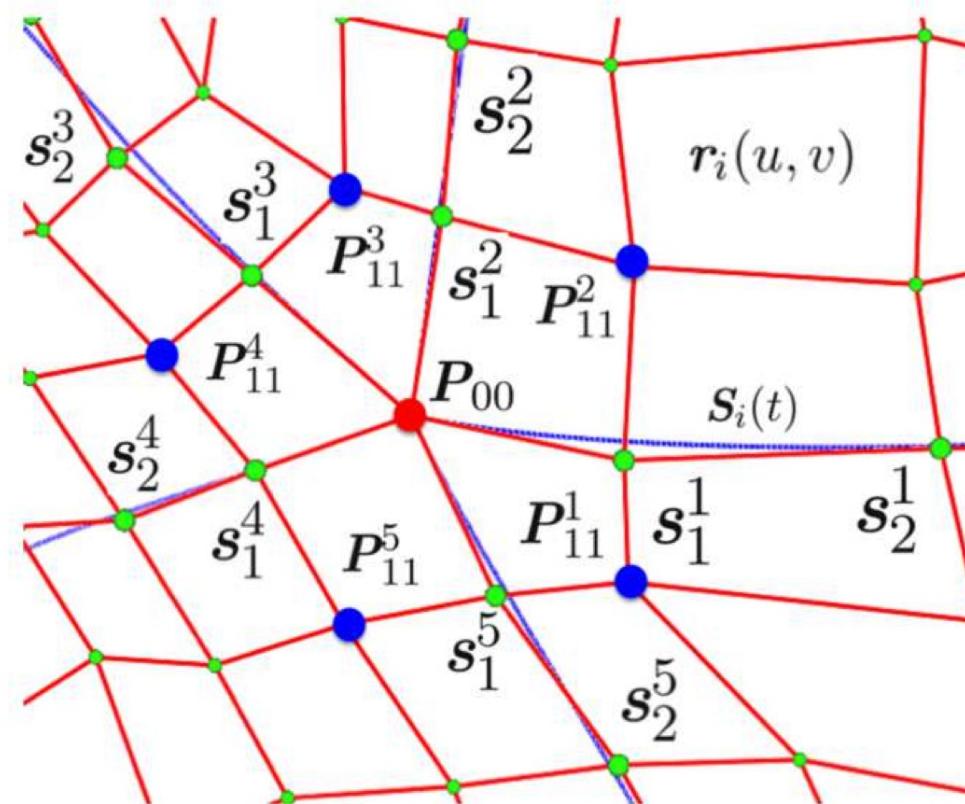
$$\text{Min} \sum_{k=1}^N \sum_{j=0}^n (\|P_j^k - \bar{P}_j^k\|^2 + \|Q_j^k - \bar{Q}_j^k\|^2)$$

the Lagrange function:

$$L = \sum_{i=0}^N \sum_{j=0}^n (\|P_j^k - \bar{P}_j^k\|^2 + \|Q_j^k - \bar{Q}_j^k\|^2) + \sum_{i=0}^N \sum_{j=0}^n \lambda_{k,j} (2s_j^k - P_j^k - Q_j^k)$$

Imposition of G¹-continuity around irregular vertex

- Some special treatments should be done achieve G¹-continuity at the irregular vertices.



G¹-continuity

Imposition of G¹-continuity (Mourrain et al, CAGD 2016)

- The G¹-continuity constraints around the irregular vertex can be described as follows:

$$(\mathbf{s}_1^i - \mathbf{P}_{00}) = \alpha_i(\mathbf{s}_1^{i+1} - \mathbf{P}_{00}) + \beta_i(\mathbf{s}_1^{i-1} - \mathbf{P}_{00}),$$

$$\mathbf{0} = n\alpha_i(\mathbf{P}_{11}^i - \mathbf{s}_1^i) + n\beta_i(\mathbf{P}_{11}^{i-1} - \mathbf{s}_1^i) - (n-1)(\mathbf{s}_2^i - \mathbf{s}_1^i) + (\mathbf{s}_1^i - \mathbf{P}_{00})$$

$$\begin{pmatrix} \alpha_1 & 0 & \dots & 0 & \beta_1 \\ \beta_2 & \alpha_2 & \dots & 0 & 0 \\ 0 & \beta_3 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & \alpha_{M-1} & 0 \\ 0 & 0 & \dots & \beta_M & \alpha_M \end{pmatrix} \begin{pmatrix} \mathbf{P}_{11}^1 \\ \mathbf{P}_{11}^2 \\ \mathbf{P}_{11}^3 \\ \vdots \\ \mathbf{P}_{11}^{M-1} \\ \mathbf{P}_{11}^M \end{pmatrix} = \begin{pmatrix} \mathbf{H}_1 \\ \mathbf{H}_2 \\ \mathbf{H}_3 \\ \vdots \\ \mathbf{H}_{M-1} \\ \mathbf{H}_M \end{pmatrix}$$

There exists
unique solution for
 $M=3$ and $M=5$

High-quality patch parameterization

Step 1: Construction of boundary second-layer control points with orthogonality optimization and continuity constraints.

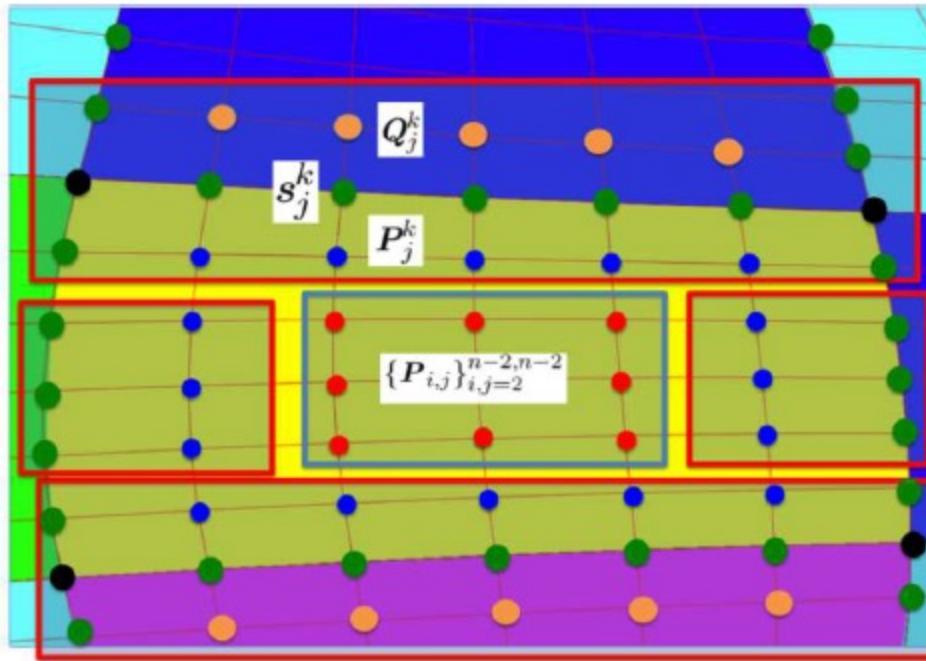
Step 2: Local C1 linear-energy-minimizing method for constructing inner control points.

Step 3: Find out the invalid patches on the parameterization, then recover patch validity.

Local C¹ linear-energy-minimizing

- Constructing interior (n-3)×(m-3) control points of each patch

$$E(\mathbf{r}) = \int_{\Omega} \tau_1 (\|\mathbf{r}_u\|^2 + \|\mathbf{r}_v\|^2) + \tau_2 (\|\mathbf{r}_{uu}\|^2 + 2\|\mathbf{r}_{uv}\|^2 + \|\mathbf{r}_{vv}\|^2) dudv$$



Inner control points construction

- A tensor product Bézier surface $r(u,v)$ has minimal energy $E(r)$ if and only if remaining inner control points satisfy

$$\begin{aligned} \theta = & \frac{\tau_1}{4(n-1)} \left(\sum_{k=0}^{n-1} \sum_{l=0}^n \frac{\binom{n}{l}}{\binom{2n}{l+j}} C_{n,i}^k \Delta^{1,0} \mathbf{P}_{kl} + \sum_{k=0}^n \sum_{l=0}^{n-1} \frac{\binom{n}{k}}{\binom{2n}{i+k}} C_{n,j}^l \Delta^{0,1} \mathbf{P}_{kl} \right) \\ & + \frac{2\tau_2}{(2n-1)^2} \sum_{k=0}^{n-1} \sum_{l=0}^{n-1} \frac{\binom{n-1}{k} \binom{n-1}{l}}{\binom{2n-2}{i+k-1} \binom{2n-2}{l+j-1}} B_{n,i}^k B_{n,j}^l \Delta^{1,1} \mathbf{P}_{kl} \\ & + \frac{\tau_2}{(2n-3)(2n+1)} \left(\sum_{k=0}^{n-2} \sum_{l=0}^n \frac{\binom{n-2}{k} \binom{n}{l}}{\binom{2n-4}{i+k-2} \binom{2n}{l+j}} A_{n,i}^k \Delta^{2,0} \mathbf{P}_{kl} + \sum_{k=0}^n \sum_{l=0}^{n-2} \frac{\binom{n}{k} \binom{n-2}{l}}{\binom{2n}{i+k} \binom{2n-4}{l+j-2}} A_{n,j}^l \Delta^{0,2} \mathbf{P}_{kl} \right) \end{aligned} \quad (29)$$

- A linear system with $(n-3) \times (n-3)$ equations and $(n-3) \times (m-3)$ variables

$$\boxed{\mathbf{M}\mathbf{P} = \mathbf{B}}.$$

High-quality patch parameterization

Step 1: Construction of boundary second-layer control points with orthogonality optimization and continuity constraints.

Step 2: Local C1 linear-energy-minimizing method for constructing inner control points.

Step 3: Find out the invalid patches on the parameterization, then recover patch validity.

Step 3: Injective parameterization

- Jacobian

$$J(u, v) = \sum_{i=0}^{2n-1} \sum_{j=0}^{2n-1} \alpha_{ij} B_i^{2n-1}(u) B_j^{2n-1}(v)$$

$$\min_{0 \leq i, j \leq 2n-1} \alpha_{ij} \leq J(u, v) \leq \max_{0 \leq i, j \leq 2n-1} \alpha_{ij}$$

$$\min_{0 \leq i, j \leq 2n-1} \alpha_{ij} > 0$$



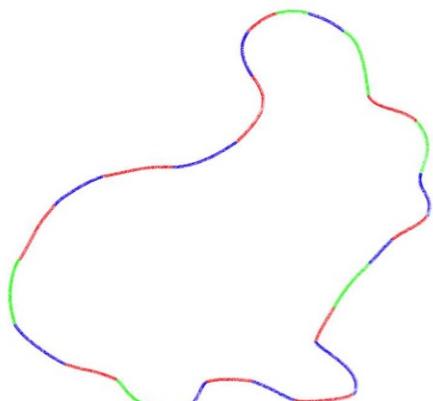
Valid patch

- Logarithmic-barrier method for invalid patch

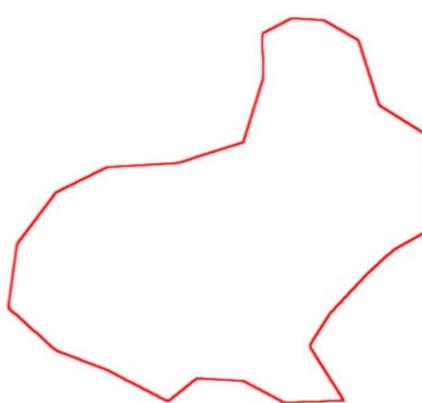
$$\min E(\mathbf{r}(u, v)) \text{ s.t. } \alpha_{ij} > 0$$

$$\arg \min_{P_{i,j}} E(\mathbf{r}(u, v)) - \mu \sum_{i=0}^{2n-1} \sum_{j=0}^{2n-1} \ln(\alpha_{ij})$$

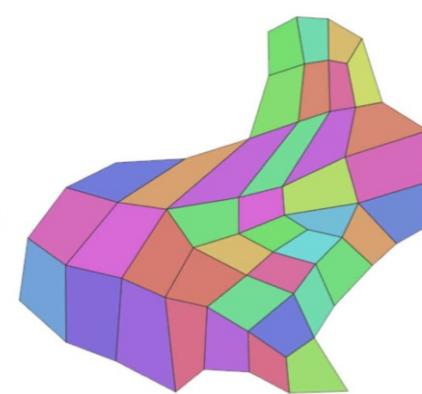
Example I



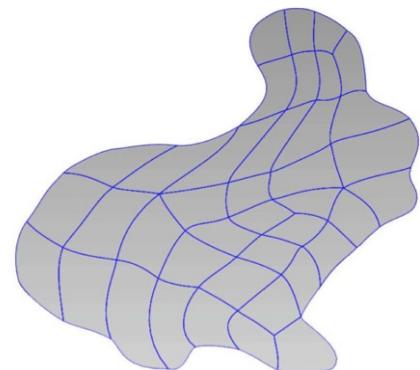
(a) boundary Bézier curves



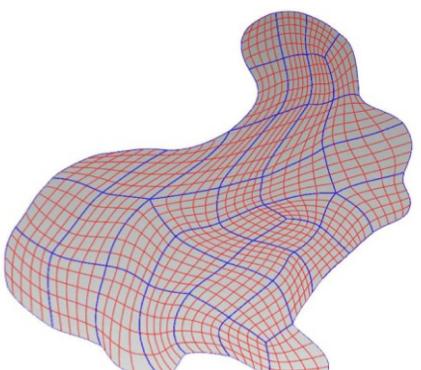
(b) discrete boundary



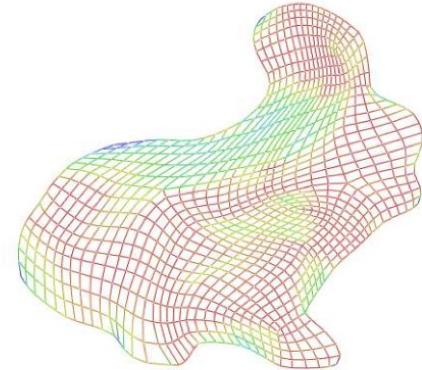
(c) quad meshing result



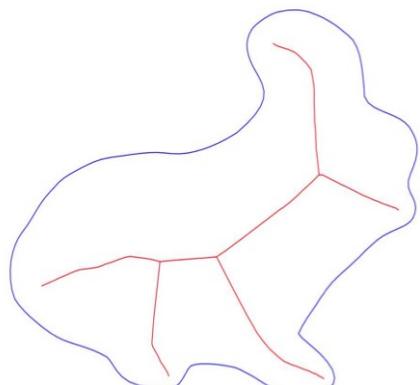
(d) segmentation curves



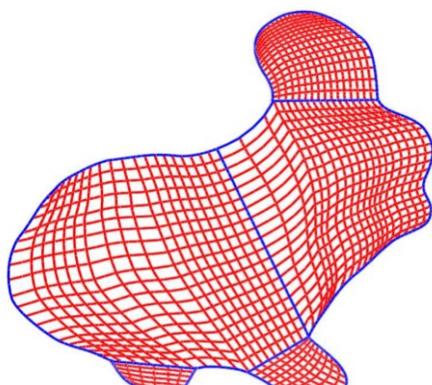
(e) parameterization result



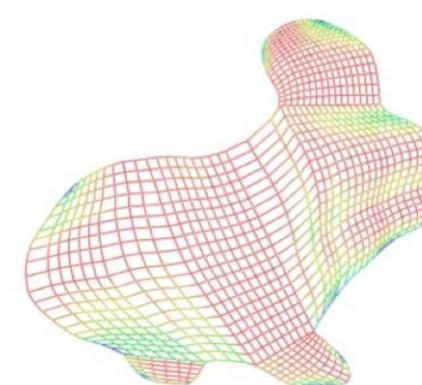
(f) Jacobian colormap



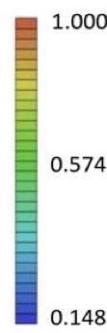
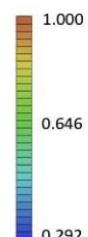
(g) extracted skeleton [42]



