智能可视建模与仿真实验室
Intelligent Visual Modeling & Simulation (iGame) Lab

第四讲 等几何分析中的计算域参数化

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目录

1. 计算域参数化问题的由来

Isogeometric analysis（等几何分析）

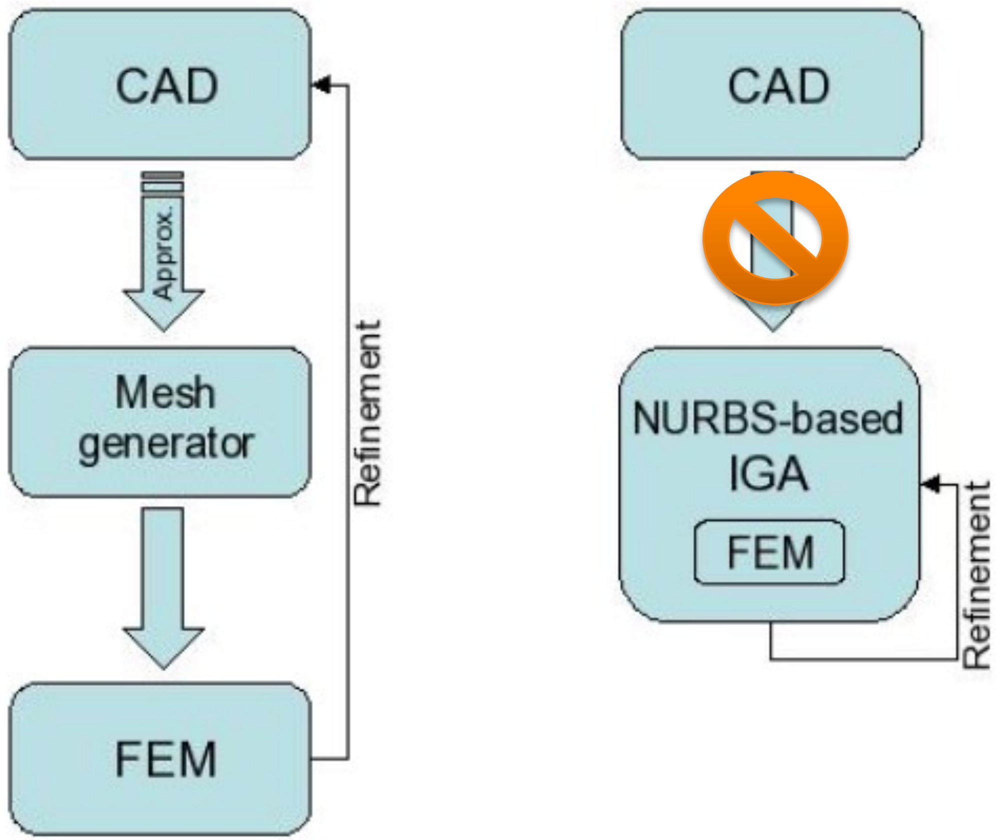
- IGA is an isoparametric, exact geometry approach, which is recently providing very promising results as an alternative to FEA.
- proposed by Prof. T. Hughes et al. from University of Texas at Austin in 2005
- motivation:
    - seamless integration of CAD and CAE.
    - avoid geometry approximations of mesh generation in FEA
    - high regularity and refinement of B-spline functions.
- basic idea: use the same standard mathematical representation as in CAD systems (such as NURBS) for both the geometry and the solution field (such as thermal conduction).

Difference between IGA and FEA

	等几何分析	有限元分析
温度场表示形式	样条曲面（体）	网格面（体）
计算单元	节点区间所对应的子面片	三角形（四面体）
基函数	样条基函数	**Lagrange** 基函数
细化方法	插入节点，升阶，保持几何	三角形细分，几何发生改变

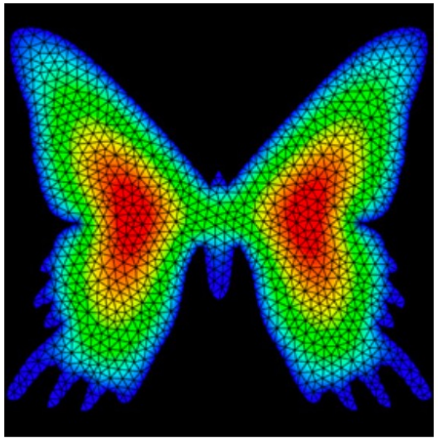
Parameterization of computational domain

- Open problem



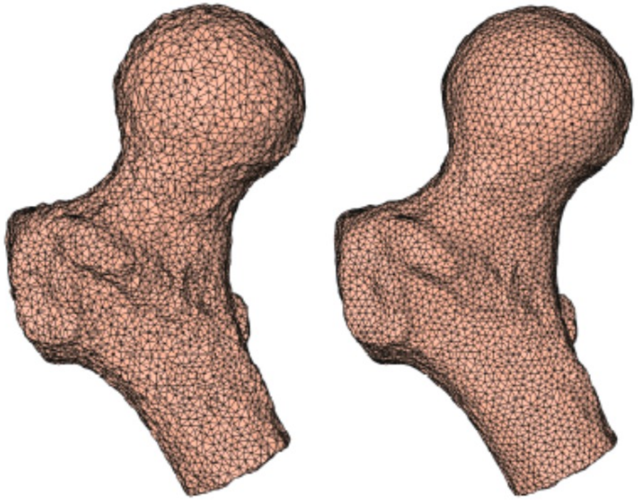
Two main differences between FEA and IGA

- computational domain:
  - FEA: discrete mesh
  - IGA: smooth domain(spline form)
- basis function:
  - FEA: Lagrange interpolation polynomial
  - IGA: B-spline basis function



Computational domain for analysis

- Mesh(computational domain) quality is an important issue in FEA
- Improvement of mesh quality in FEA by remesh

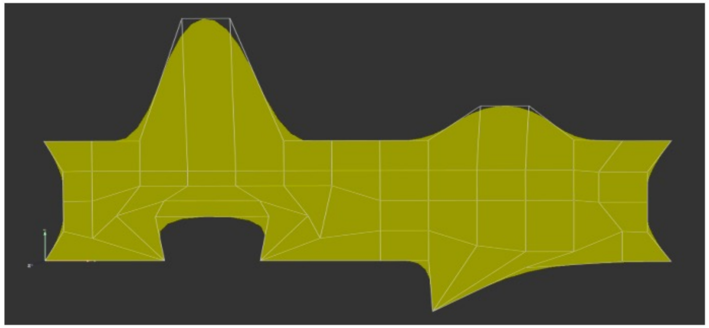


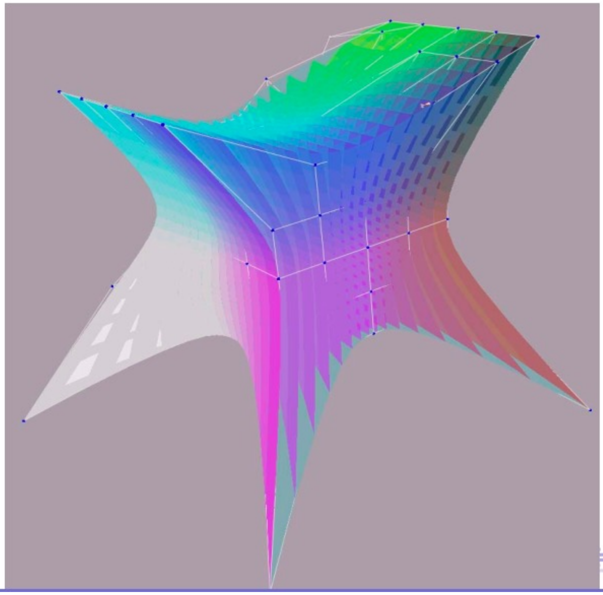
IGA-meshing

- IGA is a spline-version of FEA
- Mesh generation in FEA
- CAD models usually define only the boundary of a solid, but the application of isogeometric analysis requires a volumetric representation
- As it is pointed by Cotrell et al., the most significant challenge facing isogeometric analysis is developing three- dimensional spline parameterizations from boundary information

Representation in IGA

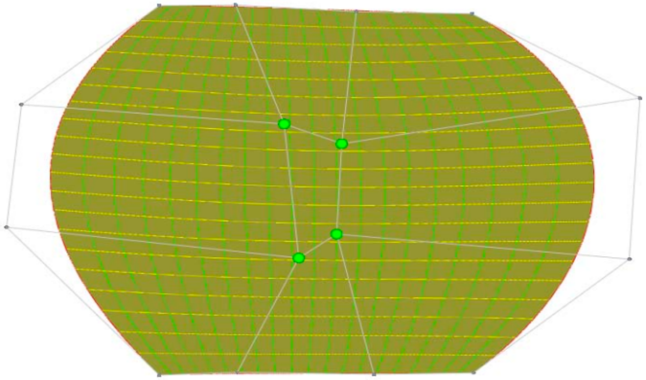
- computational domain:
  - 2D: planar B-spline surface
  - 3D: B-spline volume
- solution field :
  - 2D: space B-spline surface
  - 3D: B-spline volume





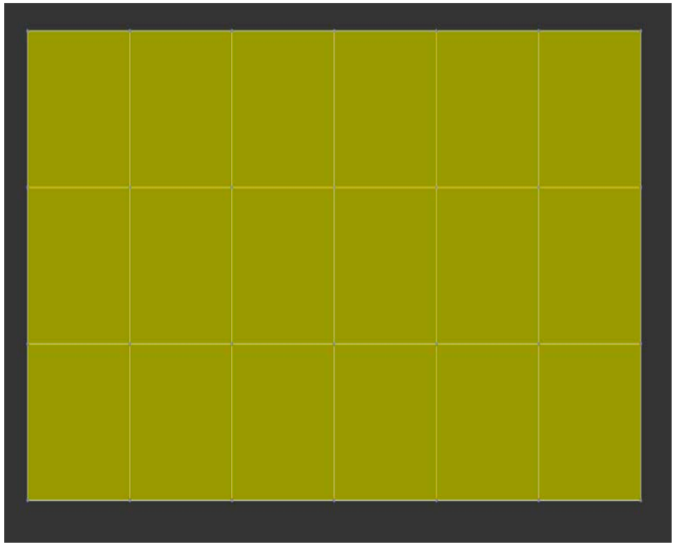
Parametrization of computational domain is more important for IGA