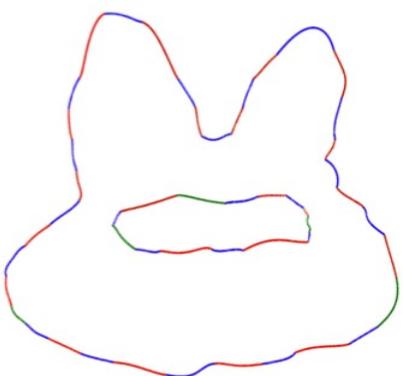
(h) skeleton-based



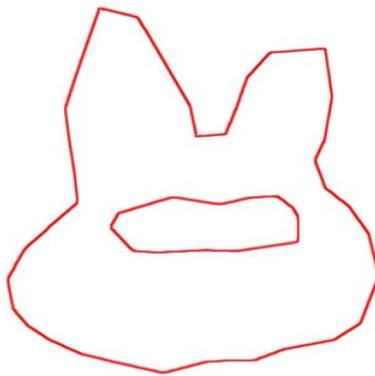
(i) Jacobian colormap of (h)



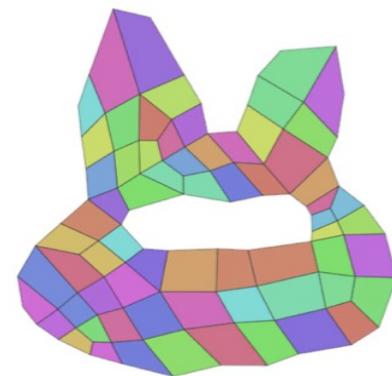
Example II



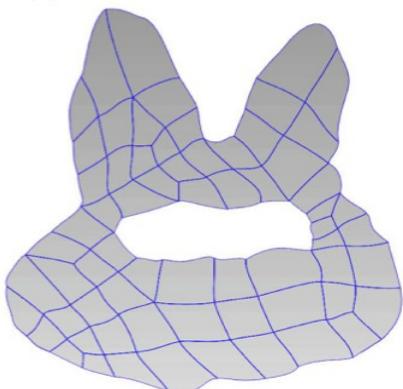
(a) boundary Bézier curves



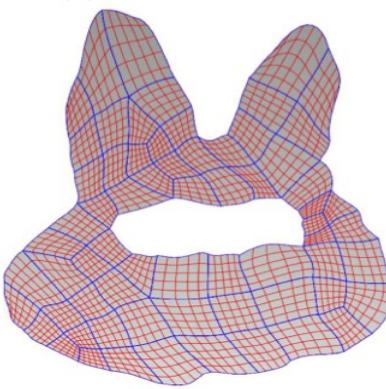
(b) discrete boundary



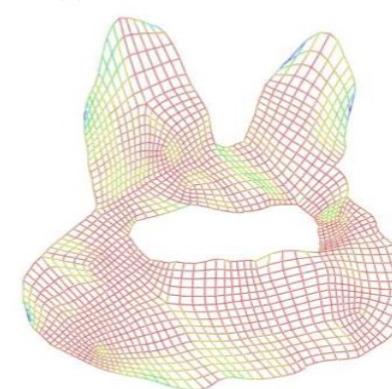
(c) quad meshing result



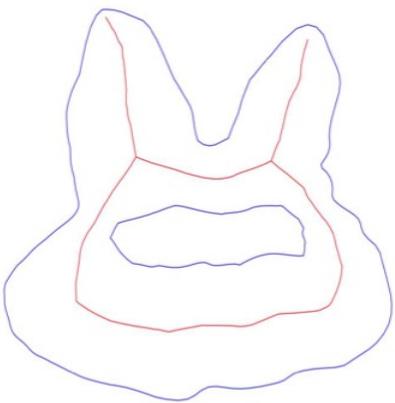
(d) segmentation curves



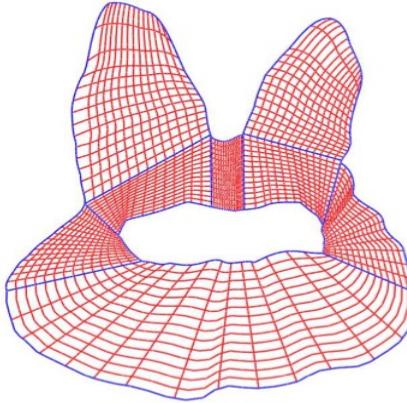
(e) parameterization result



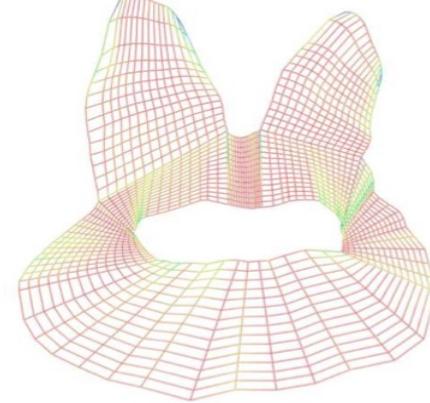
(f) Jacobian colormap



(g) extracted skeleton [42]

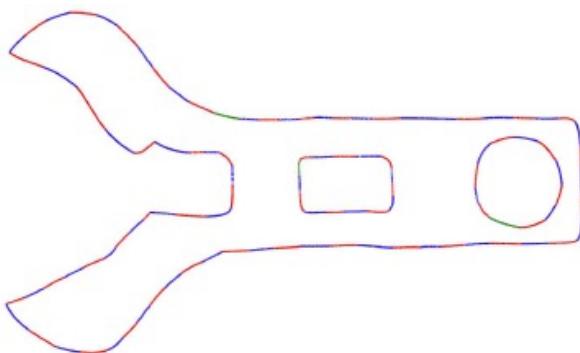


(h) skeleton-based parameterization [42]

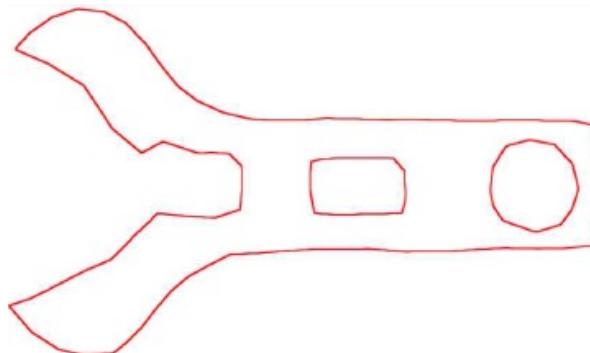


(i) Jacobian colormap of (h)

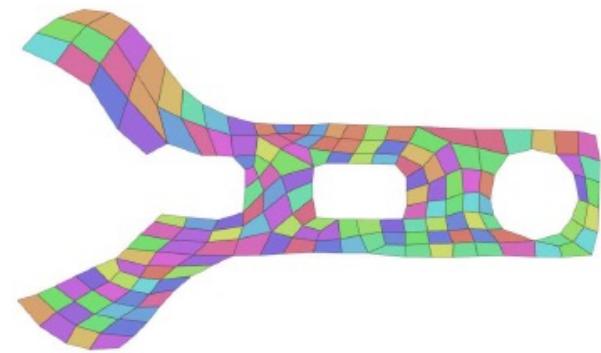
Example III



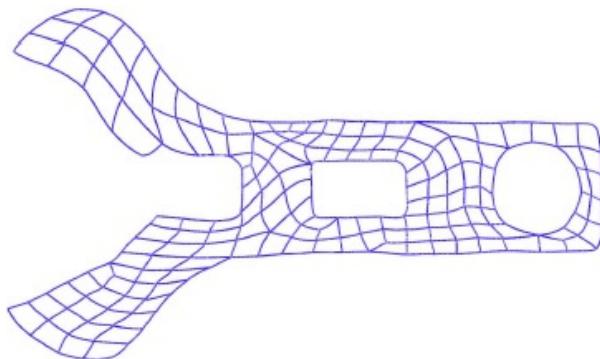
(a) boundary Bézier curves



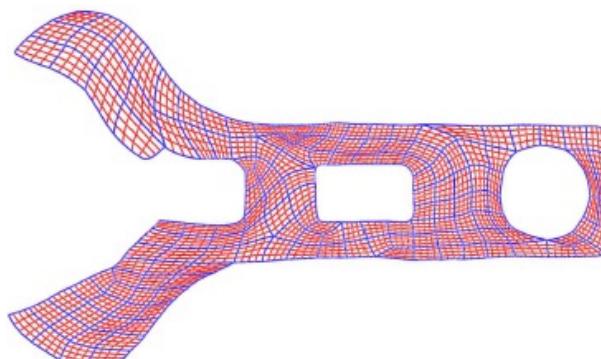
(b) discrete boundary



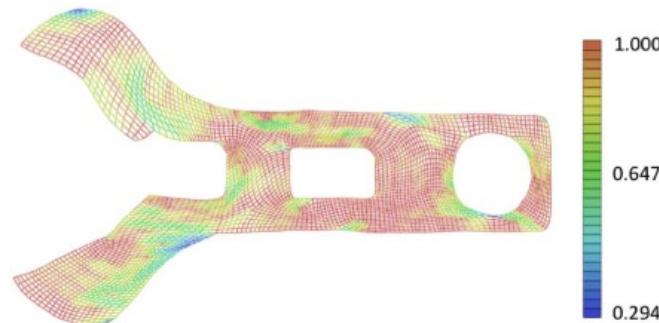
(c) quad meshing result



(d) segmentation curves

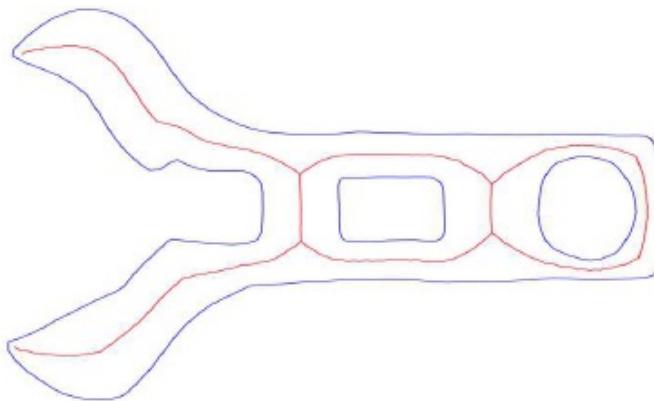


(e) parameterization result

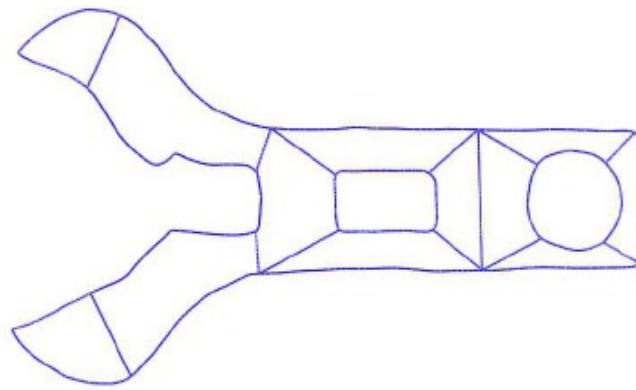


(f) Jacobian colormap

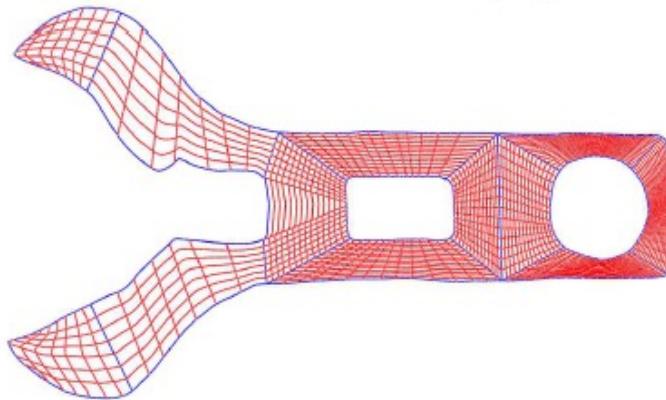
Example III with the skeleton-based decomposition



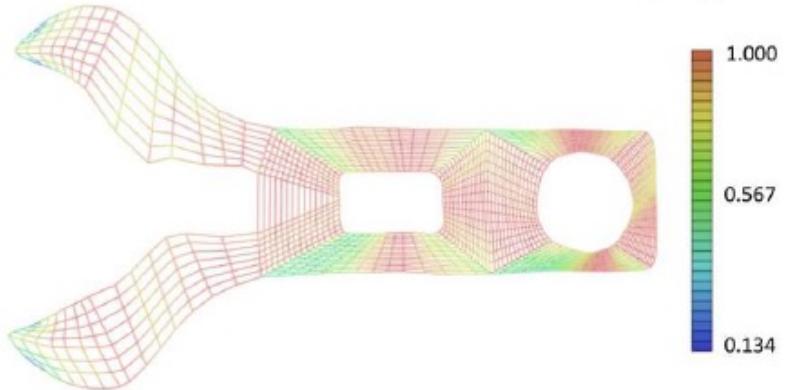
(g) extracted skeleton [42]



(h) skeleton-based domain partition[42]



(i) skeleton-based parameterization [42]



(j) Jacobian colormap of (i)

Fig. 11. Example V.

Quality comparison

Table 2: Quantitative data for planar parameterization in Fig. 9, Fig. 10 and Fig. 11. p : degree of planar parameterization; # SD: number of subdomains by domain decomposition; # Patch: number of Bézier patches; # Con.: number of control points.