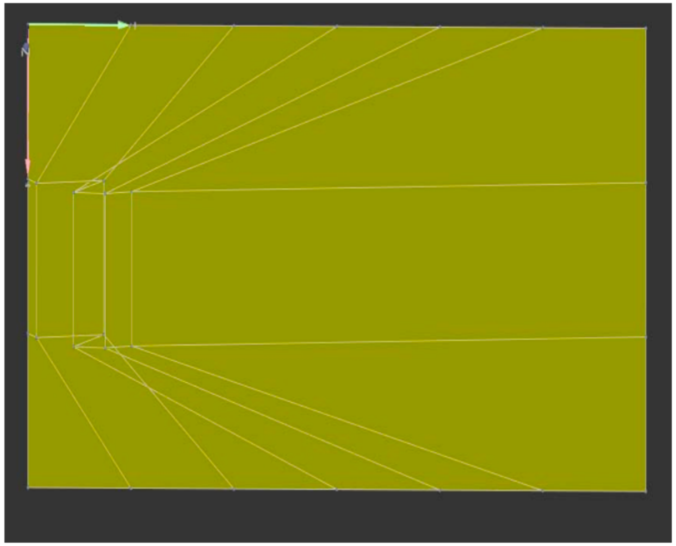
- First step in IGA(Mesh generation in FEA)
- Quality of parametrization is determined by fewer variables(control points)
- Refinement operation is not arbitrary
- Good parametrization is more important for IGA



Parametrization of computational domain for 2D IGA

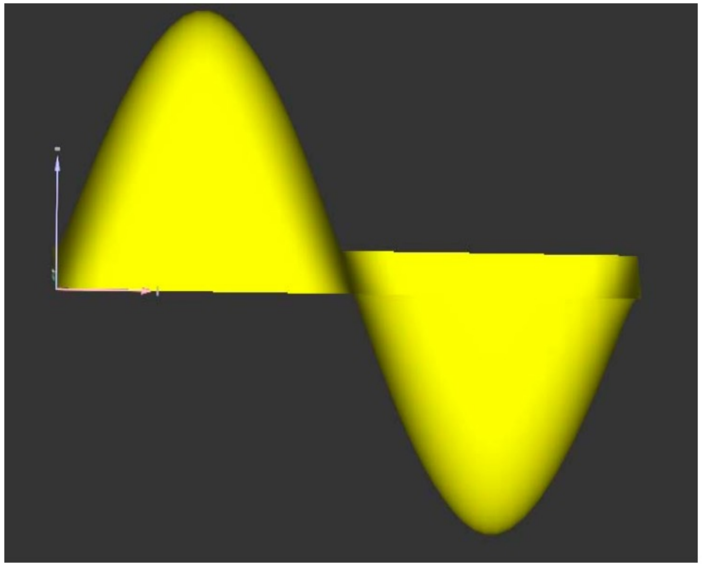
- Given planar closed boundary which consists of four B-spline curves, it has various different parametrization

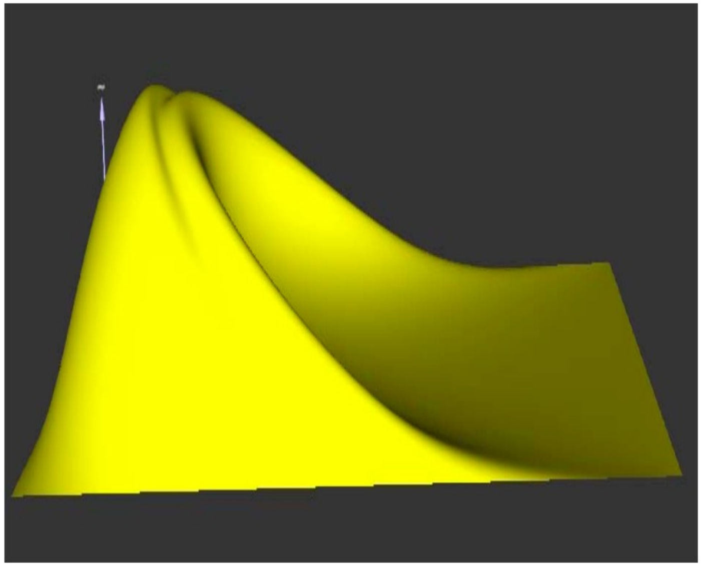




Impact of different parametrization

- Impact on the analysis results
- Impact on the convergence speed





Impact of different parametrization

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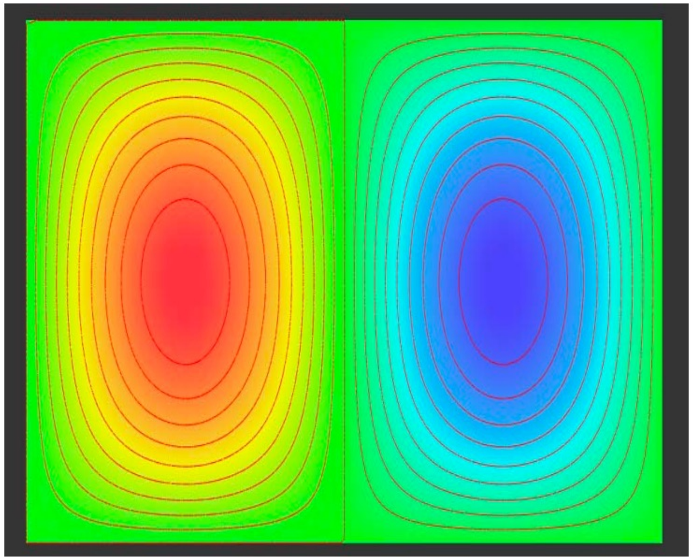
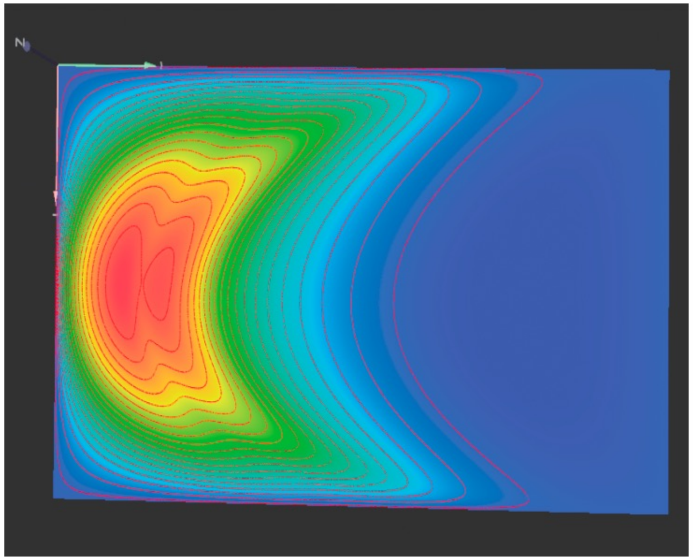
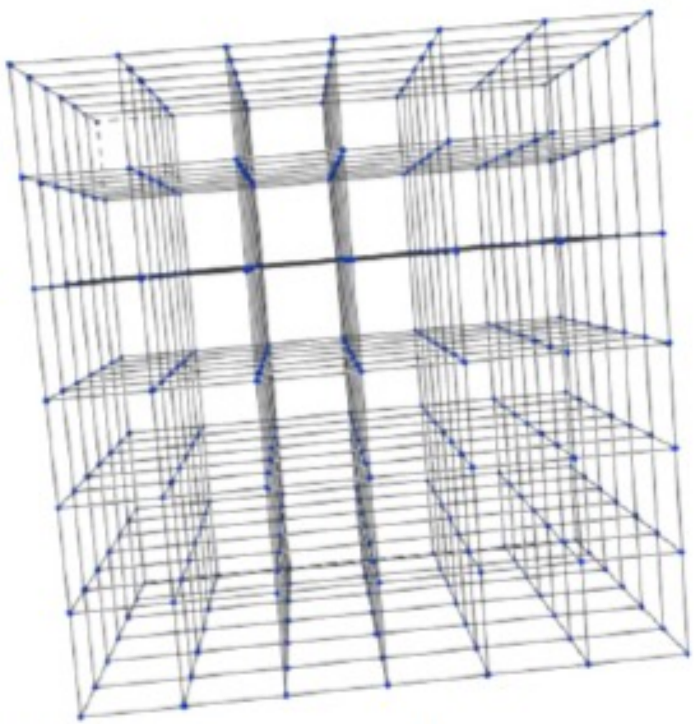
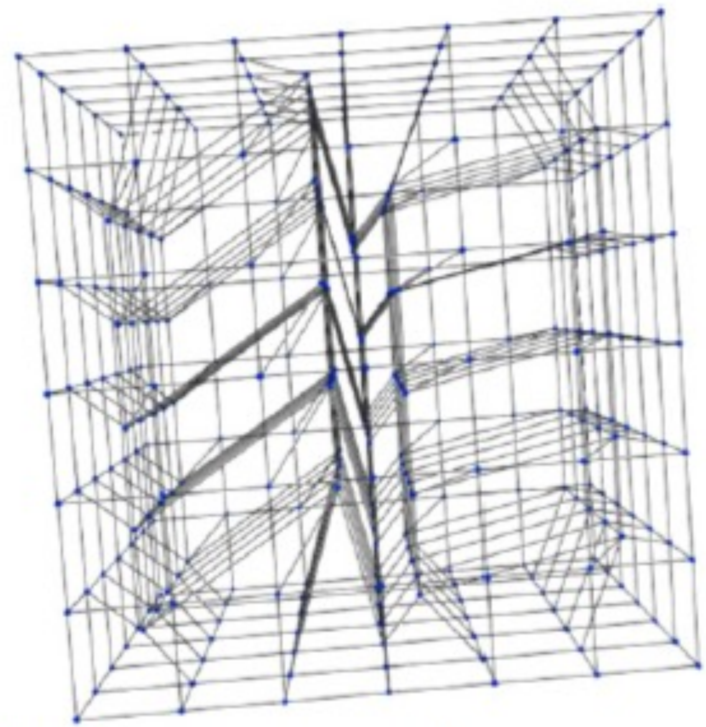




计算域参数化质量对分析结果影响 CAD2013

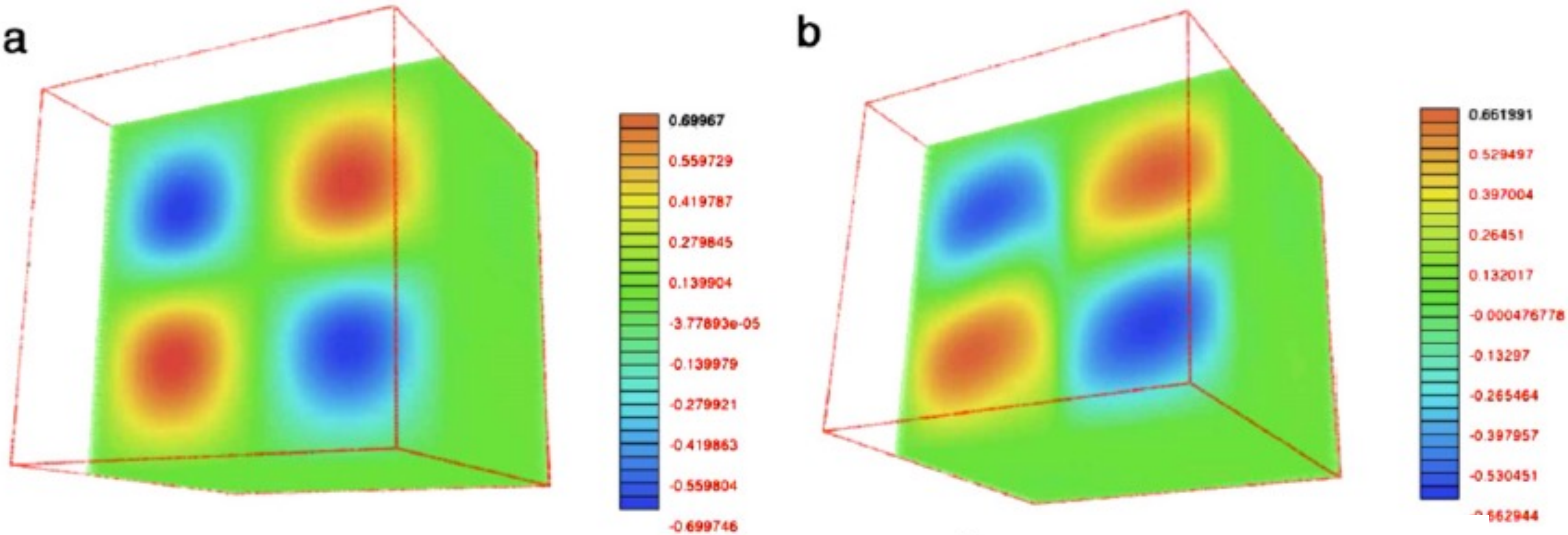


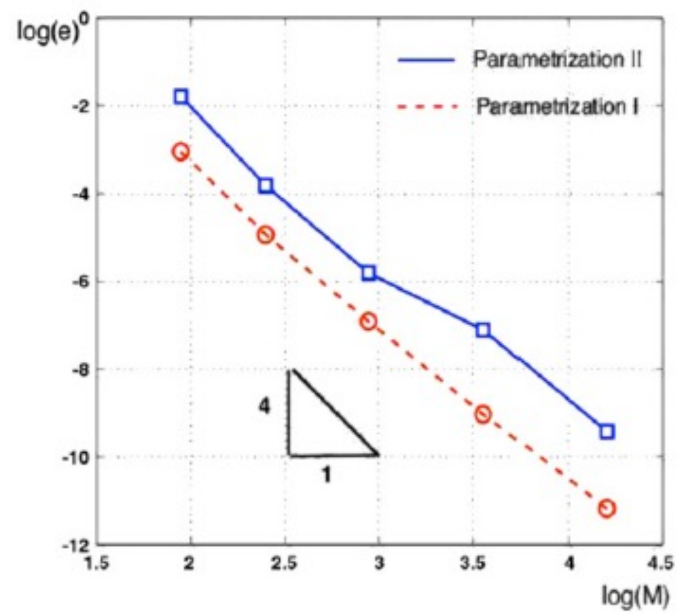
(a) Control point placement I.



(b) Control point placement II.

计算域参数化质量对分析结果影响 CAD2013

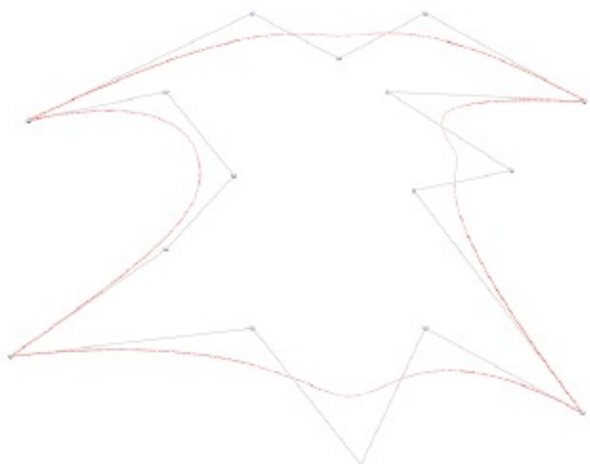




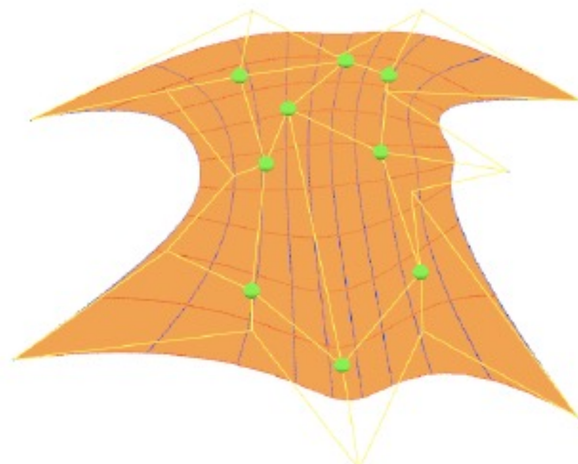
Problem statement

Construction of computational domain from boundary

given boundary control points of computational domain, construct the inner control points to generate analysis-suitable parameterization of computational domain



boundary curves

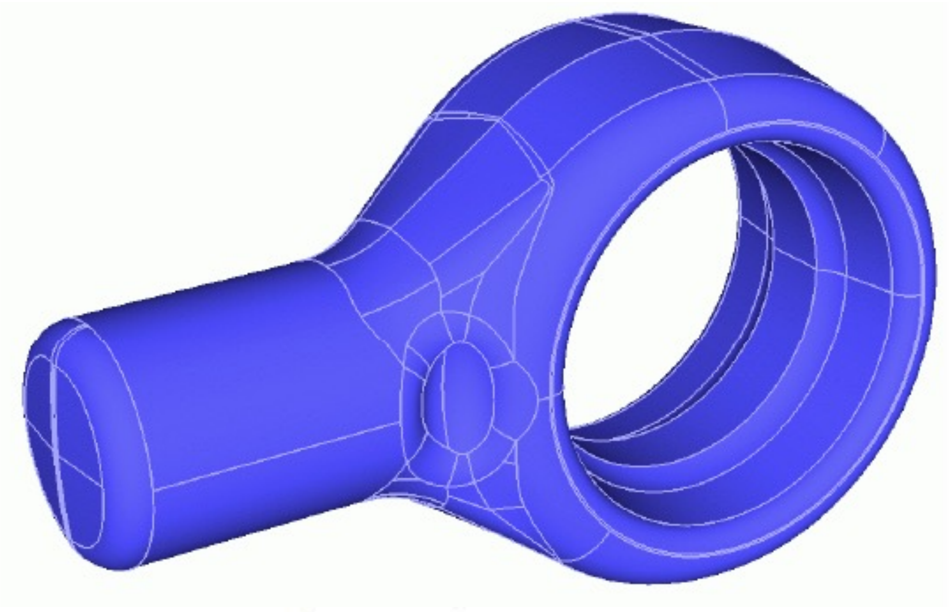


computational domain

Main difficulties

- Trimmed surface
- Complex topology
- Analysis-suitable

- 2D problem: given four planar boundary B-spline curves of computational domain, find the best planar B-spline surface as parametrization of computational domain.
- 3D problem: given six boundary B-spline surfaces of computational domain, find the best B-spline volume as parametrization of computational domain.
- Unknown variables: inner control points of B-spline surface(volume)



Related work on parameterization for IGA

- Analysis-aware optimal parameterization

  E. Cohen et al.(CMAME, 2010) , Xu et al.(CMAME,2011), Pilgerstorfer et al (CMAME, 2013)

- Volumetric spline parameterization from boundary triangulation

  T. Martin et al.(CMAME, 2009), Zhang et al.(CMAME, 2012).

- Analysis-suitable planar parameterization from spline boundary

  Xu et al.(CAD, 2013), Gravessen et al.(CMAME, 2014), Xu et al. (CMAME, 2015),

  Nian (CMAME, 2016), Kapl M. et al. (CMAME, 2016) , Buchegger and Jüttler (CAD, 2017)

- Analysis-suitable volume parameterization from spline boundary

  Xu et al.(JCP, 2013), Zhang et al.(CM, 2012), Chan et al (CAD, 2017) ,

  Haberleitner and Jüttler (CAD, 2017)

Gang Xu(徐岗)

フォロー中

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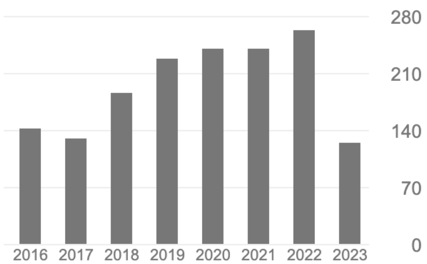
Isogeometric Analysis   Computer-Aided Design   Computer Aided Geometric…
Computational Geometry   Mesh Generation

タイトル	引用先	年
Parameterization of computational domain in isogeometric analysis: methods and comparison G Xu, B Mourrain, R Duvigneau, A Galligo Computer Methods in Applied Mechanics and Engineering 200 (23-24), 2021-2031	235	2011
Analysis-suitable volume parameterization of multi-block computational domain in isogeometric applications G Xu, B Mourrain, R Duvigneau, A Galligo Computer-Aided Design 45 (2), 395-404	196	2013
Constructing analysis-suitable parameterization of computational domain from CAD boundary by variational harmonic method G Xu, B Mourrain, R Duvigneau, A Galligo Journal of Computational Physics 252, 275-289	122	2013
Optimal analysis-aware parameterization of computational domain in 3D isogeometric analysis G Xu, B Mourrain, R Duvigneau, A Galligo Computer-Aided Design 45 (4)	111	2013

引用先   すべて表示

	すべて	2018 年以来
引用	2126	1289
h 指標	23	20
i10 指標	41	31



オープン アクセス   すべて表示

18 件の論文   25 件の論文

利用不可   利用可能

助成機関の要件に基づく

Volumetric parameterization with truncated hierarchical B-splines for isogeometric analysis
Y Zheng, F Chen - Computer Methods in Applied Mechanics and …, 2022 - Elsevier
Constructing spline parameterizations for computational domains is one of the fundamental problems in isogeometric analysis. In this paper, we present an efficient method for …
☆ 保存 99 引用 被引用数: 4 関連記事 全 2 バージョン

IGA-suitable planar parameterization with patch structure simplification of closed-form polysquare
S Wang, J Ren, X Fang, H Lin, G Xu, H Bao… - Computer Methods in …, 2022 - Elsevier
A primary challenge for isogeometric analysis (IGA)-suitable parameterization is to efficiently decompose a complex computational domain into a small number of high-quality IGA …
☆ 保存 99 引用 被引用数: 5 関連記事 全 3 バージョン