Example	Method	p	# SD	# Patch	# Con.	Scaled Jacobian		Conditional number	
						Average	Min	Average	Max
Fig. 9	Our method	6	39	39	1467	0.8843	0.292	2.76	8.06
	Xu et al.[42]	6	5	35	1309	0.5172	0.148	5.36	16.31
Fig. 10	Our method	5	66	66	1768	0.9194	0.276	2.42	10.18
	Xu et al.[42]	5	8	56	1507	0.7801	0.075	4.35	18.23
Fig. 11	Our method	5	155	155	3720	0.9017	0.294	2.57	7.86
	Xu et al.[42]	5	12	132	3282	0.7894	0.134	4.23	15.64

Parameters and computing time

Example	F_{shape} in (14)		F in (16)			$E(r)$ in (28)		# T_1	# T_2	# T
	σ_1	σ_2	ω_1	ω_2	ω_3	τ_1	τ_2			
Fig. 1	2.0	1.0	2.0	1.0	50.0	2.0	1.5	73.22	177.86	251.08
Fig. 8(c)	1.0	1.0	1.0	1.0	50.0	1.0	1.5	22.68	26.64	49.32
Fig. 8(g)	1.0	1.0	1.0	1.0	50.0	1.0	1.5	22.96	27.18	50.14
Fig. 8(k)	1.0	1.0	1.0	1.0	50.0	1.0	1.5	27.68	36.32	63.90
Fig. 9	1.0	2.0	1.0	2.0	50.0	1.0	2.0	32.74	53.84	86.58
Fig. 10	2.0	1.0	1.0	2.0	50.0	2.0	1.0	41.02	61.36	102.38
Fig. 11	2.0	1.0	2.0	2.0	50.0	2.0	1.0	75.70	209.68	285.38

标架场引导的区域参数化



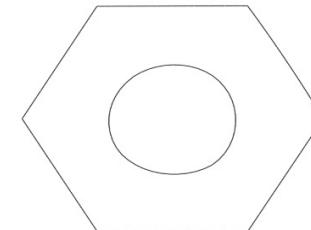
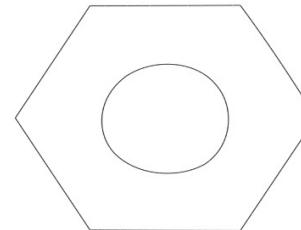
基于标架场理论的复杂区域参数化方法

The Research on Parameterization Method of Complex Domain Based on Frame Field



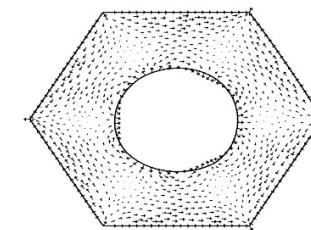
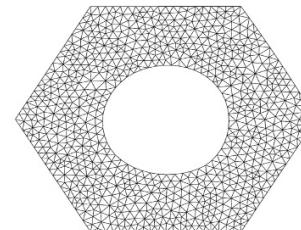
模型边界的转化

将由多条B样条曲线组成的边界转化为易于生成四边形网格结构的多边形边界



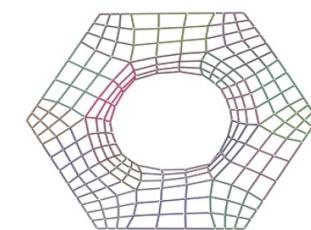
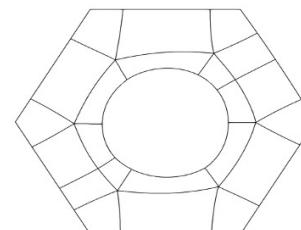
Delaunay三角化

使用Delaunay三角化算法在区域内部生成背景网格



四边形区域的生成

分析奇异点的位置，利用奇异点构造流线并简化，计算域内部分成若干个合理的四边形区域



计算域参数化

对每个四边形区域构造Coons面完成参数化构造

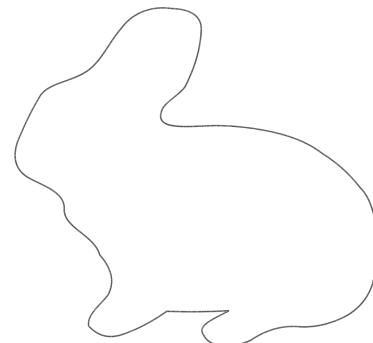
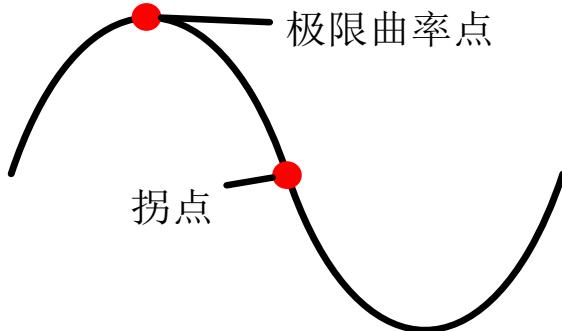


基于标架场理论的复杂区域参数化方法

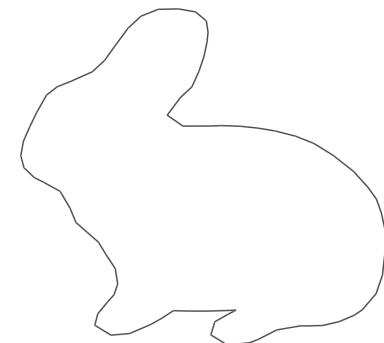
Research Process

模型边界的转化

对曲线边界进行检测，找寻到曲线特征点，依据曲线特征点分割曲线，转化为多边形边界



曲线边界



多边形边界



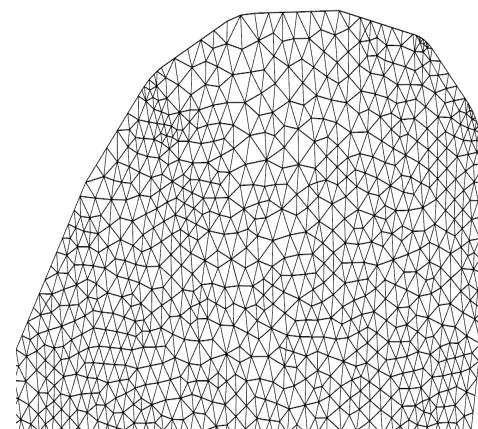
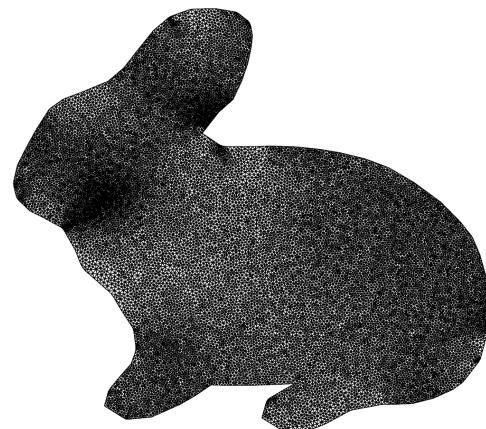
基于标架场理论的复杂区域参数化方法

Research Process



Delaunay三角化

使用Delaunay三角化将多边形边界区域内部生成背景网格



Delaunay三角化



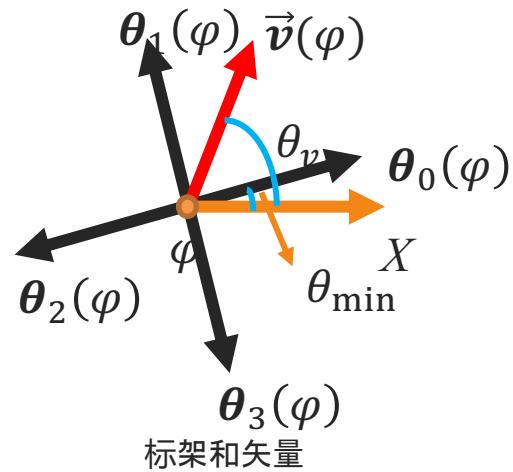
基于标架场理论的复杂区域参数化方法

Research Process

建立拉普拉斯方程

根据边界区域标架场和矢量场信息，
定义关于区域内部的光滑矢量场方程

$$\begin{cases} \nabla^2 \mathbf{v}_i(\varphi) = 0, \quad \varphi \in D \\ \mathbf{v}_i(\varphi) = \mathbf{v}_{i,0}(\varphi), \quad \varphi \in \partial D \end{cases}, (i = x, y)$$





矢量场方程的求解

利用Meyer提出的Laplace-Beltrami算子离散化对矢量场方程进行求解

$$\Delta_m \mathbf{v}_i(\varphi_n) = \frac{1}{A_T(\varphi_n)} \sum_{k \in N(\varphi_n)} \frac{\cot \alpha_{nk} + \cot \beta_{nk}}{2} (\mathbf{v}_i(\varphi_k) - \mathbf{v}_i(\varphi_n))$$

区域矢量场

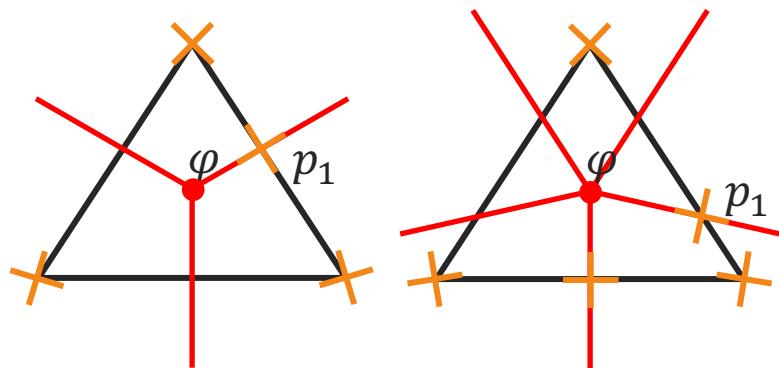


基于标架场理论的复杂区域参数化方法

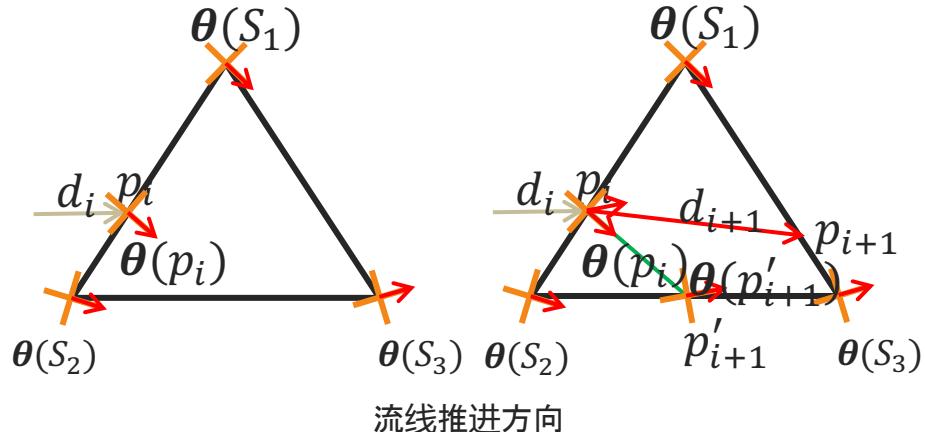
Research Process

构造流线

通过所求的矢量场信息，定位出奇异点位置，判断出奇异点的价，
并通过奇异点构造初始流线



三价点和五价点流线流出方向



流线推进方向

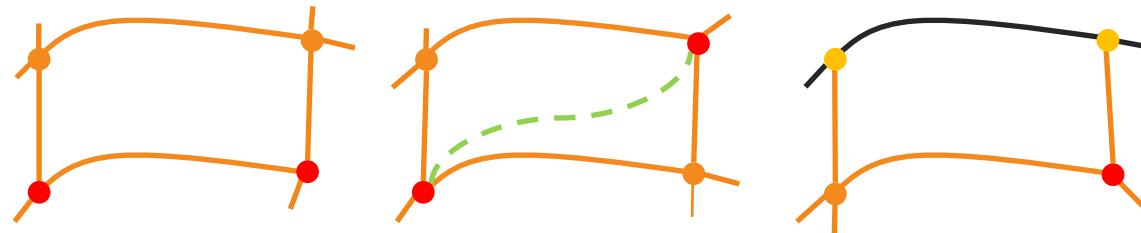


基于标架场理论的复杂区域参数化方法

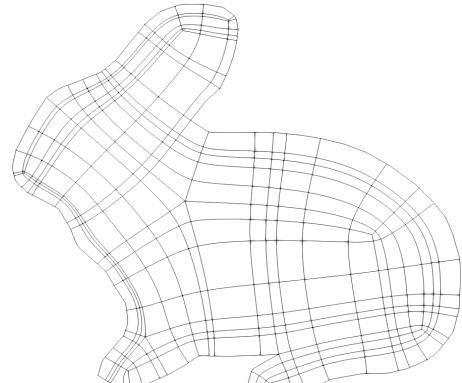
Research Process

流线简化

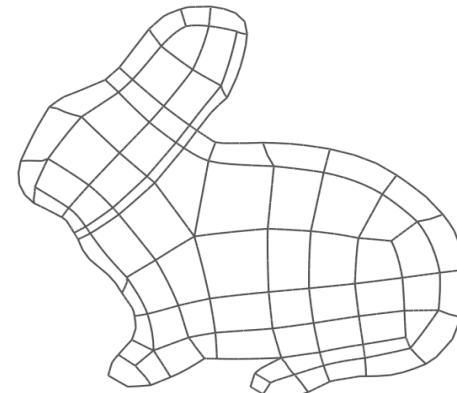
根据流线简化规则对构造的流线进行简化操作



流线简化的三种情况



初始流线

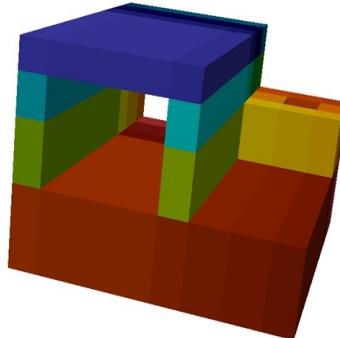


流线简化

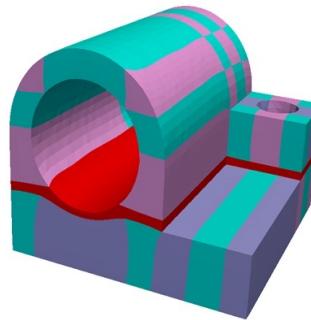
Paramterization with Polycube/Polysquare (CMAME 2019)



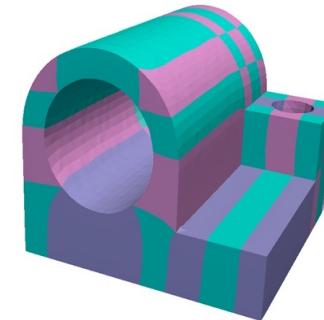
(a)



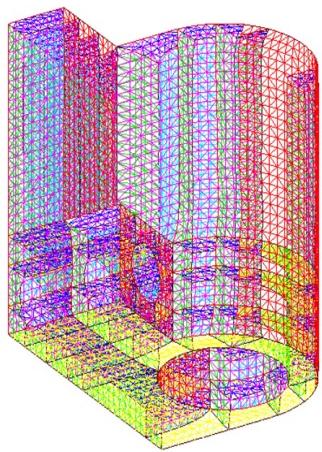
(b)



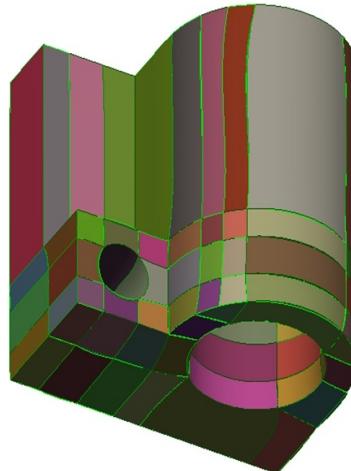
(c)



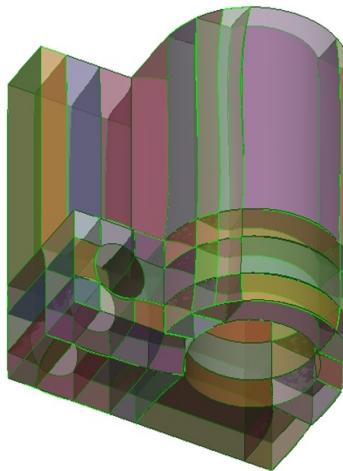
(d)