TCB-spline-based isogeometric analysis method with high-quality parameterizations
Z Wang, J Cao, X Wei, YJ Zhang - Computer Methods in Applied Mechanics …, 2022 - Elsevier
Isogeometric analysis (IGA) was introduced to integrate methods for analysis and computer-aided design (CAD) into a unified process. High-quality parameterization of a physical …
☆ 保存 99 引用 被引用数: 4 関連記事 全 3 バージョン

Penalty function-based volumetric parameterization method for isogeometric analysis
Y Ji, MY Wang, MD Pan, Y Zhang, CG Zhu - Computer Aided Geometric …, 2022 - Elsevier
In isogeometric analysis, constructing bijective and low-distorted parameterizations is a fundamental task. Compared with the planar problem, the volumetric case is more …
☆ 保存 99 引用 被引用数: 5 関連記事

Curvature-Based *r*-Adaptive Isogeometric Analysis with Injectivity-Preserving Multi-Sided Domain Parameterization

Y Ji, M Wang, Y Yu, C Zhu - Journal of Systems Science and Complexity, 2023 - Springer

Inspired by the r-refinement method in isogeometric analysis, in this paper, the authors propose a curvature-based r-adaptive isogeometric method for planar multi-sided …

☆ 保存 99 引用 被引用数: 2 関連記事

Constructing planar domain parameterization with HB-splines via quasi-conformal mapping

M Pan, F Chen - Computer Aided Geometric Design, 2022 - Elsevier

Constructing a high-quality parameterization of a computational domain is a fundamental research problem in isogeometric analysis, which has been extensively investigated so far …

☆ 保存 99 引用 被引用数: 2 関連記事 全 2 バージョン

h-Refinement method for toric parameterization of planar multi-sided computational domain in isogeometric analysis

Y Ji, JG Li, YY Yu, CG Zhu - Computer Aided Geometric Design, 2022 - Elsevier

Toric surface patches are a class of multi-sided surface patches that can represent multi-sided domains without mesh degeneration. In this paper, we propose an improved …

☆ 保存 99 引用 被引用数: 2 関連記事

On an improved PDE-based elliptic parameterization method for isogeometric analysis using preconditioned Anderson acceleration

Y Ji, K Chen, M Möller, C Vuik - Computer Aided Geometric Design, 2023 - Elsevier

Constructing an analysis-suitable parameterization for the computational domain from its boundary representation plays a crucial role in the isogeometric design-through-analysis …

☆ 保存 99 引用 全 3 バージョン

Boundary Correspondence for Iso-Geometric Analysis Based on Deep Learning

Z Zhan, Y Zheng, W Wang, F Chen - Communications in Mathematics and …, 2023 - Springer

One of the key problems in isogeometric analysis (IGA) is domain parameterization, ie, constructing a map between a parametric domain and a computational domain. As a …

☆ 保存 99 引用

Construction of IGA-suitable Volume Parametric Models by the Segmentation–Mapping–Merging Mechanism of Design Features

L Chen, N Bu, Y Jin, G Xu, B Li - Computer-Aided Design, 2022 - Elsevier

Volumetric parameterization is the key and bottleneck issue in the current research of constructing complex models for isogeometric analysis (IGA). Many researchers used …

☆ 保存 99 引用 被引用数: 1 関連記事 全 2 バージョン

Curvature-based R-Adaptive Planar NURBS Parameterization Method for Isogeometric Analysis Using Bi-Level Approach

Y Ji, MY Wang, Y Wang, CG Zhu - Computer-Aided Design, 2022 - Elsevier

Localized and anisotropic features extensively exist in various physical phenomena. The present work focuses on the r-adaptive parameterization technique for isogeometric analysis …

☆ 保存 99 引用 被引用数: 2 関連記事 全 2 バージョン

[PDF] Skeleton-Based Volumetric Parameterizations for Lattice Structures

L Chen, S Liang, N Yan, X Yang… - … IN ENGINEERING & …, 2023 - cdn.techscience.cn

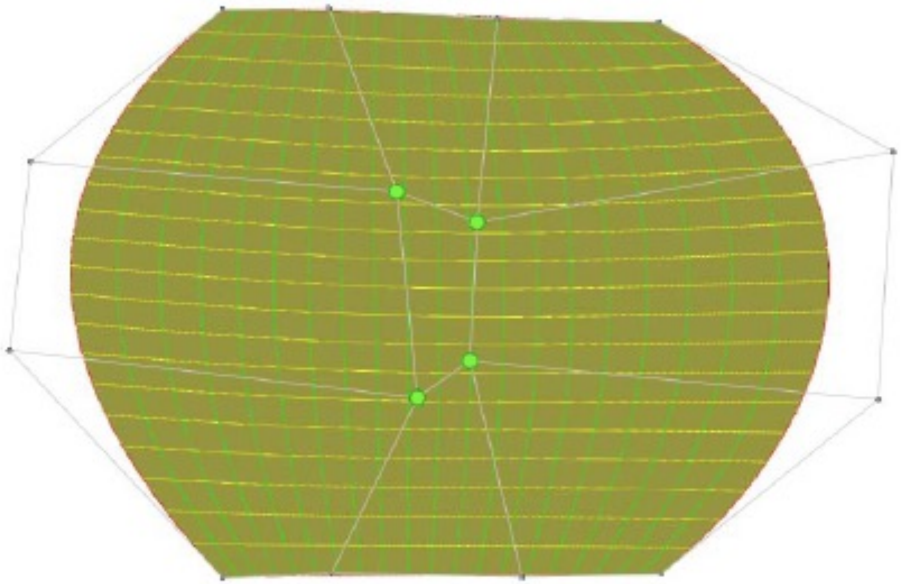
Lattice structures with excellent physical properties have attracted great research interest. In this paper, a novel volume parametric modeling method based on the skeleton model is …

☆ 保存 99 引用 関連記事 »

2. 计算域参数化的质量评价

Analysis-suitable parameterization（XU 2011）

- injective (no self-intersections)
- as uniform as possible
- orthogonal isoparametric curves

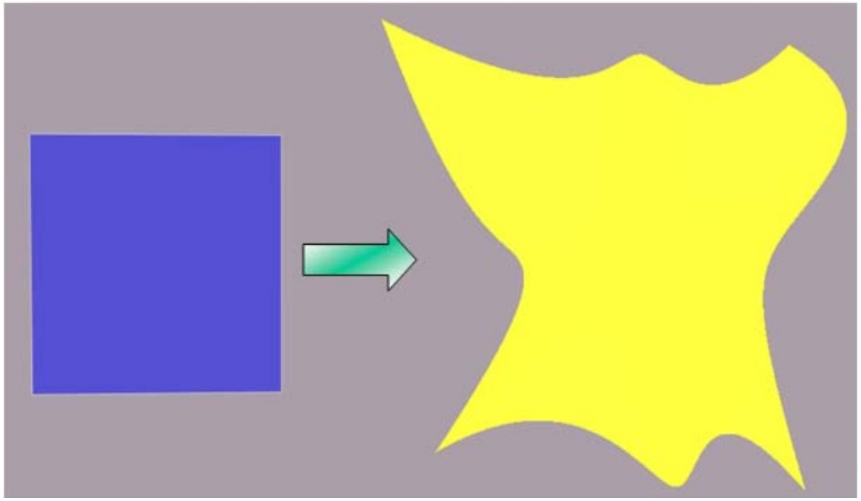


Analysis-suitable parameterization （XU 2011）

- no self-intersections: injective
- injective condition

Theorem 1

Suppose that ***F*** is a mapping from connected domain Ω_0 to domain Ω_1, if ***F*** is injective on the connteced boundary $\partial\Omega_0$ and the Jacobian determinant of ***F*** is non-zero, then ***F*** is injective on the interior domain of Ω_0.



Analysis-suitable parameterization （XU 2011）

- planar B-spline surface $\boldsymbol{F}(u, v)$

$$\boldsymbol{F}(u, v) = \sum_{i=0}^{m} \sum_{j=0}^{n} B_i^p(u) B_j^q(v) \boldsymbol{P}_{ij} \tag{1}$$

with $\boldsymbol{U} = \{0, \ldots, 0, u_{p+1}, \ldots, u_{r-p-1}, 1, \ldots, 1\}$
$\boldsymbol{V} = \{0, \ldots, 0, v_{p+1}, \ldots, v_{r-p-1}, 1, \ldots, 1\}$
$\boldsymbol{P}_{ij} = (P_{ij}^x, P_{ij}^y)$

$$\boldsymbol{J}(\boldsymbol{F}) = \begin{vmatrix} \boldsymbol{F}_u^x & \boldsymbol{F}_v^x \\ \boldsymbol{F}_u^y & \boldsymbol{F}_v^y \end{vmatrix} \tag{2}$$

Lemma 1

If the Jacobian determinant $\boldsymbol{J}(\boldsymbol{F})$ of planar B-spline surface satisfies $\boldsymbol{J}(\boldsymbol{F}) > 0$, $\boldsymbol{F}(u, v)$ has no self-intersections.

Analysis-suitable parameterization（XU 2011）

Computation of Jacobian determinant

$$\boldsymbol{F}_u(u,v) = \sum_{i=0}^{m-1}\sum_{j=0}^{n} \boldsymbol{P}_{ij}^{(1,0)} N_{i,p-1}(u) N_{i,q}(v)$$

$$\boldsymbol{F}_v(u,v) = \sum_{i=0}^{m}\sum_{j=0}^{n-1} \boldsymbol{P}_{ij}^{(0,1)} N_{i,p}(u) N_{i,q-1}(v)$$

$$\begin{aligned}\boldsymbol{J}(\boldsymbol{F}) &= \boldsymbol{F}_u^x \boldsymbol{F}_v^y - \boldsymbol{F}_v^x \boldsymbol{F}_u^y \\ &= \sum_{i=0}^{2m-1}\sum_{j=0}^{2n-1} G_{ij} N_{i,2p-1}(u) N_{i,2q-1}(v)\end{aligned}$$

Theorem 3

From the convex hull property of B-spline surface and regularity at four corners, if $G_{ij} \geq 0$, then $\boldsymbol{J}(\boldsymbol{F}) > 0$, that is, $\boldsymbol{F}(u,v)$ has no self-intersections.

IGA刚度矩阵条件数

条件数是一个表征矩阵稳定特性的标志,条件数越大,说明矩阵越不稳定，对数值计算求解器的性能有较大影响

The condition number of the stiffness matrix, which is a key factor for the stability of the linear system, depends strongly on the quality of the domain parameterization

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journal homepage: www.elsevier.com/locate/cma

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Bounding the influence of domain parameterization and knot spacing on numerical stability in Isogeometric Analysis

CrossMark

Elisabeth Pilgerstorfer *, Bert Jüttler

Institute of Applied Geometry, Johannes Kepler University, Linz, Austria

$$\kappa(\mathbf{S}) \leqslant C\,(\mathrm{diam}(\Omega))^2 \frac{\max\limits_{\ell}\left(\lambda_{\max}(\mathbf{N}^{\ell}_{\max})H^{\ell}_2\right)}{\min\limits_{\ell}\left(h^{\ell_1}_1 h^{\ell_2}_2 \omega^{\ell}_{\min}\right)},$$

1. In order to obtain a small bound, the lengths of both partial derivatives should be close to equal, and the angle between them should be close to 90.
2. Furthermore, elements with a small area should be avoided, i.e., the area should not vary too much between elements in the physical domain

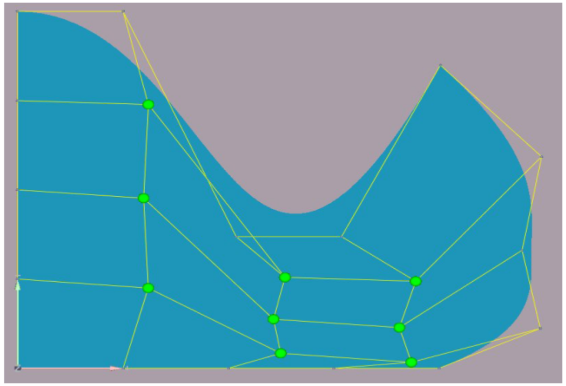
3. 计算域参数化方法

Initial construction of inner control points

- Discrete Coons method [G.Farin, 1999]

  Given the boundary control points $\boldsymbol{P}_{0,j}, \boldsymbol{P}_{n,j}, \boldsymbol{P}_{i,0}, \boldsymbol{P}_{i,m}$, the inner contro points $\boldsymbol{P}_{i,j}$ can be constructed as follows:

$$
\begin{aligned}
\boldsymbol{P}_{i,j} &= (1-\frac{i}{n})\boldsymbol{P}_{0,j} + \frac{i}{n}\boldsymbol{P}_{n,j} + (1-\frac{j}{m})\boldsymbol{P}_{i,0} + \frac{j}{m}\boldsymbol{P}_{i,m} \\
&\quad -[1-\frac{i}{n} \quad \frac{i}{n}]\begin{pmatrix} \boldsymbol{P}_{0,0} & \boldsymbol{P}_{0,m} \\ \boldsymbol{P}_{n,0} & \boldsymbol{P}_{n,m} \end{pmatrix}\begin{pmatrix} 1-\frac{j}{m} \\ \frac{j}{m} \end{pmatrix}
\end{aligned}
$$



Discrete Coons volumes (Xu, 2011)

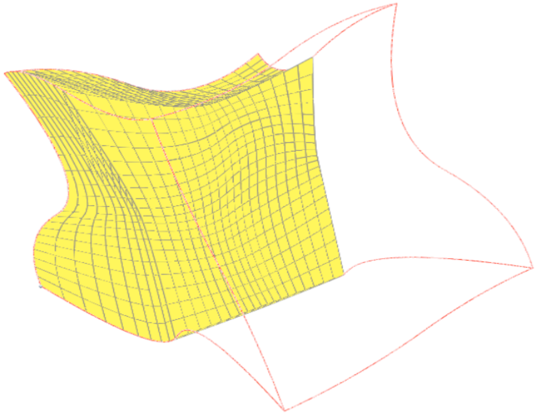
The interior control points $\boldsymbol{P}_{i,j,k}$ can be constructed as linear combinations of points described by the following formulas:

$$
\begin{aligned}
\boldsymbol{P}_{i,j,k} \quad = \quad & (1-i/l)\boldsymbol{P}_{0,j,k} + i/l\boldsymbol{P}_{l,j,k} + (1-j/m)\boldsymbol{P}_{i,0,k} + j/m\boldsymbol{P}_{i,m,k} \\
& +(1-k/n)\boldsymbol{P}_{i,j,0} + k/n\boldsymbol{P}_{i,j,n} - [1-i/l, i/l]\begin{bmatrix} \boldsymbol{P}_{0,0,k} & \boldsymbol{P}_{0,m,k} \\ \boldsymbol{P}_{l,0,k} & \boldsymbol{P}_{l,m,k} \end{bmatrix}\begin{bmatrix} 1-j/m \\ j/m \end{bmatrix} \\
& -[1-j/m, j/m]\begin{bmatrix} \boldsymbol{P}_{i,0,0} & \boldsymbol{P}_{i,0,n} \\ \boldsymbol{P}_{i,m,0} & \boldsymbol{P}_{i,m,n} \end{bmatrix}\begin{bmatrix} 1-k/n \\ k/n \end{bmatrix} \\
& -[1-k/n, k/n]\begin{bmatrix} \boldsymbol{P}_{0,j,0} & \boldsymbol{P}_{l,j,0} \\ \boldsymbol{P}_{0,j,n} & \boldsymbol{P}_{l,j,n} \end{bmatrix}\begin{bmatrix} 1-i/l \\ i/l \end{bmatrix} \\
& +(1-k/n)\left[[1-i/l, i/l]\begin{bmatrix} \boldsymbol{P}_{0,0,0} & \boldsymbol{P}_{0,m,0} \\ \boldsymbol{P}_{l,0,0} & \boldsymbol{P}_{l,m,0} \end{bmatrix}\begin{bmatrix} 1-j/m \\ j/m \end{bmatrix}\right] \\
& +k/n\left[[1-i/l, i/l]\begin{bmatrix} \boldsymbol{P}_{0,0,n} & \boldsymbol{P}_{0,m,n} \\ \boldsymbol{P}_{l,0,n} & \boldsymbol{P}_{l,m,n} \end{bmatrix}\begin{bmatrix} 1-j/m \\ j/m \end{bmatrix}\right]
\end{aligned}
$$

Problem statement

given six boundary B-spline surfaces, find the placement of inner control points such that the resulted trivariate B-spline parametric volume is a good computational domain for 3D isogeometric analysis

boundary curves



computational domain

约束优化方法 CMAME 2011, CAD 2013

Input: six boundary B-spline surfaces

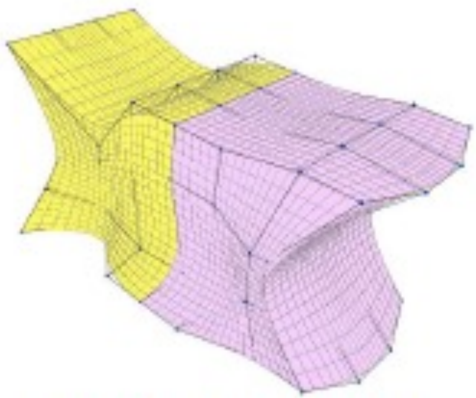
Output: inner control points and the corresponding B-spline volume parameterization

- Construct the initial inner control points by discrete Coons method;
- Construct the constraint condition from boundary B-spline surfaces;
- Solve the following constraint optimization problem by using sequential quadratic programming (SQP for short) method

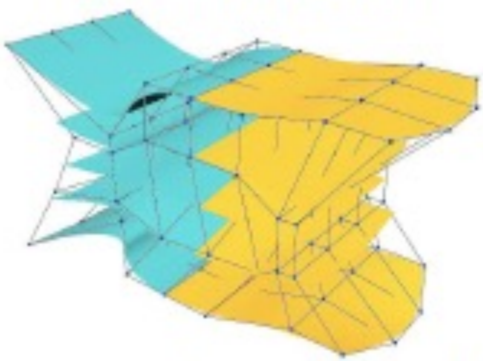
$$\begin{aligned}
&min \iiint (\| \sigma_\xi \|^2 + \| \sigma_\eta \|^2 + \| \sigma_\zeta \|^2) \\
&+\omega(\| \sigma_{\xi\xi} \|^2 + \| \sigma_{\eta\eta} \|^2 + \| \sigma_{\zeta\zeta} \|^2 \\
&+2 \| \sigma_{\xi\eta} \|^2 + 2 \| \sigma_{\xi\zeta} \|^2 + 2 \| \sigma_{\eta\zeta} \|^2) d\xi d\eta d\zeta. \\
&s.t. \quad G_{ijk} > 0
\end{aligned}$$

- Generate the corresponding B-spline volume parameterization $\sigma(\xi, \eta, \zeta)$ as computational domain.

Multi-block case



(a) C^1 B-spline blocks



(b) Isoparametric surfaces and control lattices in C^1 B-spline blocks

$$\frac{\partial}{\partial \xi}\sigma_1(\xi,\eta,\zeta)|_{\xi=\xi_1} = \frac{\partial}{\partial \xi}\sigma_2(\xi,\eta,\zeta)|_{\xi=\xi_1}$$

$$\sum_{\substack{0\leq j\leq m \\ 0\leq k\leq n}} \omega^{1,1}_{i,j,k}\Delta^{1,1}_{i,j,k}N^q_jN^r_k = \sum_{\substack{0\leq j\leq m \\ 0\leq k\leq n}} \omega^{1,2}_{i,j,k}\Delta^{1,2}_{i,j,k}N^q_jN^r_k.$$

$$\omega^{1,1}_{i,j,k}\Delta^{1,1}_{i,j,k} = \omega^{1,2}_{i,j,k}\Delta^{1,2}_{i,j,k}, i = 0,...,l,$$

Variational harmonic method
(Journal of Computational Physics, 2013)

- Given: computational domain $\mathcal{S}$, parametric domain $\mathcal{P}$,

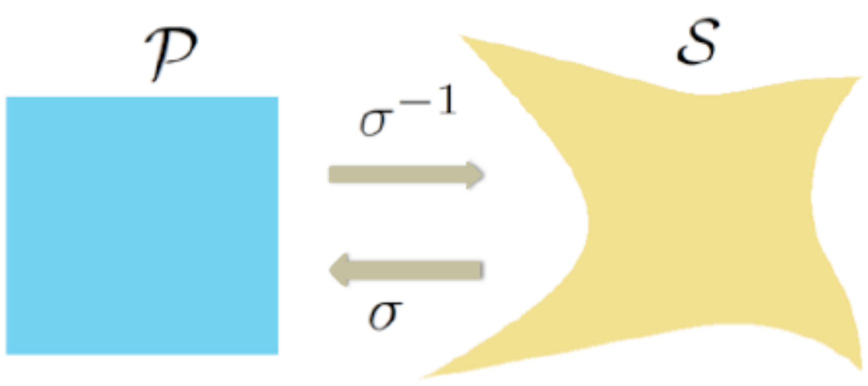
$$S(\xi,\eta) = (x(\xi,\eta), y(\xi,\eta)) = \sum_{i=0}^{n}\sum_{j=0}^{m} N_i^p(\xi) N_j^q(\eta) p_{i,j}$$