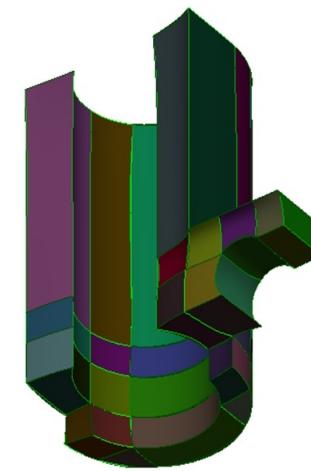
(e)



(f)



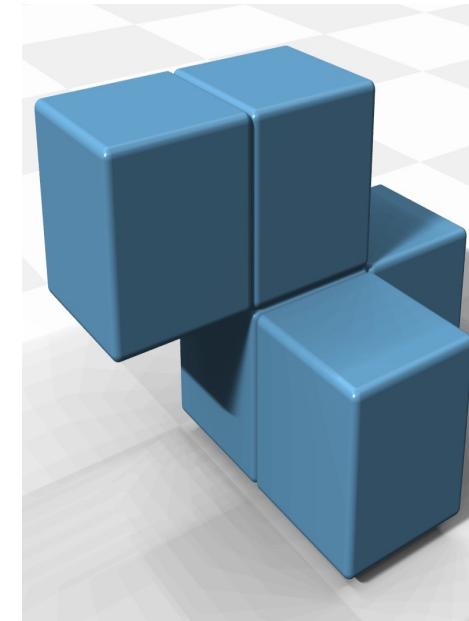
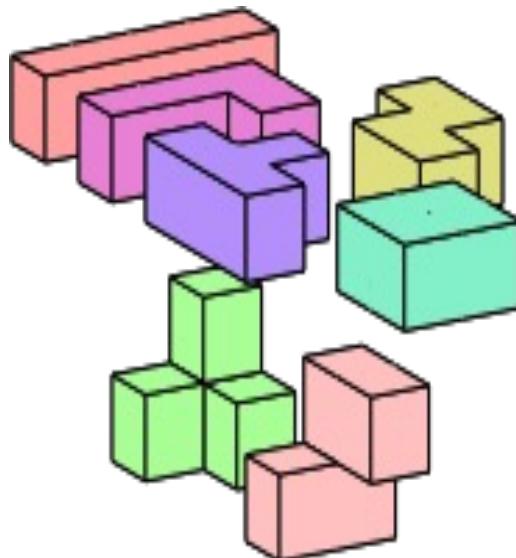
(g)



(h)

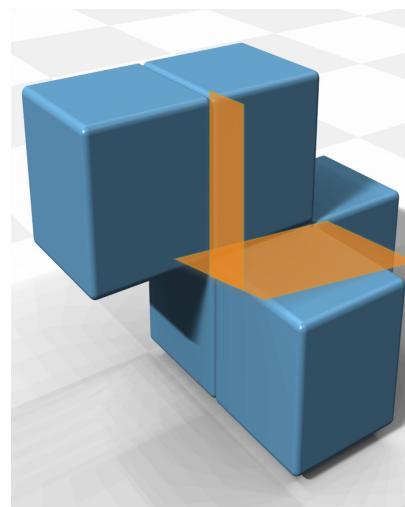
Polycube Structure

- A polycube is a solid figure formed by joining equal cubes face to face.
- Polycubes are the three-dimensional analogues of the planar polyominoes.



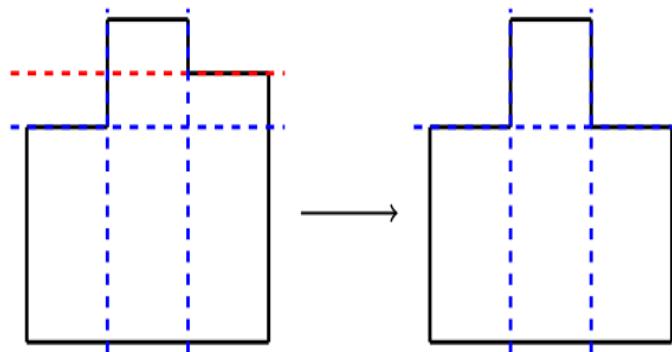
Main Idea

- Get the polycube structure of input models using the method in [Huang 2014].
- Generate the IGA blocks by slicing the polycube along the axis aligned boundary faces.



Main Idea

- Block number is usually positively related to the number of cuts along each direction.
- Deform the polycube to get a simpler structure.

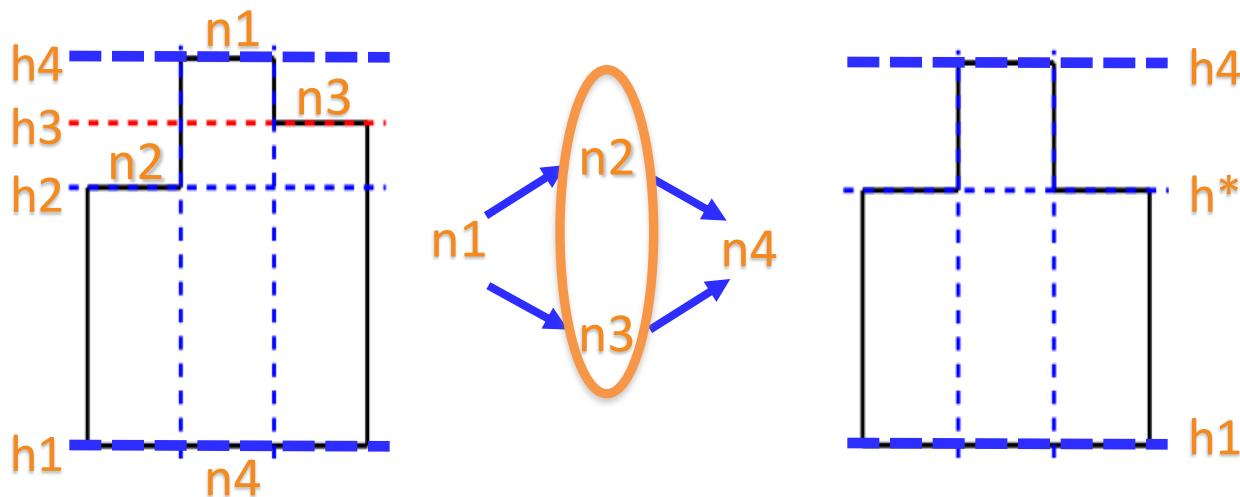


6 blocks

4 blocks

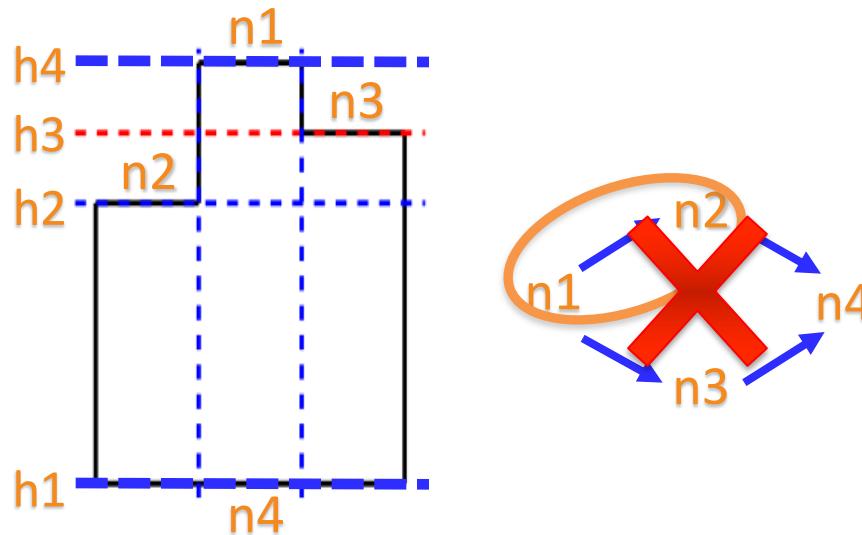
Directed Graph Simplification

- Reduce the number of blocks.
 - Each node in the graph has different height.
 - Cluster node pairs into a same height.



Directed Graph Simplification

- Non-degenerate constraints.
 - Edge can not degenerate to a point.
 - Can not cluster two connected nodes.
 - Find all disconnected nodes as candidate pairs.



Directed Graph Simplification

- Distortion Control.
 - Use the stretch of graph edge length to evaluate the distortion.
 - Find a new height to minimize the distortion.

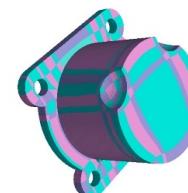
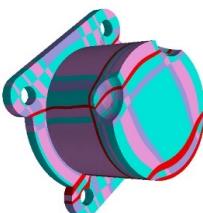
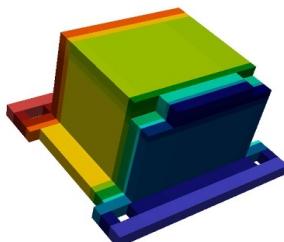
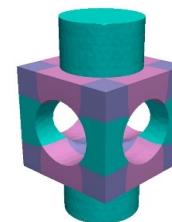
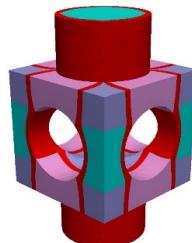
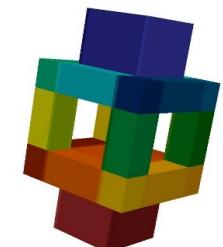
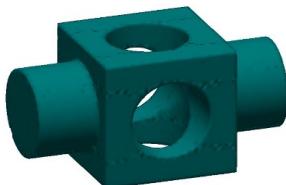
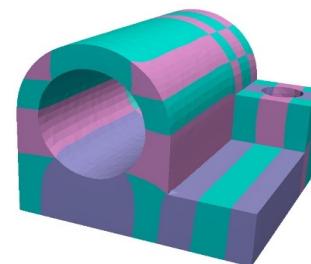
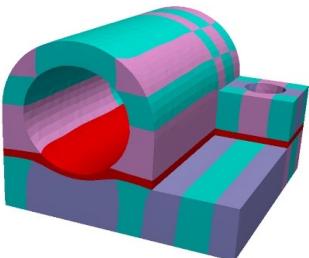
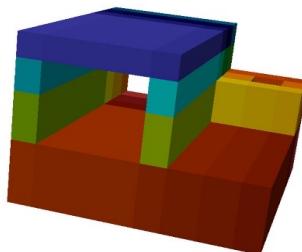
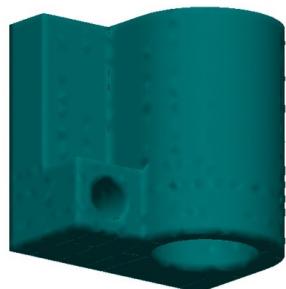
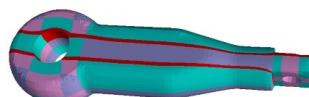
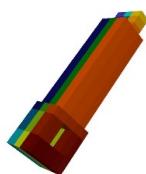
$$D_{ij} = \min_{h_{ij}^*} \sum_{l \in T(n_i) \cup T(n_j)} d(l, h^k),$$

s.t. $d(l, h) < D, \forall l \in T(n_i) \cup T(n_j),$

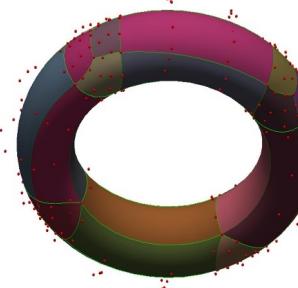
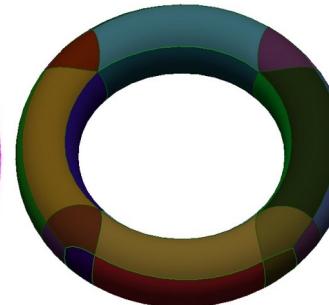
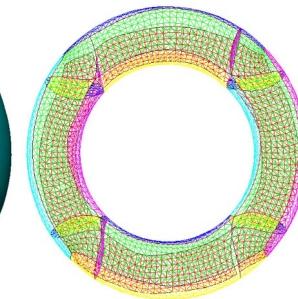
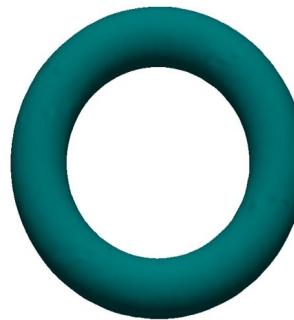
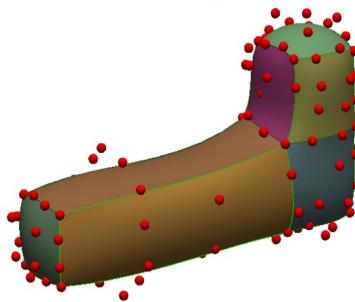
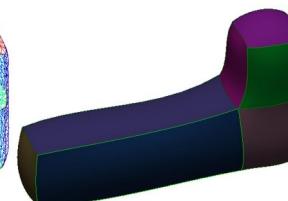
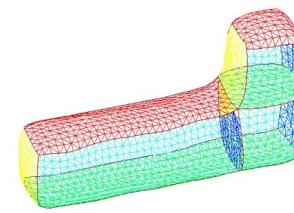
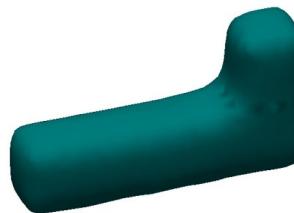
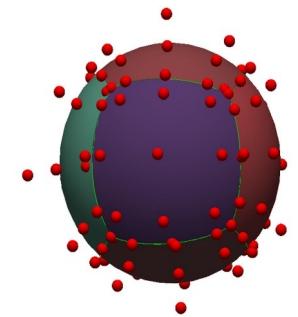
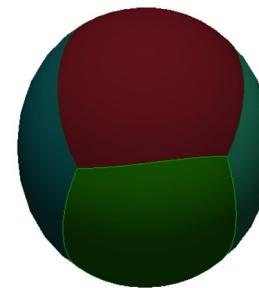
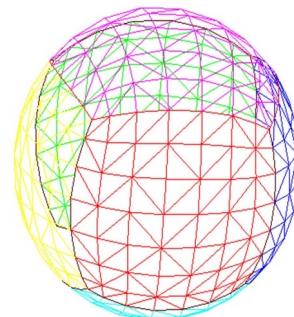
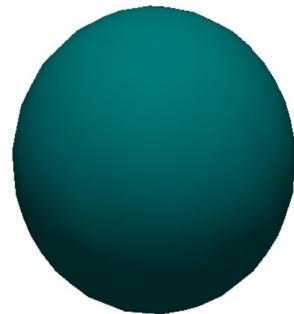
$$h_m^k = h_{ij}^*, \forall p_m \in n_i \cup n_j.$$