- Harmonic mapping: $\sigma : \mathcal{S} \mapsto \mathcal{P}$

$$\Delta\xi(x,y) = \xi_{xx} + \xi_{yy} = 0$$

$$\Delta\eta(x,y) = \eta_{xx} + \eta_{yy} = 0$$

- $\sigma^{-1} : \mathcal{P} \mapsto \mathcal{S}$ is one-to-one



Main method

- Convert the harmonic condition into some constraints on parametric domain by chain rules:

$$\xi_{xx} + \xi_{yy} = 0, \quad \eta_{xx} + \eta_{yy} = 0$$

$$(x_\eta^2 + y_\eta^2)\frac{\partial^2 U}{\partial \xi^2} - 2(x_\xi x_\eta + y_\xi y_\eta)\frac{\partial^2 U}{\partial \xi \eta} + (x_\xi^2 + y_\xi^2)\frac{\partial^2 U}{\partial \eta^2} = 0$$

- $U = x(\xi, \eta)$ and $U = y(\xi, \eta)$
- Relaxation solving method
- Drawback: Non-uniform elements near convex(concave) boundary region

Variational harmonic method

- Convert the harmonic condition into some constraints on parametric domain by chain rules:

$$\xi_{xx} + \xi_{yy} = 0, \quad \eta_{xx} + \eta_{yy} = 0$$

$$Lx(\xi, \eta) = Ly(\xi, \eta) = 0$$

$$\| LS(\xi, \eta) \|^2 = (Lx)^2 + (Ly)^2 = 0$$

- Objective function combining uniform and orthogonal term:

$$\iint \| LS(\xi, \eta) \|^2 + \lambda_1 (\| S_{\xi\xi} \|^2 + \| S_{\eta\eta} \|^2$$
$$+2 \| S_{\xi\eta} \|^2) + \lambda_2 (\| S_\xi \|^2 + \| S_\eta \|^2) dudv$$

- $$L = (x_\eta^2 + y_\eta^2) \frac{\partial^2}{\partial \xi^2} - 2(x_\xi x_\eta + y_\xi y_\eta) \frac{\partial^2}{\partial \xi \eta} + (x_\xi^2 + y_\xi^2) \frac{\partial^2}{\partial \eta^2}$$

Overview

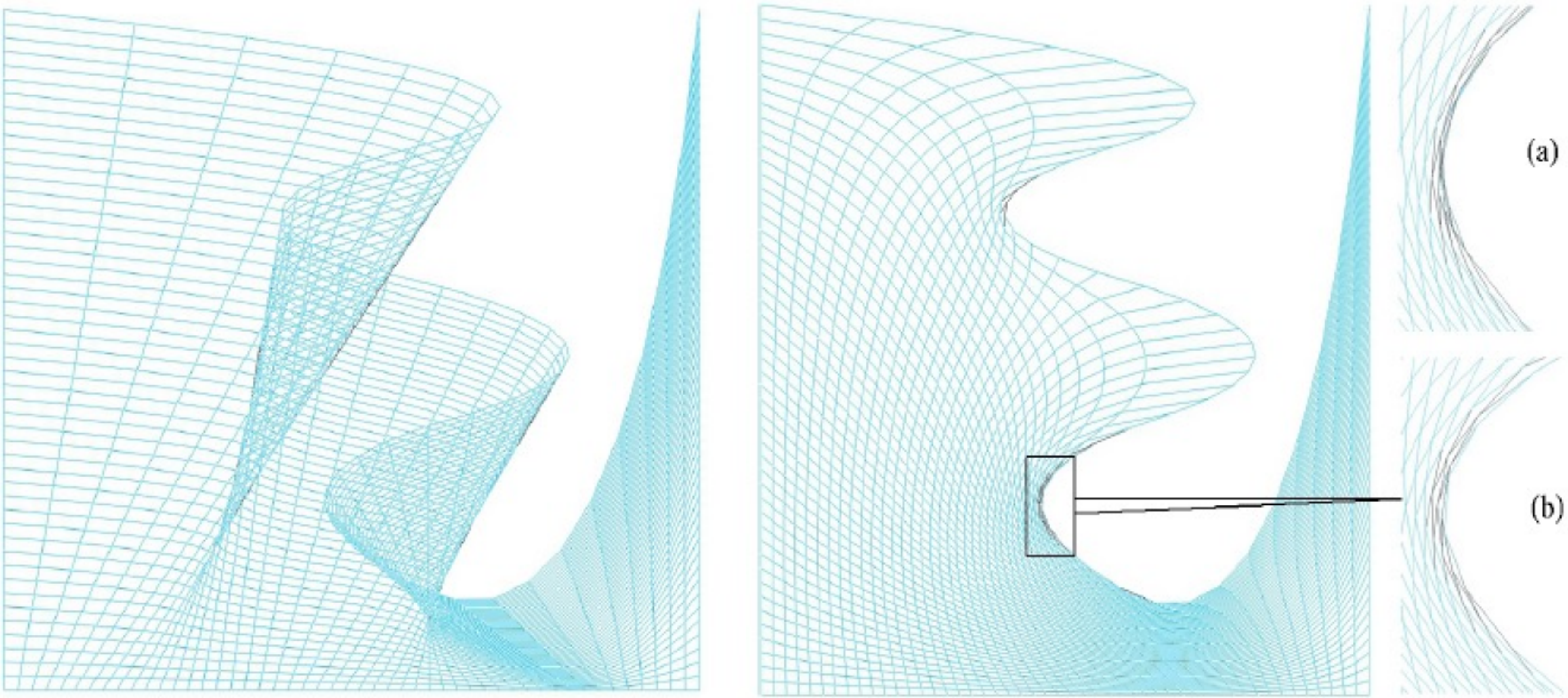
Input: four coplanar boundary B-spline curves
Output: inner control points and the corresponding planar B-spline surfaces

- Construct the initial inner control points by Discrete Coons method;
- Solve the following optimization problem by using steepest-descent method

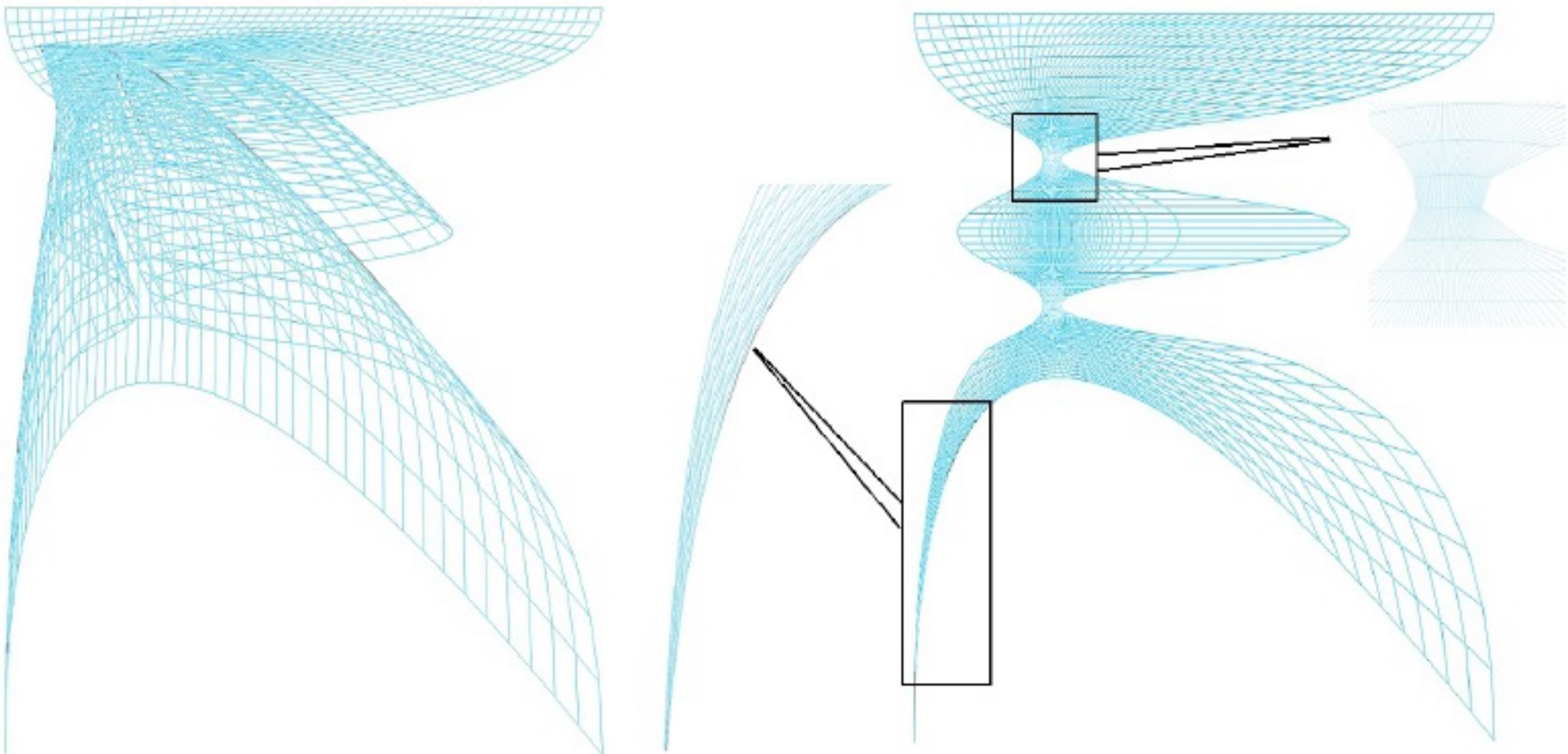
$$MIN \quad \iint \| LS(\xi,\eta) \|^2 + \lambda_1(\| S_{\xi\xi} \|^2 + \| S_{\eta\eta} \|^2 + 2 \| S_{\xi\eta} \|^2) + \lambda_2(\| S_{\xi} \|^2 + \| S_{\eta} \|^2) dudv$$

- Generate the corresponding planar B-spline surface $S(\xi,\eta)$ as computational domain.

Two examples（1/2）



Two examples（2/2）



Volume parameterization with variational harmonic method

Harmonic condition

Convert the harmonic condition into some constraints on parametric domain by chain rules:

$$Lx(\xi, \eta, \zeta) = Ly(\xi, \eta, \zeta) = Lz(\xi, \eta, \zeta) = 0 \tag{1}$$

where

$$L = a^{11}\frac{\partial^2}{\partial \xi^2} + 2a^{12}\frac{\partial^2}{\partial \xi \eta} + 2a^{13}\frac{\partial^2}{\partial \xi \zeta} + a^{22}\frac{\partial^2}{\partial \eta^2} + 2a^{23}\frac{\partial^2}{\partial \eta \zeta} + +a^{33}\frac{\partial^2}{\partial \zeta^2}, \tag{2}$$

$$\begin{aligned} a^{11} &= a_{22}a_{33} - a_{23}^2, a^{12} = a_{13}a_{23} - a_{12}a_{33}, a^{13} = a_{12}a_{23} - a_{13}a_{22}, \\ a^{22} &= a_{11}a_{33} - a_{13}^2, a^{23} = a_{13}a_{12} - a_{11}a_{23}, a^{33} = a_{11}a_{22} - a_{12}^2, \end{aligned}$$

and

$$\begin{aligned} a_{11} &= (S_\xi, S_\xi), a_{12} = (S_\xi, S_\eta), a_{13} = (S_\xi, S_\zeta), \\ a_{22} &= (S_\eta, S_\eta), a_{23} = (S_\eta, S_\zeta), a_{33} = (S_\zeta, S_\zeta). \end{aligned}$$

Overview

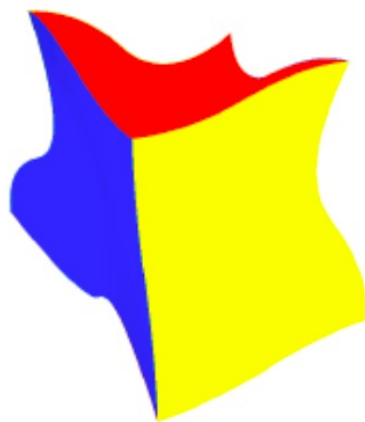
Input: six boundary B-spline surfaces
Output: inner control points and the corresponding B-spline volumes

- Construct the initial inner control points by discrete Coons method;
- Solve the following optimization problem by using steepest-descent method

$$\textbf{Min} \iint \| L\mathcal{S}(\xi,\eta,\zeta) \|^2 + \lambda_1(\| \mathcal{S}_{\xi\xi} \|^2 + \| \mathcal{S}_{\eta\eta} \|^2 + 2 \| \mathcal{S}_{\xi\eta} \|^2 + 2 \| \mathcal{S}_{\eta\zeta} \|^2 + 2 \| \mathcal{S}_{\xi\zeta} \|^2 + \| \mathcal{S}_{\zeta\zeta} \|^2) + \lambda_2(\| \mathcal{S}_{\xi} \|^2 + \| \mathcal{S}_{\eta} \|^2 + \| \mathcal{S}_{\zeta} \|^2) dudv$$

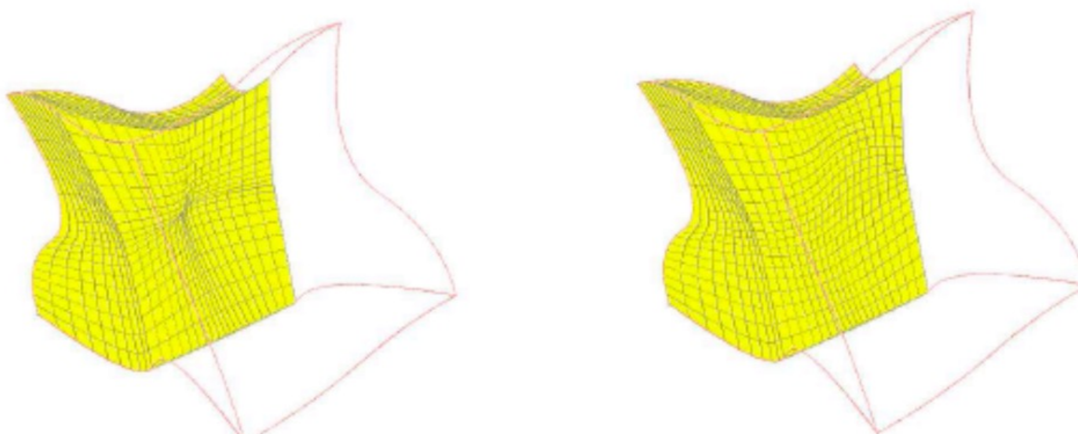
- Generate the corresponding B-spline volume $\mathcal{S}(\xi,\eta,\zeta)$ as computational domain.