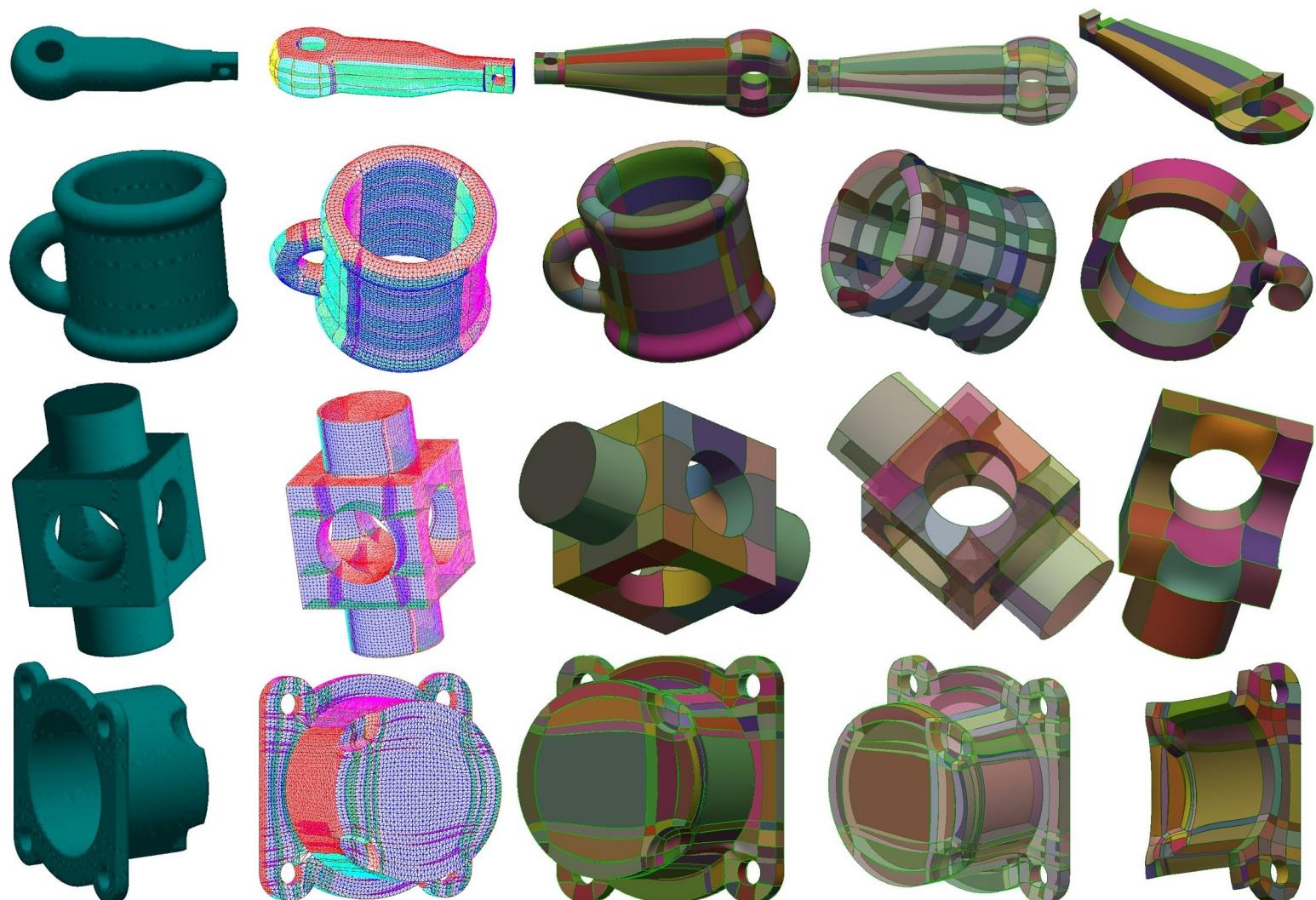
$$d(l, h) = \frac{\bar{h}_{l,+} - \bar{h}_{l,-}}{h_{l,+} - h_{l,-}} + \frac{h_{l,+} - h_{l,-}}{\bar{h}_{l,+} - \bar{h}_{l,-}}.$$

Block optimization



Examples





Parameterization with closed-form polysquare



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IGA-suitable planar parameterization with patch structure simplification of closed-form polysquare

Shiyi Wang^a, Jingwen Ren^b, Xianzhong Fang^{a,c}, Hongwei Lin^{a,b}, Gang Xu^d, Hujun Bao^a,
Jin Huang^{a,*}

^a State Key Laboratory of CAD & CG, Zhejiang University, Hangzhou, China

^b School of Mathematics Science, Zhejiang University, Hangzhou, China

^c Faculty of Electrical Engineering and Computer Science, Ningbo University, Ningbo, China

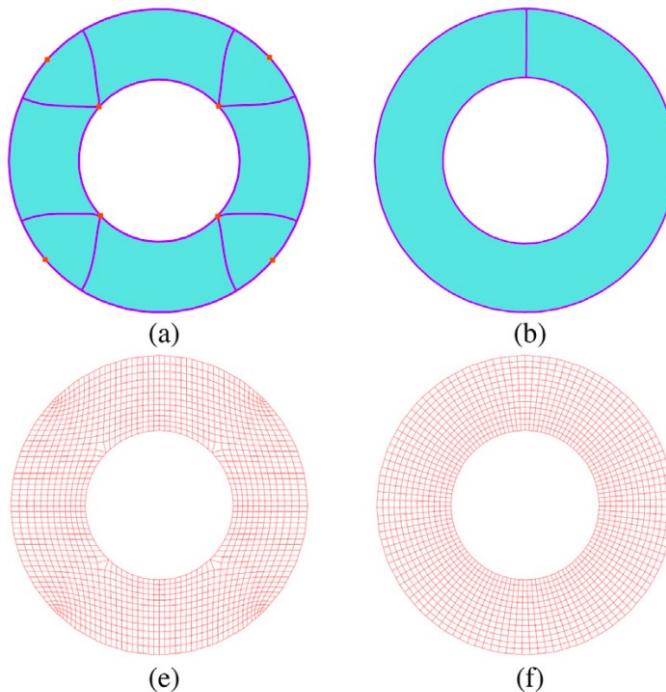
^d School of Computer Science and Technology, Hangzhou Dianzi University, Hangzhou, China

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Available online xxxx

Parameterization with closed-form polysquare

- By integrating the frame field technique, a polycube structure can be extended into a more general structure i.e. a closed-form polycube.
- Such a structure has no internal singularity as well, but it has a more flexible topology than the common polycube.
- Such flexibility helps to generate better meshes for complex models in the sense of having fewer undesired boundary corners and lower distortion



Framework

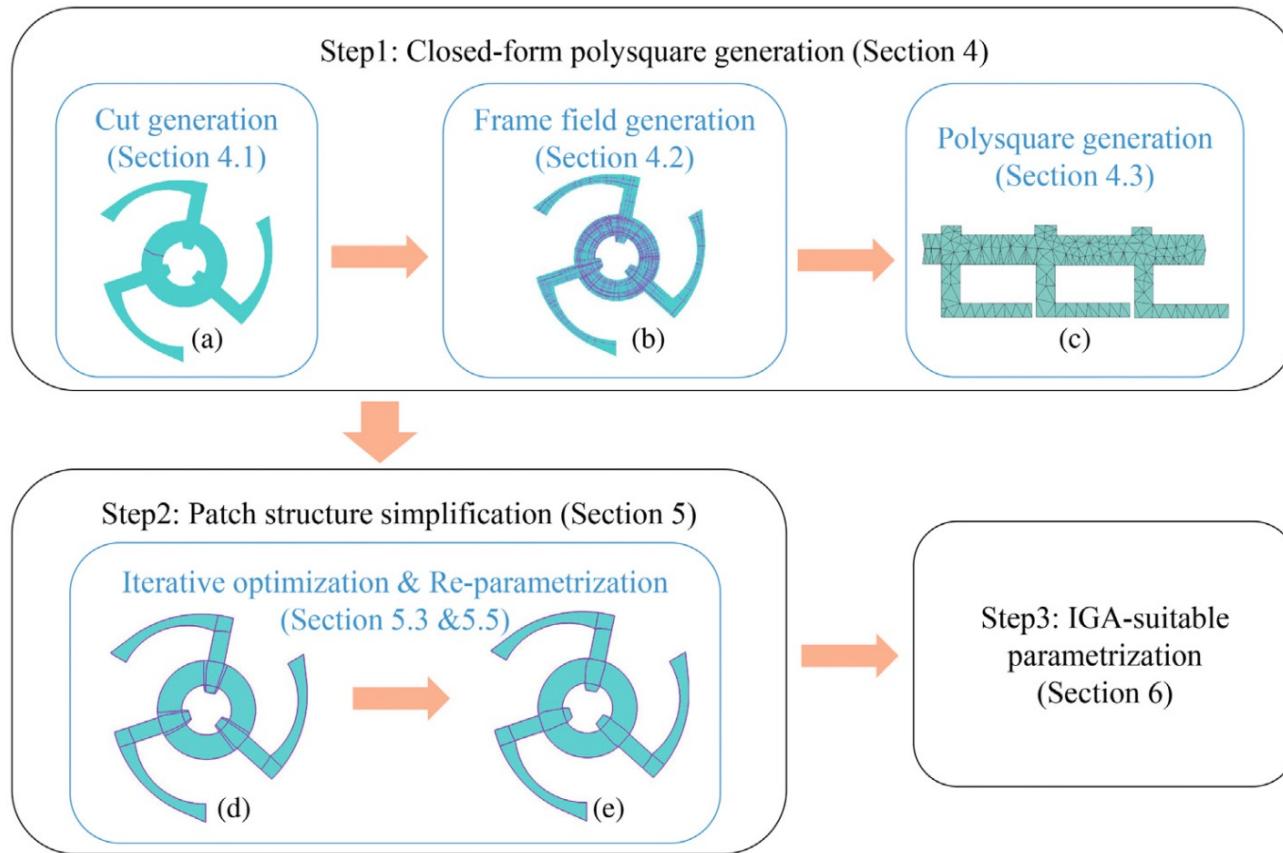
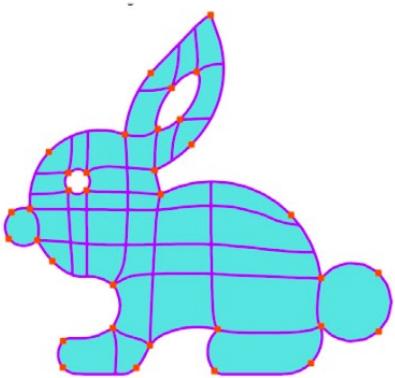
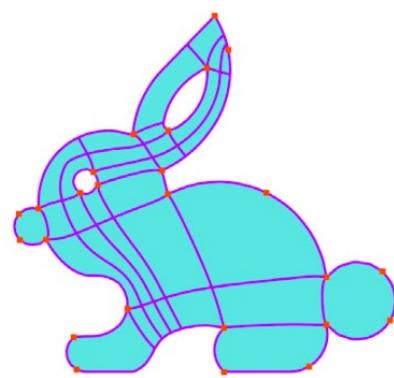


Fig. 1. Pipeline of our method. Cut the input mesh into a disk-like one (a) where the cut edges are marked in purple. Internal singularity-free cross-frame field generation (b). Closed-form polysquare parameterization (c). Simplify the original patch structure (d) and obtain a simpler one (e). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

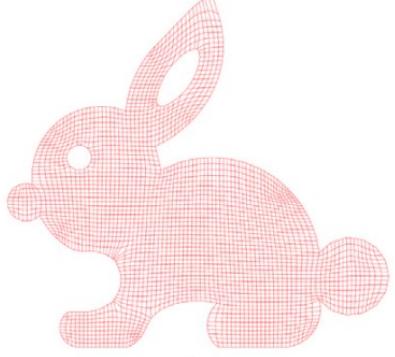
Comparison Results



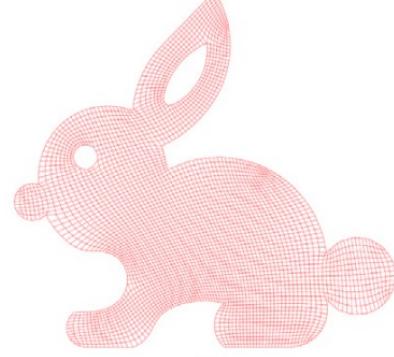
(c)



(d)



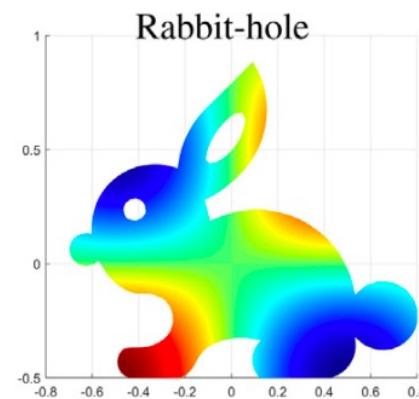
(g)



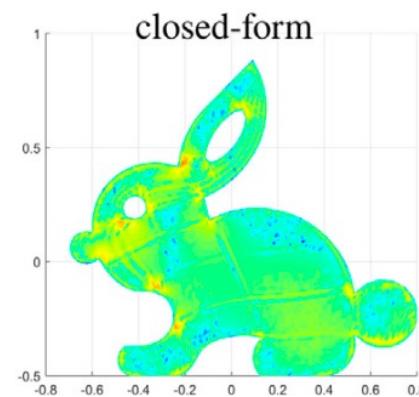
(h)