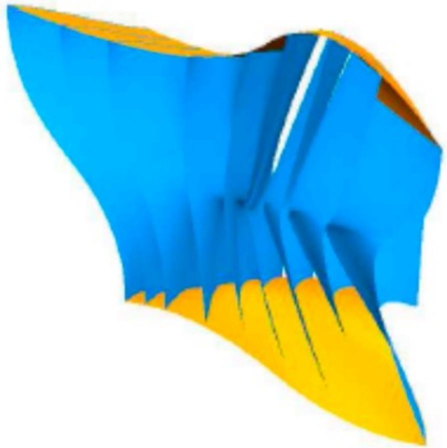
Example I



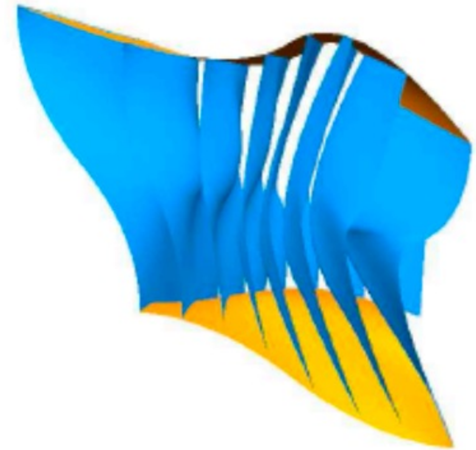
(a) boundary surfaces

(b) boundary curves

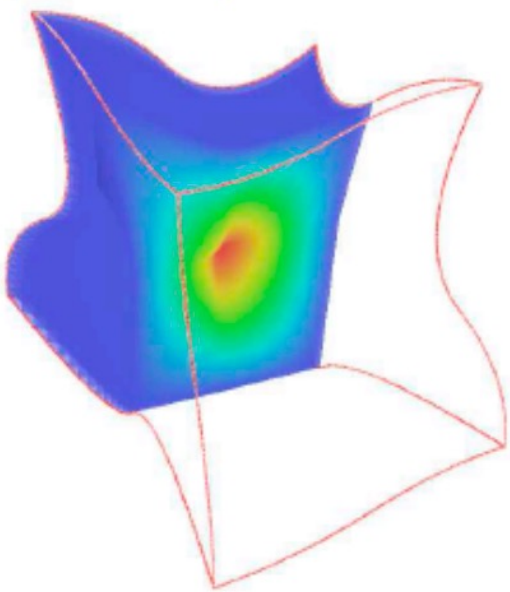


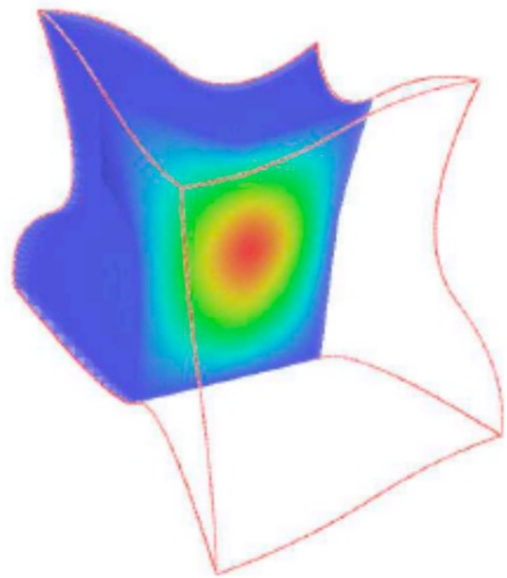


(e) initial iso-parametric surface

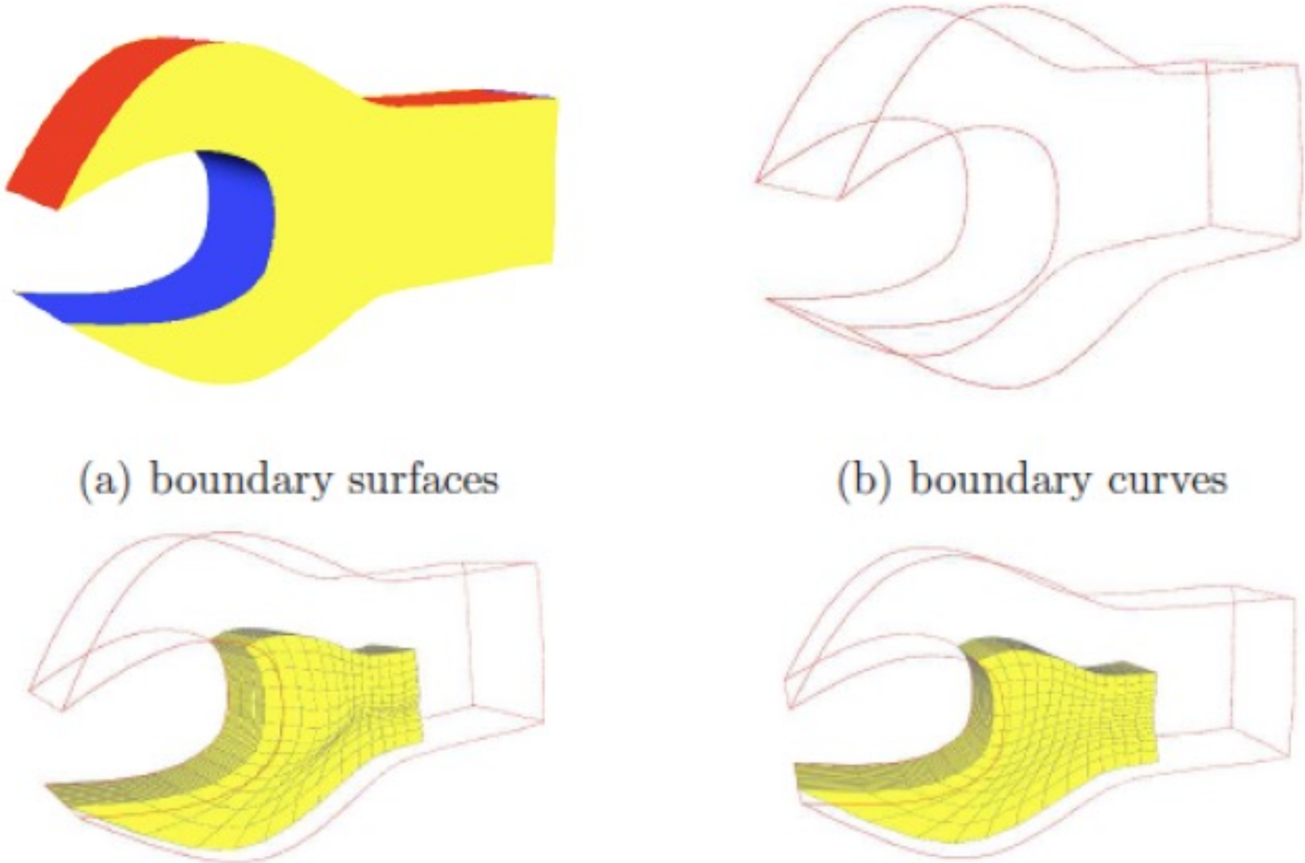


(f) final iso-parametric surface





3D example I

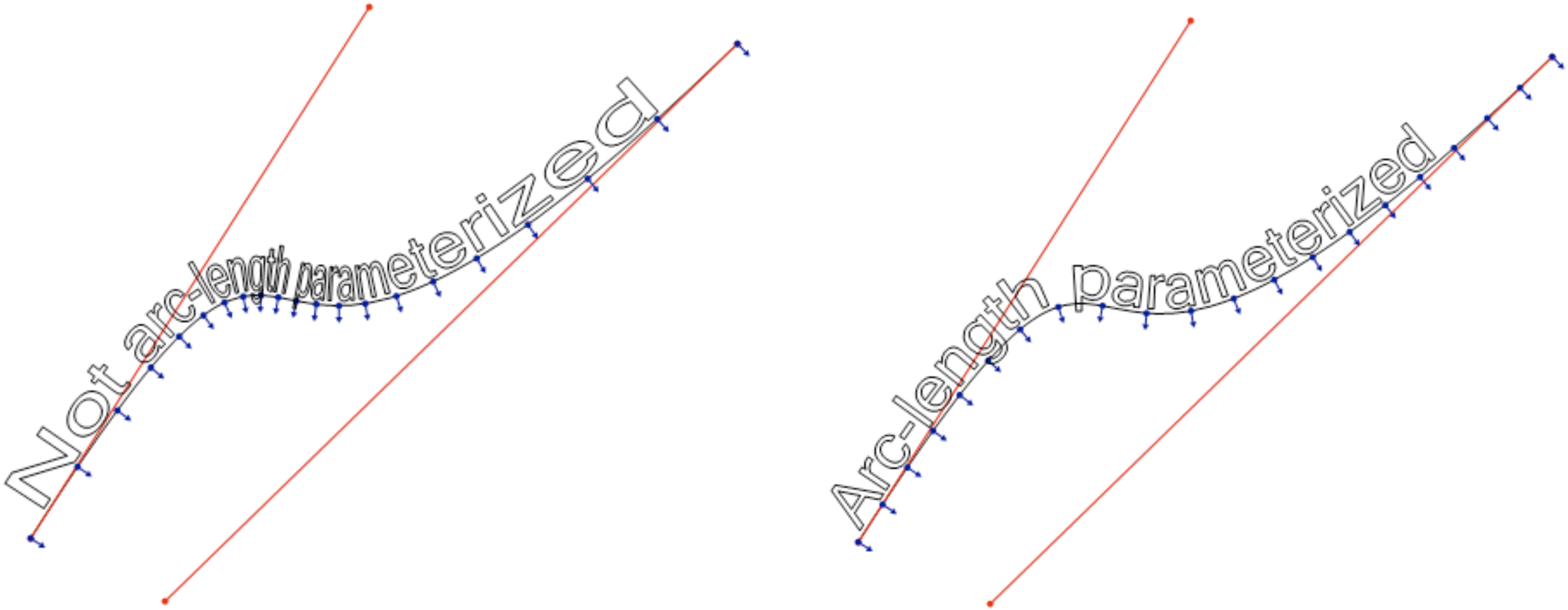


(a) boundary surfaces (b) boundary curves

(c) Coons volume (d) final volume parameterization

Boundary reparameterization for volumetric parameterization (Computational Mechanics, 2014)

Goal: construct optimal Möbius reparameterization of boundary surfaces to achieve high-quality isoparametric structure without changing the boundary geometry



Möbius reparameterization

$$\boldsymbol{R}(u,v) = \frac{\sum_{i=0}^{n}\sum_{j=0}^{m}\lambda_{i,j}\boldsymbol{C}_{i,j}N_i^p\boxed{(u)}N_j^q\boxed{(v)}}{\sum_{i=0}^{n}\sum_{j=0}^{m}\lambda_{i,j}N_i^p\boxed{(u)}N_j^q\boxed{(v)}},$$

$$\boxed{u = \frac{(1-\alpha)\xi}{\alpha(1-\xi)+(1-\alpha)\xi}}$$

$$\boxed{v = \frac{(1-\beta)\eta}{\beta(1-\eta)+(1-\beta)\eta}}$$

New NURBS surface with the same control points but different weights and knot vectors

$$\widetilde{\lambda}_{i,j} = \frac{\lambda_{i,j}}{\prod_{r=1}^{p}K_{i,r}\prod_{s=1}^{q}L_{j,s}}$$

Optimization method

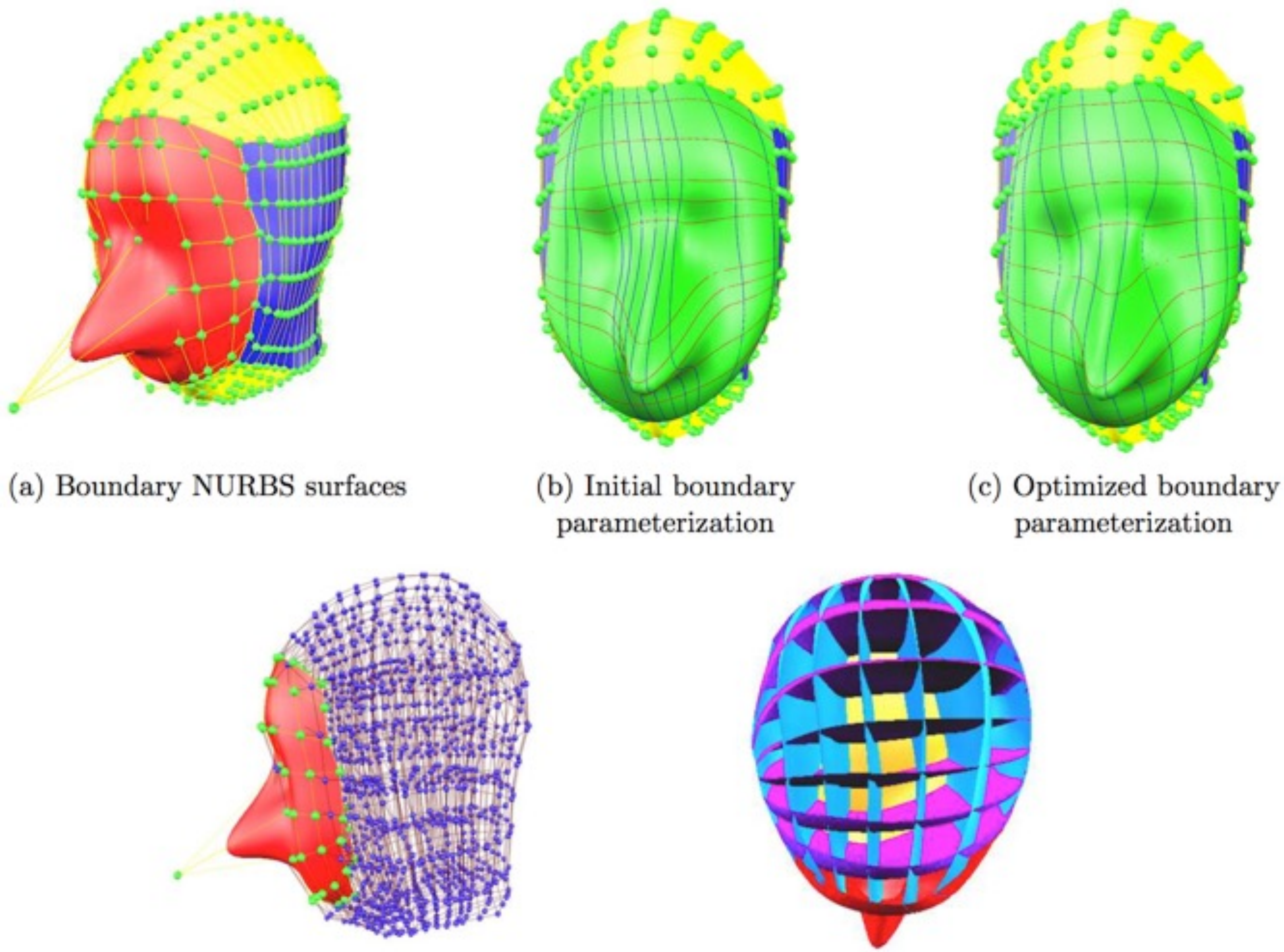
- Find the optimal

$$\alpha, \beta$$

such that the reparameterized NURBS surface minimizes the following objective function

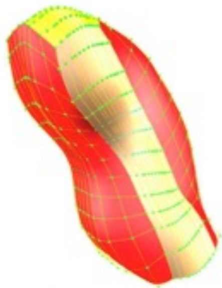
$$\int_{\mathcal{P}} (\det \widetilde{\boldsymbol{J}} - J_{avg})^2 + \omega_1 (\|\widetilde{\boldsymbol{R}}_{\xi\xi}\|^2 + \|\widetilde{\boldsymbol{R}}_{\eta\eta}\|^2)\, d\xi\, d\eta$$

Reparameterization for VP problem



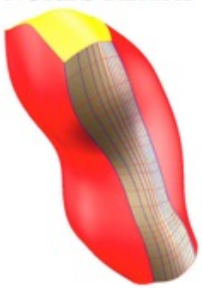
(a) Boundary NURBS surfaces (b) Initial boundary parameterization (c) Optimized boundary parameterization

(d) Control lattice (e) Final isoparametric structure (top view)

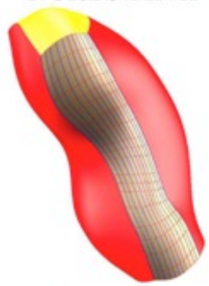


(a) Boundary NURBS surfaces

(b) Boundary NURBS curves



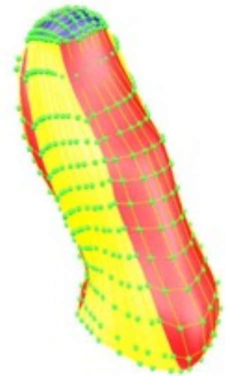
(c) Initial boundary parameterization



(d) Optimized boundary parameterization

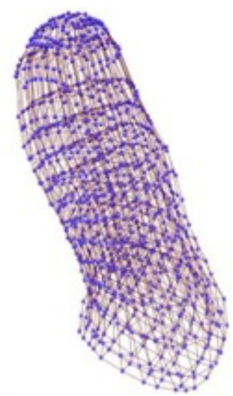
(e) Control lattice

(f) Final isoparametric structure

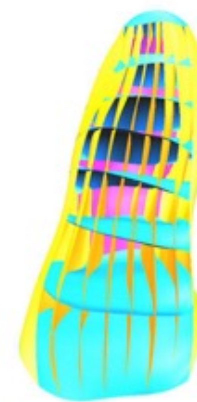


(a) Boundary NURBS surfaces and control mesh

(b) Boundary NURBS curves



(c) Resulting control lattice



(d) Final isoparametric structure

Thank you

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