exact-form

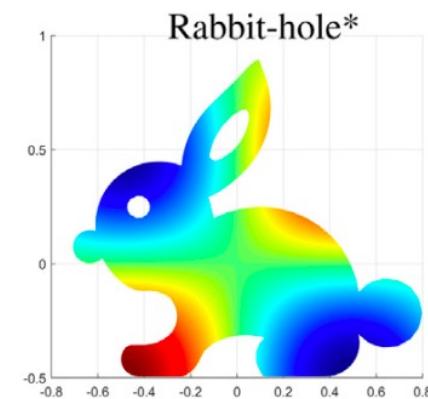
closed-form



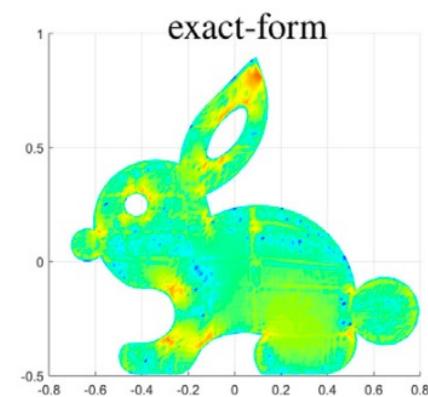
Rabbit-hole



closed-form



Rabbit-hole*

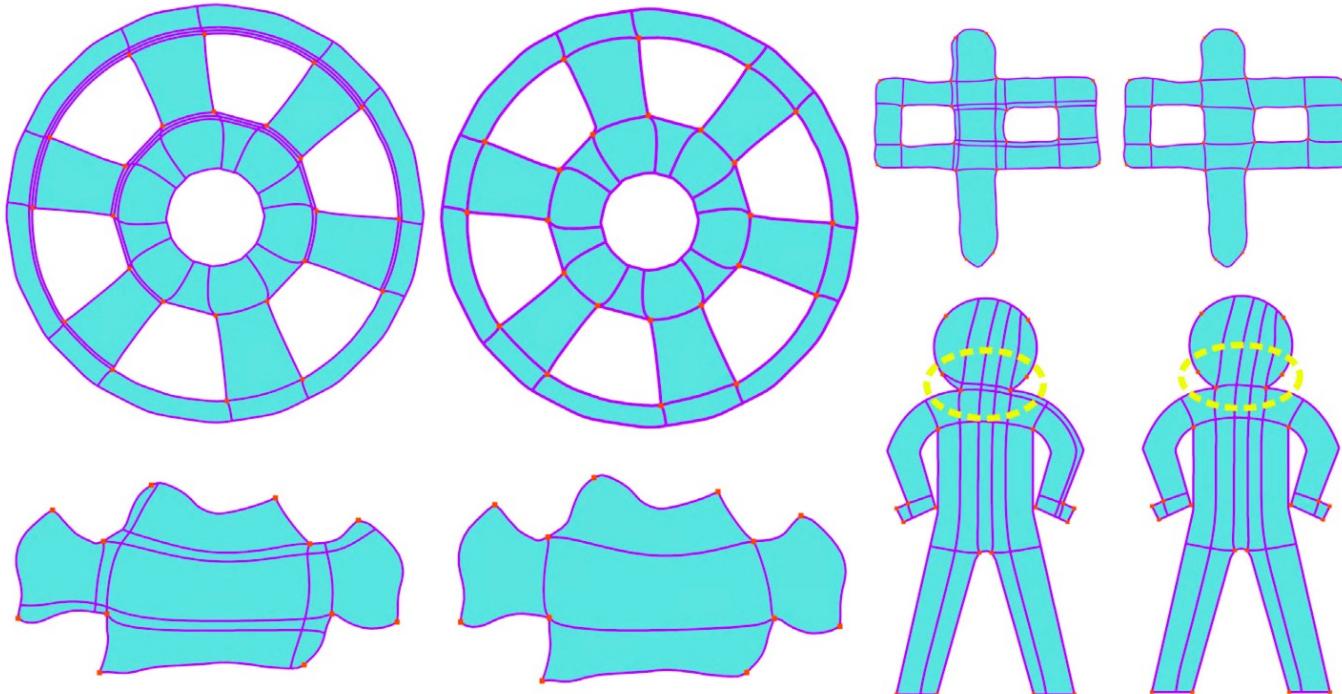


exact-form

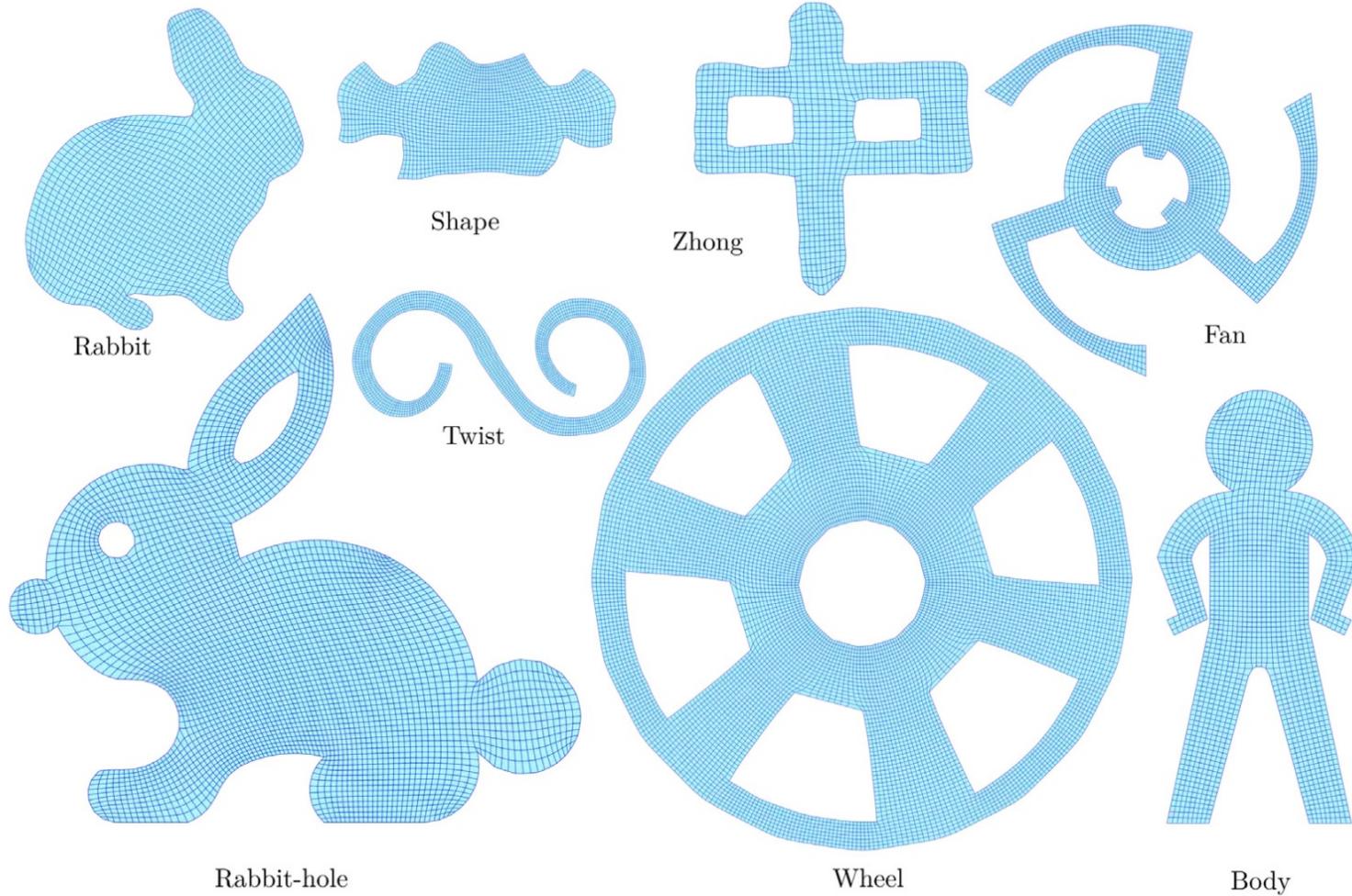
$$e_T = 1.2888 \times 10^{-5}$$

$$e_T = 2.5549 \times 10^{-5}$$

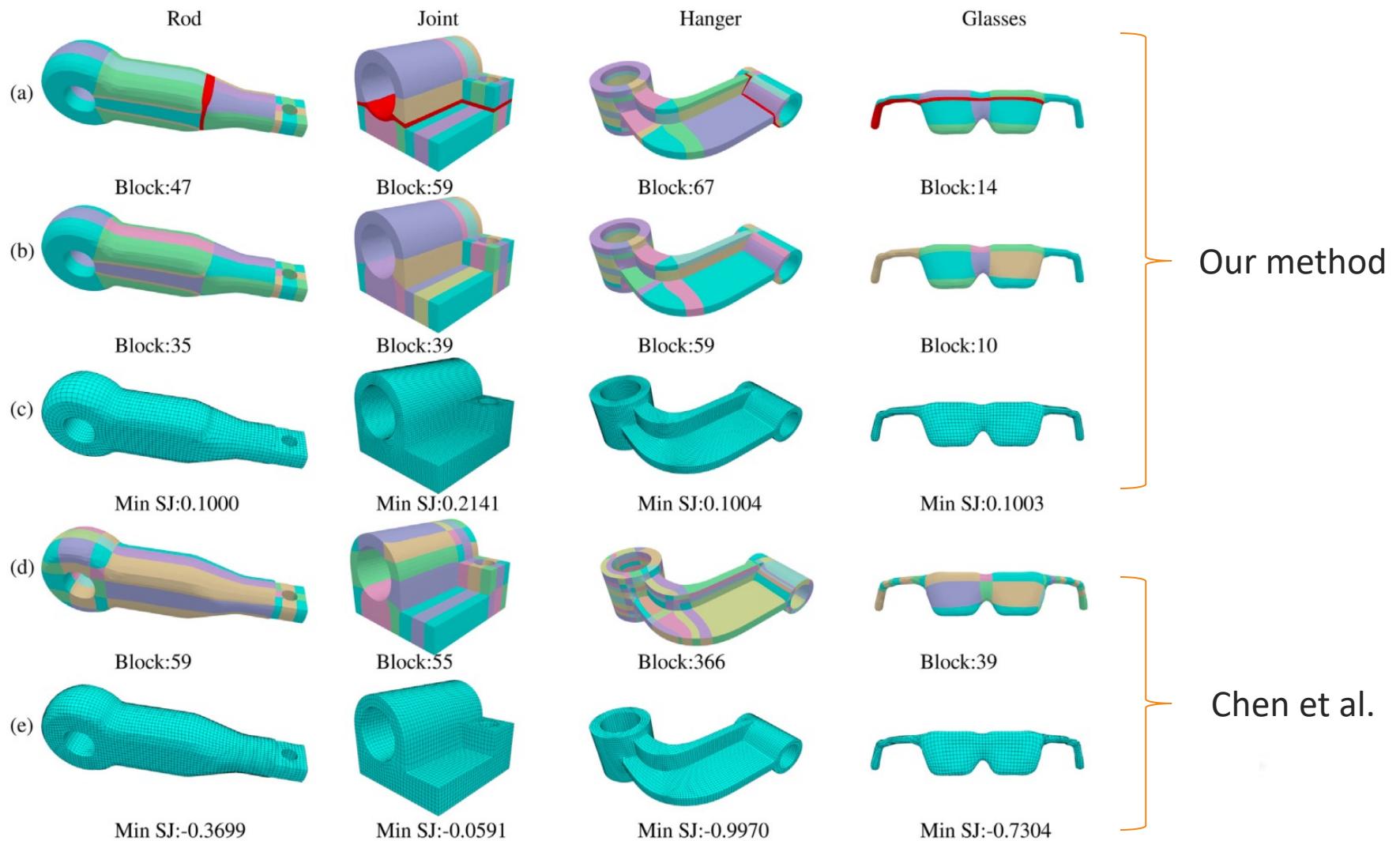
Patch simplification



2D Results



3D results



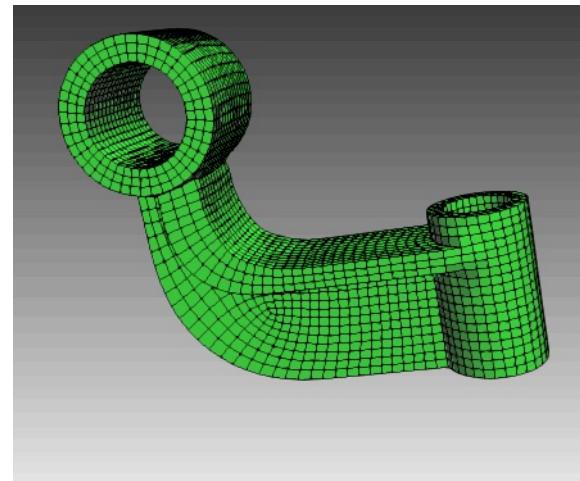
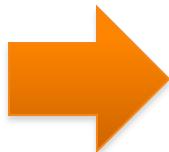
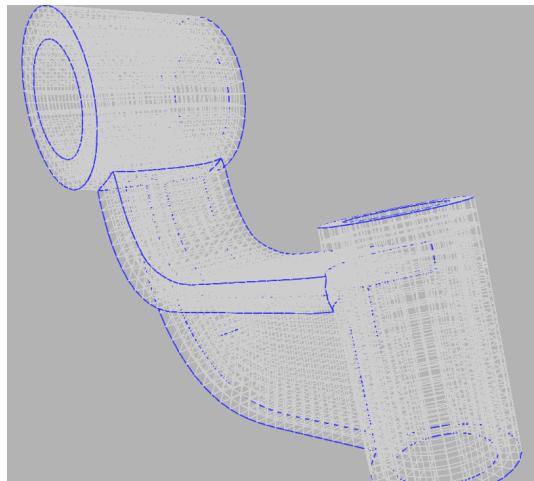
3. 一些开放性问题

挑战性问题

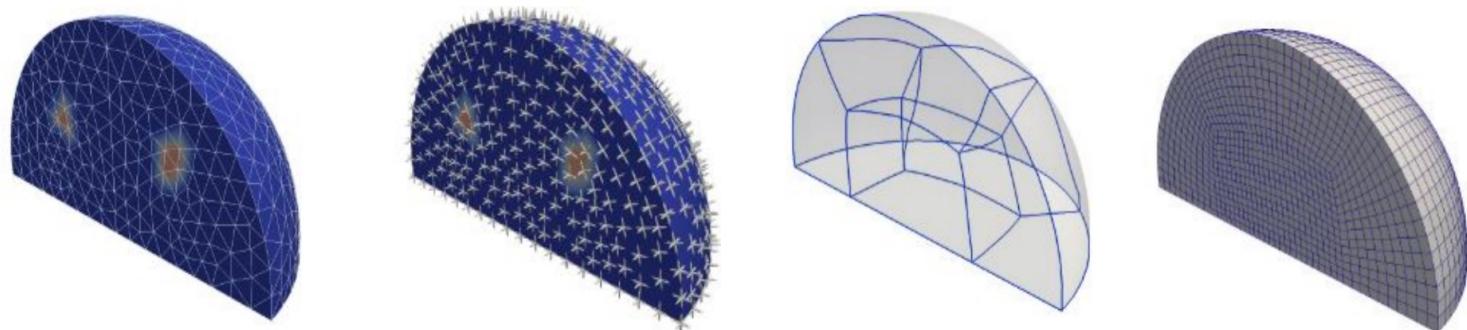
- 三维结构化体参数化拓扑构造问题
- 体参数化在奇异点/奇异边处的连续性
- 带有裁剪曲面边界的体参数化方法
- 直接基于曲体的复杂模型造型方法

Problem statement

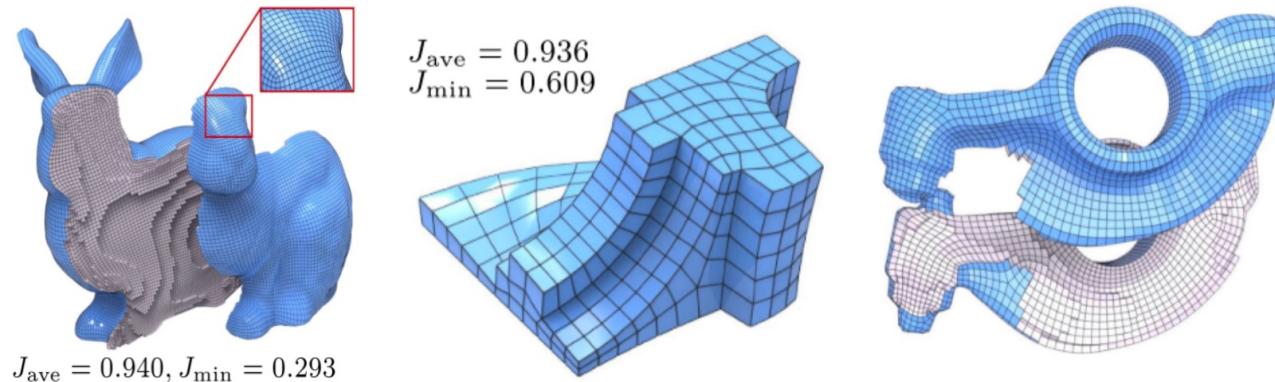
Constructing unstructured trivariate Bézier representation with geometric continuity constraints at extraordinary vertices/edges to interpolate a specified unstructured hexahedral mesh



空间区域的大块六面体分解

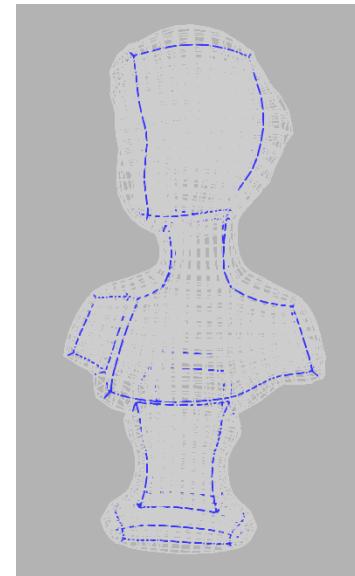


计算初始标架场 → 优化标架光滑性 → 基于标架场提取奇异图 → 生成六面体网格

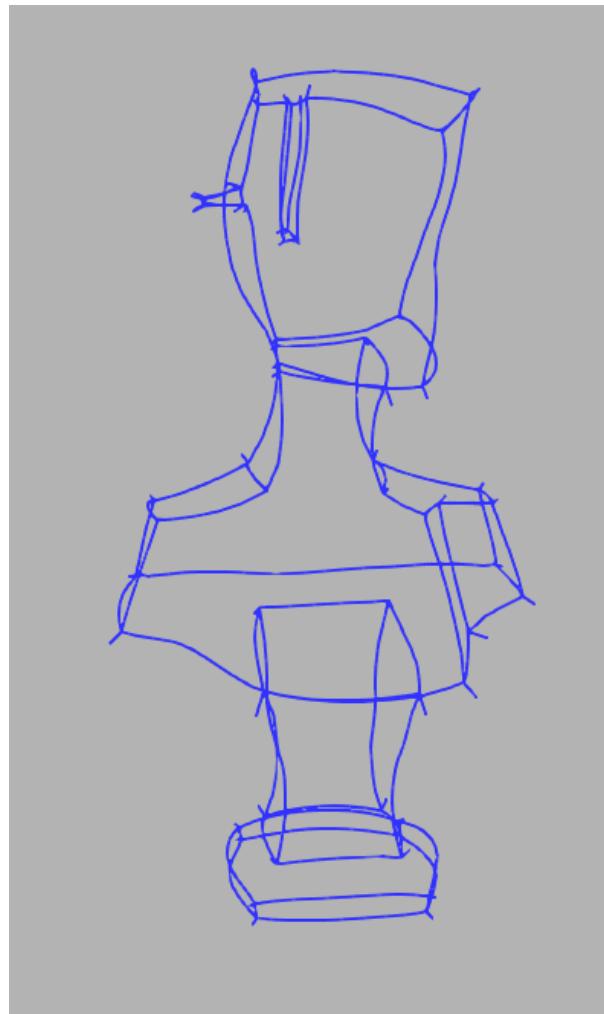
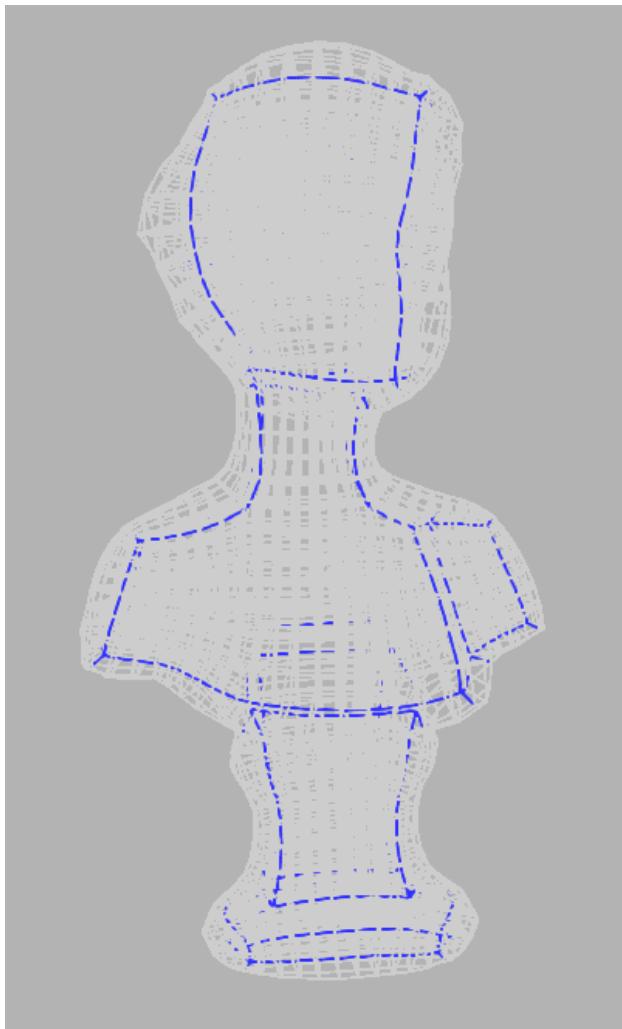


Main difficulty ---- definition of G¹ continuity

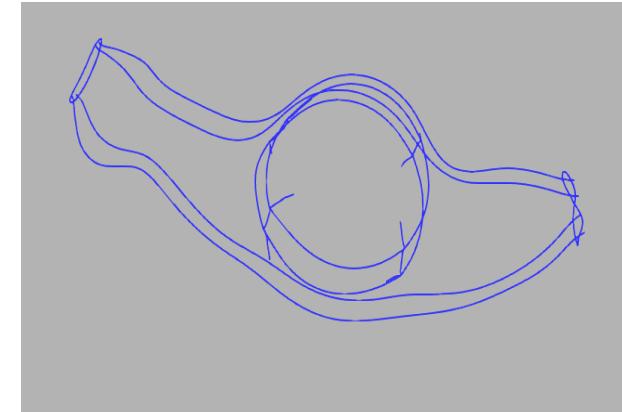
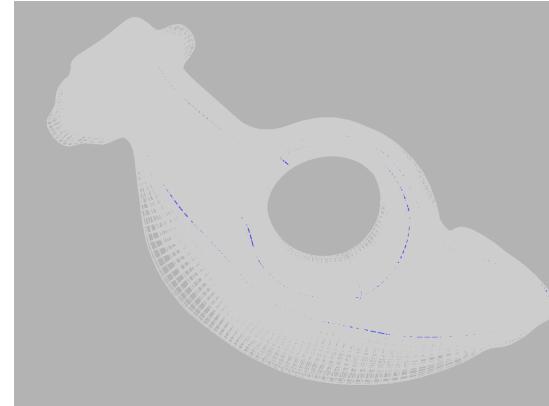
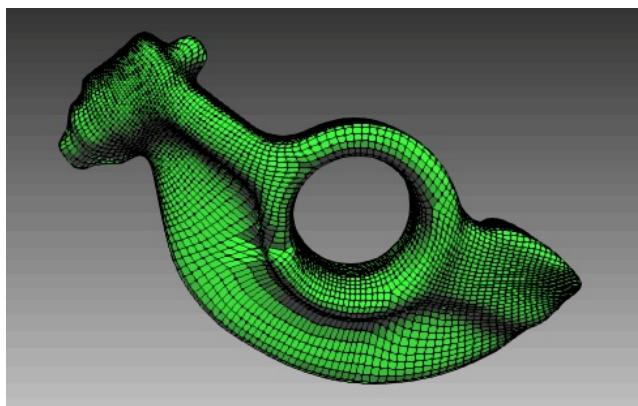
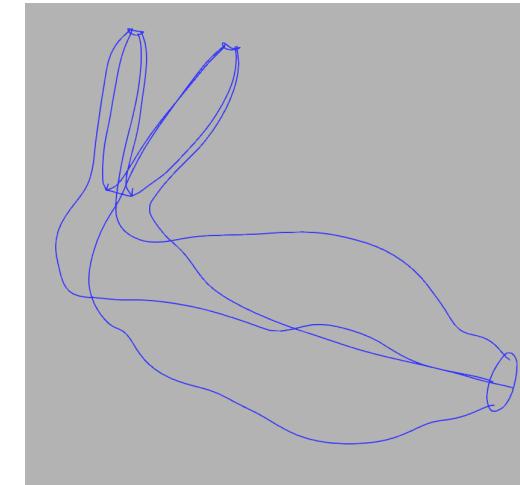
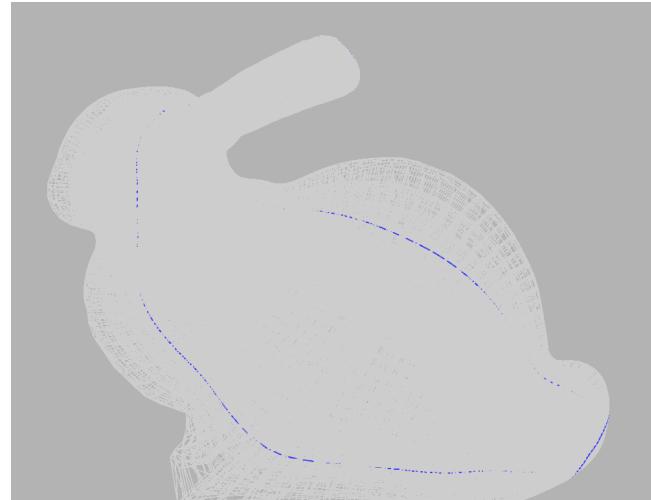
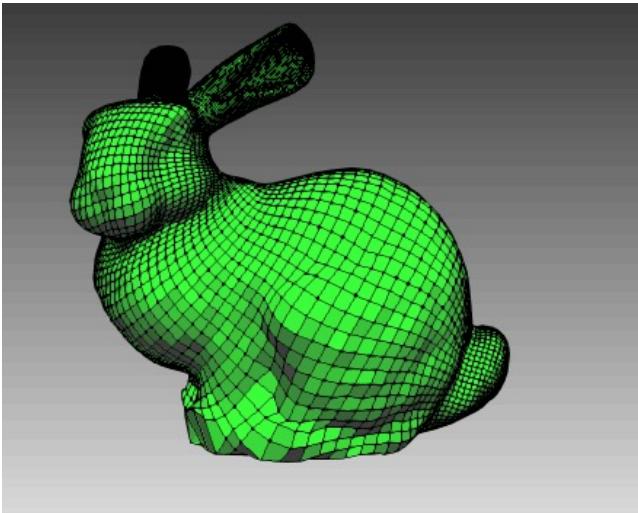
- G¹-continuity at regular parts
- How to define the G¹-continuity along the **irregular edges?**
- How to define the G¹-continuity around **irregular vertices?**



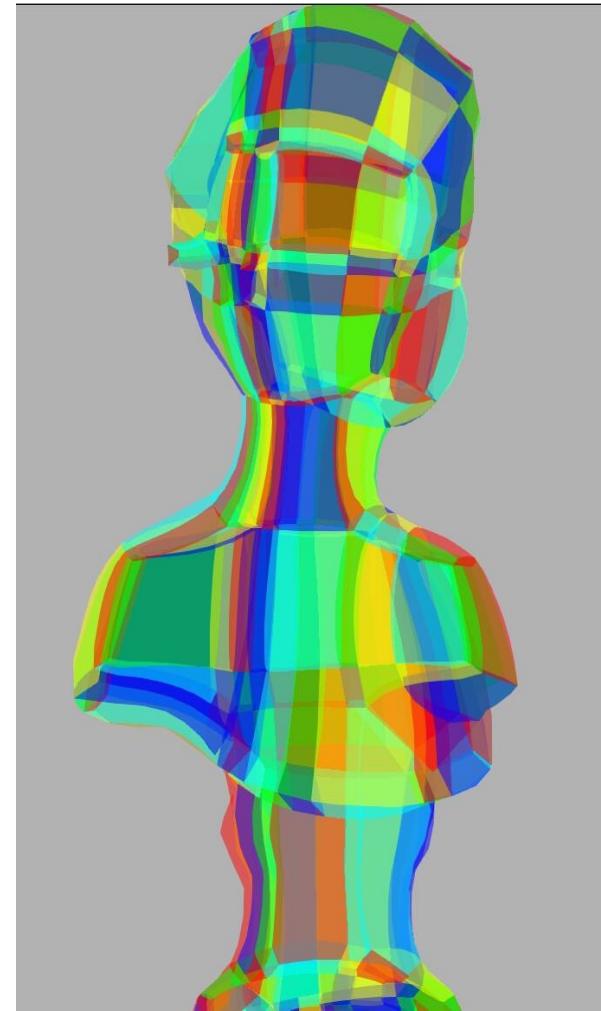
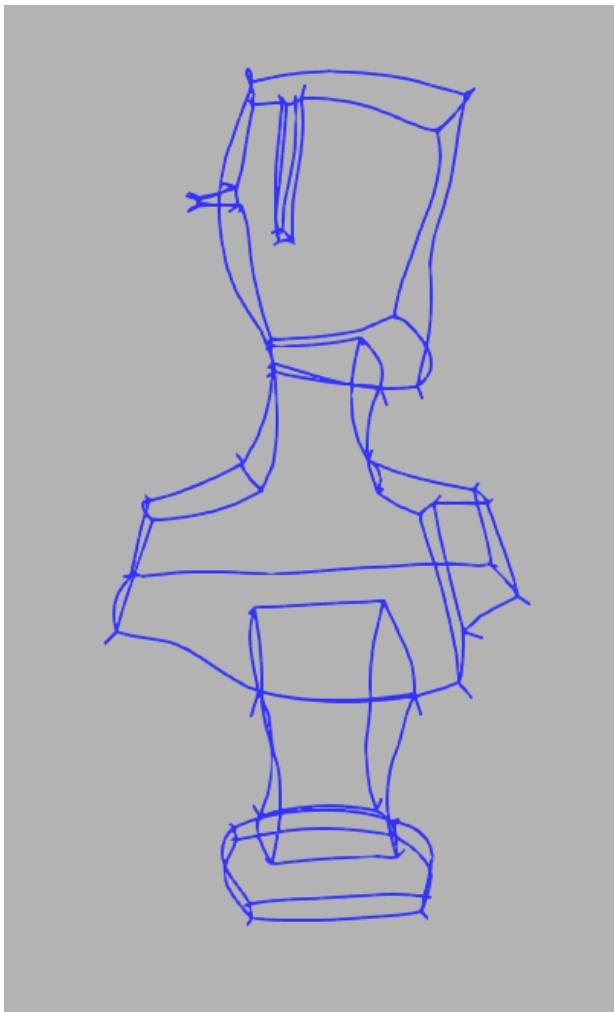
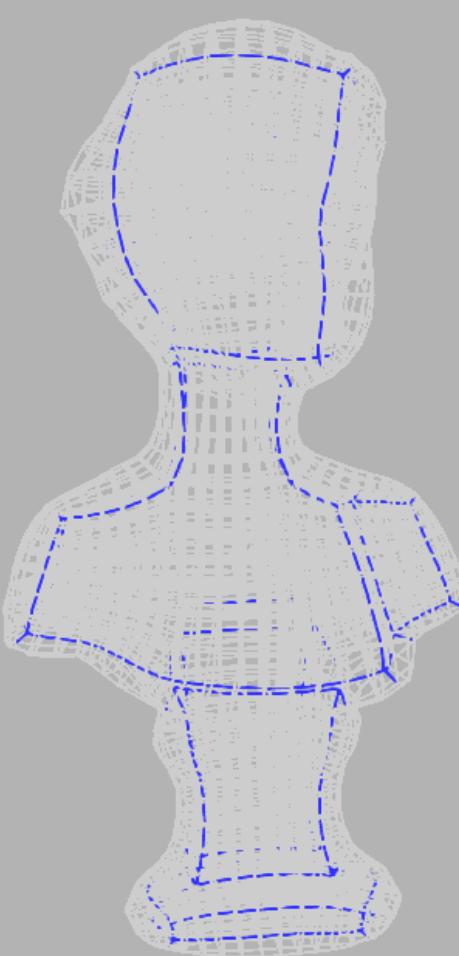
Extraction of irregular structure for hex-mesh



Extraction of irregular structure for hex-mesh

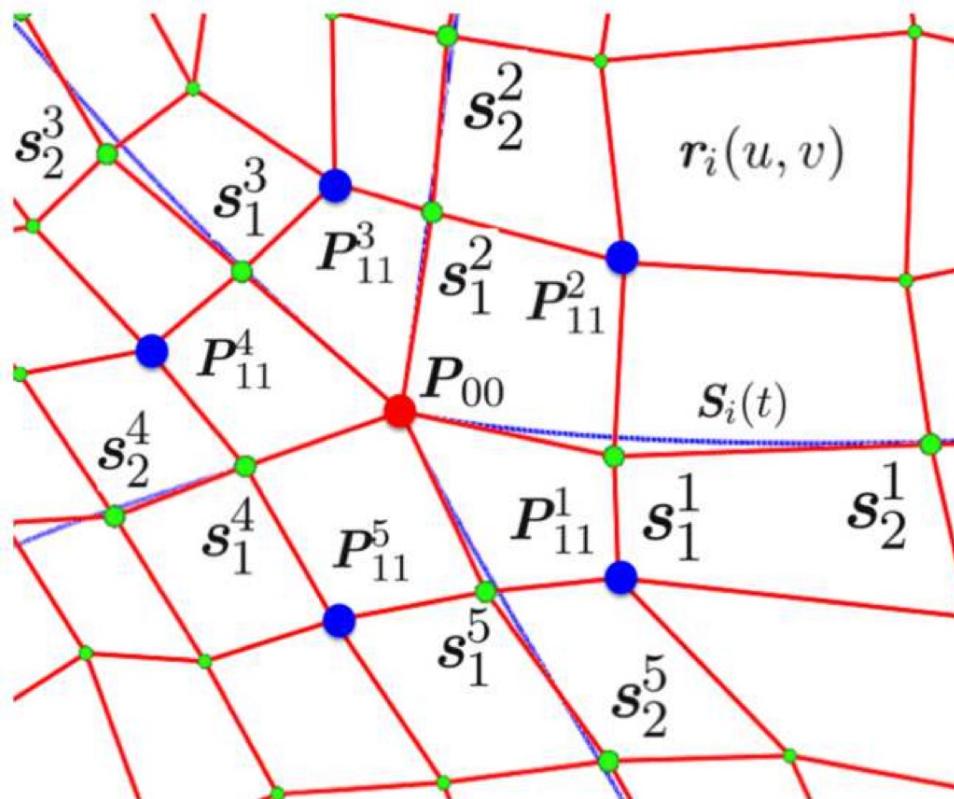


Extraction of irregular structure for hex-mesh



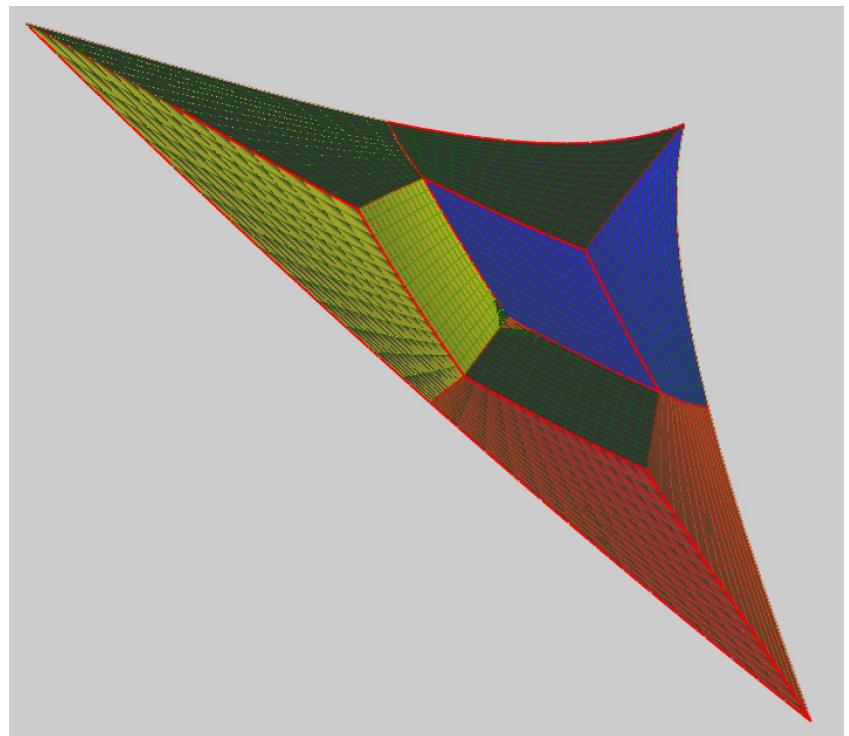
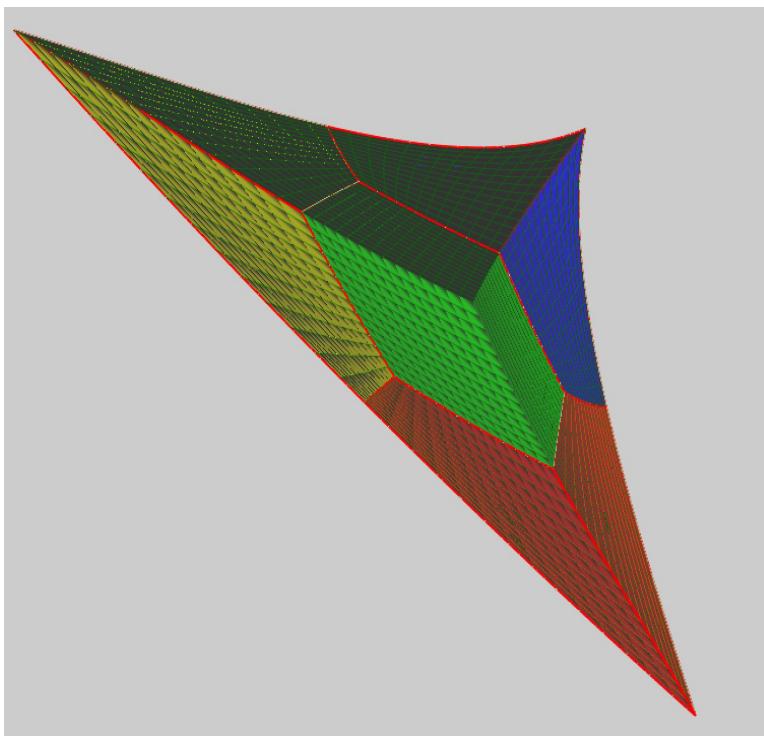
G^1 -continuity along the irregular edges

- Extension of the G^1 -continuity around the irregular vertex in planar parameterization.

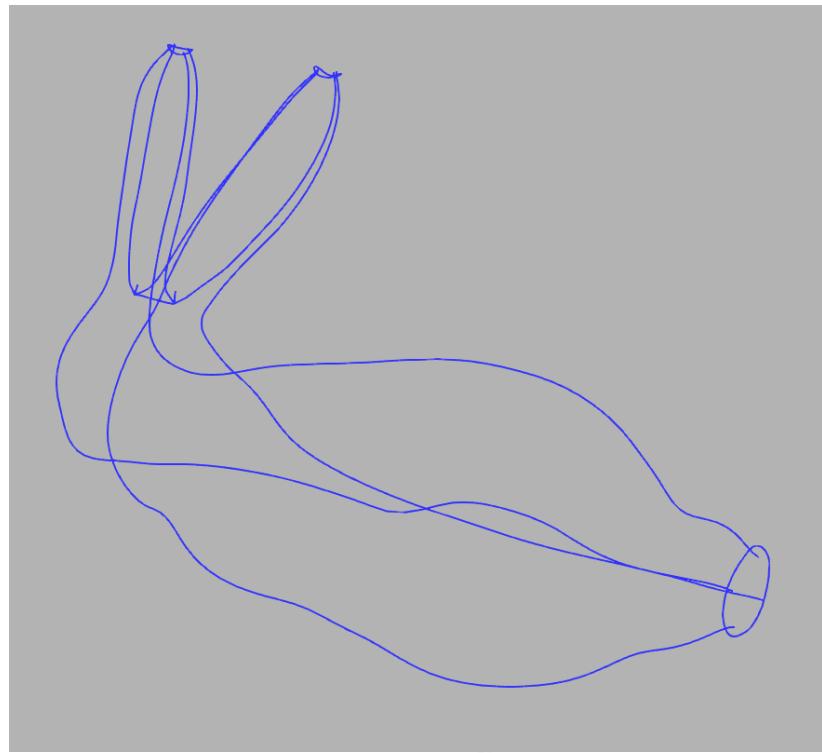
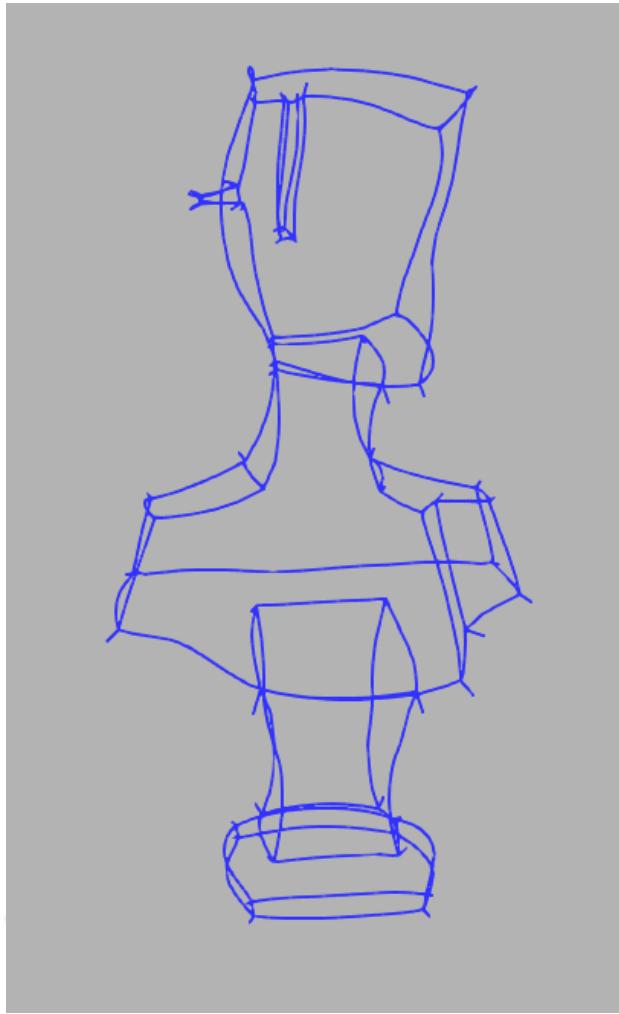


G^1 -continuity along the irregular edges

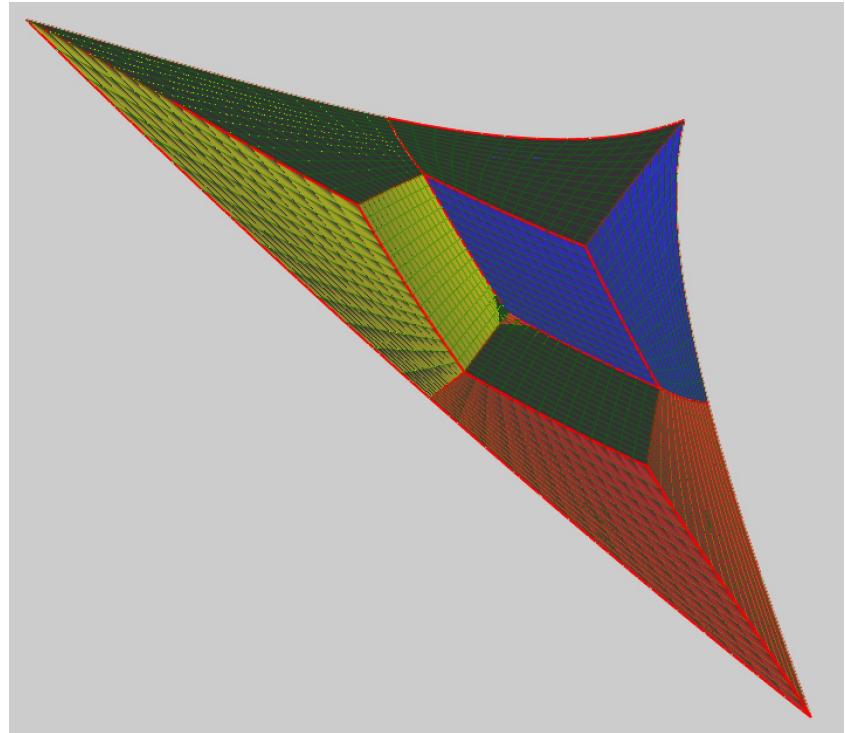
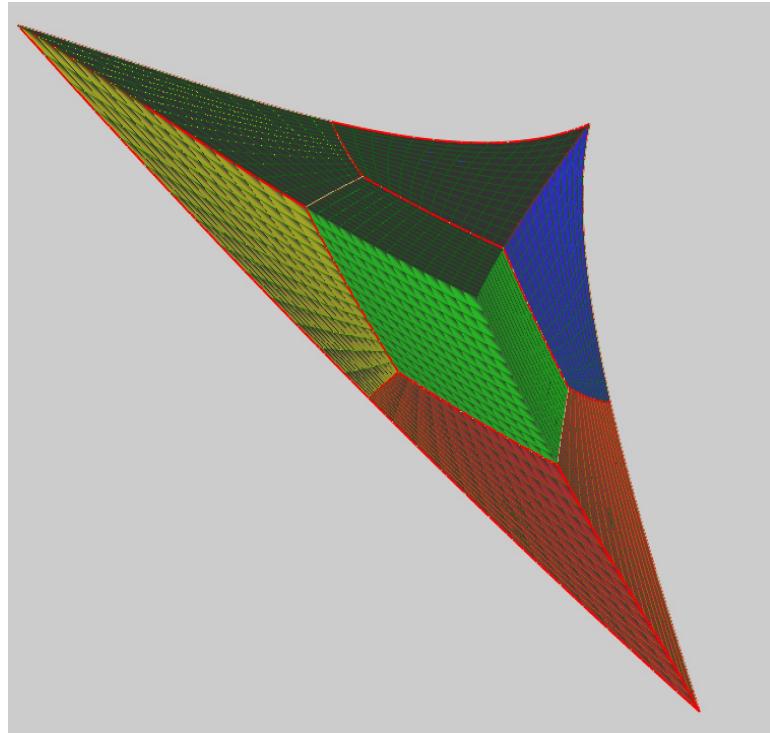
- Extension of the G^1 -continuity around the irregular vertex in planar parameterization.



G^1 -continuity around irregular vertex ?



G^1 -continuity around irregular vertex?



Thank you



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